Macroeconomic Dynamics, **19**, 2015, 508–534. Printed in the United States of America.
doi:10.1017/S1365100514000601

A "WORKING" SOLUTION TO THE QUESTION OF NOMINAL GDP TARGETING

MICHAEL T. BELONGIA
University of Mississippi

PETER N. IRELAND
Boston College

Although a number of economists have tried to revive the idea of nominal GDP targeting since the financial crisis of 2008, very little has been said about how this objective might be achieved in practice. This paper adopts and extends a strategy first outlined by Holbrook Working and later employed in the P-Star model. It presents a series of theoretical and empirical results to argue that Divisia monetary aggregates can be controlled by the Federal Reserve and that the trend velocities of these aggregates exhibit the stability required to make long-run targeting of a nominal objective feasible.

Keywords: Nominal GDP Targeting, Divisia Monetary Aggregates, P-Star Model

1. INTRODUCTION

Although stabilizing nominal GDP (NGDP) has been suggested previously as an objective for monetary policy actions, an increasing number of economists have tried to revive the idea since the financial crisis of 2008 and the apparent ineffectiveness of manipulating the federal funds rate when the zero bound constraint has been met. But although the merits of NGDP stabilization as a final objective for monetary policy have been emphasized in recent discussions, very little has been said about how this goal might be achieved in practice. Indeed, whereas earlier discussions offered explicit strategies and established linkages between, for example, nominal GNP and the monetary base [see, e.g., Meltzer (1987) and McCallum (1988)] or NGDP and a broader monetary aggregate [Feldstein and Stock (1994)], these recent discussions have been relatively strong on the goal and relatively silent on how a path to the goal might be implemented.[1] Indeed, with the recent innovations of payment of interest on reserves and unusual behavior of the monetary base in the aftermath of the financial crisis, even thoughts of reviving some of the older, well-articulated strategies have been put on hold.[2] Thus, for all

The authors wish to thank William Barnett, David Beckworth, David Laidler, and Scott Sumner for extremely helpful comments and Yan Li for able research assistance. Address correspondence to: Michael T. Belongia, Department of Economics, University of Mississippi, Box 1848, University, MS 38677, USA; e-mail: mtbelong@olemiss.edu.

of the attention that NGDP targeting has received as a potential goal for monetary policy, a practical means of achieving that end has yet to be offered.[3]

In this paper we propose a strategy for NGDP targeting based on a framework first outlined by Holbrook Working (1923) and used, with only minor modifications, by Hallman et al. (1991) in the P-Star model. In these earlier applications, a policy maker is able to evaluate whether a value for the money stock is consistent with long-run price stability. Using essentially the same derivation found in Working's original paper and making appropriate changes to the practical adaptations employed by Hallman et al., we find a path for money that is consistent with any desired long-run trajectory for nominal GDP. Unlike previous applications of this framework, we employ Divisia monetary aggregates in establishing a path for money that the central bank should try to maintain and use a one-sided filtering algorithm that can be implemented in real time to control for slow-moving trends in velocity.[4]

In what follows, we first explain the basic analytics of Working's framework and how we have adapted it to NGDP. Then, after reproducing Hallman et al.'s regression results to show that movements in the Divisia aggregates consistently anticipate movements in nominal income over a sample period that extends from 1967 through the present, we compare actual paths for the Divisia monetary aggregates with alternative trajectories that, according to our framework, would have been consistent with more stable NGDP growth since 1985. After using this comparison to discuss, in particular, the stance of current monetary policy, we examine how the Fed might control the behavior of these Divisia aggregates within an intermediate targeting strategy. Overall, we conclude that *if* NGDP is chosen as the central bank's objective (a question on which this paper takes no position), the strategy outlined in this paper has several virtues: It is transparent to outside observers, it is forward-looking, and yet it can be implemented in a fairly straightforward manner.[5]

2. WORKING'S FRAMEWORK

Working's (1923) objective was to find a value for the money supply that would be consistent with long-run price stability. At the time of his writing, many others had investigated empirically relationships implied by the Equation of Exchange.[6] From this research, strategies to stabilize the price level emerged, but even Fisher's (1920) plan did not incorporate a method for dealing with lags in the process. Thus, Working's innovation was to recognize the role of lags and to establish a policy framework that embedded a long-run desired path for price stability. A central bank then could compare the current price level with the desired long-run path and evaluate whether the stance of policy was too accommodative or too restrictive.

Working began by re-writing the basic expression for the Equation of Exchange as $(V/T) = (P/M)$. Because (P/M) did not have a "definite conception," Working dealt with its reciprocal. To find a long-run path for it, he estimated a trend value for the price level using a regression of the log value of the price level on time,

time squared, and time cubed; future values for the price level were extrapolated from this trend regression. With this information, Working then could plot, on a log scale, values for (*M*/*P*) to illustrate the value of circulating medium that would be consistent with his long-run trend path for the aggregate price level.

In adapting Working's framework for the P-Star model, Hallman et al. (1991) expressed their basic relationship as

$$P_t^* = (M2_t V_t^*)/Q_t^*. \tag{1}$$

In this expression, P_t^* is the long-run target value for the price level at time t, V_t^* is the long-run equilibrium value for velocity, taken by Hallman et al., to be the sample mean for M2 velocity, and Q_t^* is the value of potential real GDP at time t.[7] Rearranging terms to apply the framework more directly to nominal income targeting and making some desirable changes in empirical choices, the framework to be employed in this paper is

$$PQ_t^* = M_t V_t^*, \tag{2}$$

where PQ_t^* is the long-run target value for NGDP, M_t is the value of a Divisia monetary aggregate, and V_t^* is trend velocity for that chosen monetary aggregate. Equation (2) highlights one key advantage of any nominal-income-targeting scheme, relative to the price-level or inflation-targeting framework implied by the P-star model in (1): Nominal income targeting allows one to sidestep the challenge of estimating potential output accurately in real time.[8] Meanwhile, the use of a Divisia monetary aggregate in (2) in place of simple-sum M2 in (1) is motivated by Barnett's (1980) classic work, which introduced monetary economists to the logic behind, and the practical benefits of, Divisia monetary aggregation; this empirical choice also distinguishes our approach from that of Feldstein and Stock (1994), which, like the P-star model, uses simple sum M2 as an intermediate target within a NGDP targeting strategy.[9]

In (2), we also depart from the P-star framework in yet another way, by calculating trend velocity V_t^* using the one-sided version of the Hodrick–Prescott (1997) filter described by Stock and Watson (1999).[10] Figure 1 uses quarterly data to compare the actual velocities of Divisia M1, M2, MZM, and M4 with the trend values obtained with this one-sided HP filter. Data on all four of these Divisia monetary aggregates are provided by the Center for Financial Stability (CFS) and described in Barnett et al. (2013). The monetary assets included in Divisia M1 and M2 coincide with those included by the Federal Reserve in its official simple sum variants of those same aggregates: M1 accounts for currency, travelers' checks, demand deposits, and NOW accounts, whereas M2 adds savings deposits, small time deposits, and retail money market funds. The MZM aggregate—"money, zero maturity"—includes those assets in M2, less small time deposits, plus institutional money market funds. It was first discussed in detail by Motley (1988), who referred to it as "non-term M3," but later picked up the label of MZM. Finally, the M4 aggregate is the broadest compiled by the CFS, and adds

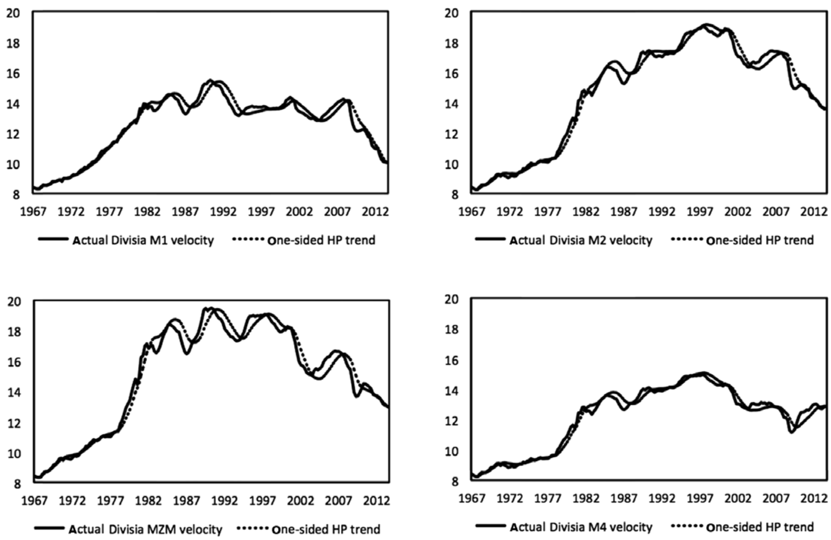

FIGURE 1. Actual and trend values of the velocities of Divisia monetary aggregates.

to the assets in M2 and MZM a variety of highly liquid money market instruments: large time deposits, overnight and term repurchase agreements, commercial paper, and Treasury bills.[11]

The graphs in Figure 1 reveal quite clearly the shifting, but slow-moving trends in velocities that, as Reynard (2007) finds, must be accounted for in identifying the long-run linkages between money and prices, not just in the United States but in Switzerland and the Euro area as well.[12] Our one-sided version of the H-P filter imposes the same setting, $\lambda = 1{,}600$, for the smoothing parameter as is commonly used in the two-sided H-P filter for quarterly data. It produces a similar, but somewhat more volatile, measure of the trend, reflecting the fact that, unlike the standard H-P filter, the one-sided variant only uses data up through period t in constructing the value for the trend at period t. This feature, however, is precisely what allows our algorithm to be implemented in real time and also makes our measure suitable for use in the forecasting equations described later. An added advantage of this one-sided filter is that once the parameter λ is fixed, no additional parameters need to be estimated or calibrated in constructing the series for trend velocity: As explained by Stock and Watson (1999, p. 301), values for the trend can be generated quickly and easily using the equations of the standard Kalman filter.

Otherwise, equation (2) parallels (1) for the P-star model by depicting the NGDP target PQ_t^* for time t as one that is implied by the level of the Divisia monetary aggregate M_t for that period, given the value of V_t^*, and by suggesting that the actual value for nominal income PQ_t should tend to gravitate, over time, toward the target PQ_t^*. To test this hypothesis, we estimate a set of regression equations that mirror Hallman et al.'s (1991, p. 847) in their specifications. Specifically, Hallman et al. find that in quarterly data running from 1955.1 through 1988.4, inflation tends to rise when the long-run price target P_t^* implied by (1) is above the actual price level P_t; likewise, inflation falls when P_t^* is below P_t. They confirm the statistical significance of this result by regressing the change in inflation on four of its own lags and the lagged value of the price gap, defined as the difference between p_t^* and p_t, the natural logarithms of P_t^* and P_t, and rejecting the null hypothesis that the coefficient on the lagged price gap equals zero. Here, similarly, we regress $\Delta^2 x_t$, the change in nominal income growth (and hence the analog to Hallman et al.'s $\Delta\pi_t = \Delta^2 p_t$, the change in the inflation rate) on four of its own quarterly lags and on the lagged value of the nominal income gap, defined as the difference between x_t^*, the natural log of the nominal income target PQ_t^* in (2), and x_t, the log of the actual value of NGDP during period t.

Although the availability of data on the Divisia monetary aggregates pushes the starting date for our own quarterly sample ahead to 1967.1, we can now extend that sample well beyond Hallman et al.'s, all the way through 2013.3. Our estimates, with absolute values of the t statistics below each coefficient, are

$$\Delta^2 x_t = \underset{(8.6)}{-0.619\Delta^2 x_{t-1}} \; \underset{(4.4)}{-0.355\Delta^2 x_{t-2}} \; \underset{(3.3)}{-0.271\Delta^2 x_{t-3}} \; \underset{(0.8)}{-0.057\Delta^2 x_{t-4}} \; \underset{(4.1)}{+0.379(x_{t-1}^* - x_{t-1})}$$

for Divisia M2 and

$$\Delta^2 x_t = \underset{(8.0)}{-0.590\Delta^2 x_{t-1}} \; \underset{(3.9)}{-0.324\Delta^2 x_{t-2}} \; \underset{(3.0)}{-0.250\Delta^2 x_{t-3}} \; \underset{(0.6)}{-0.044\Delta^2 x_{t-4}} \; \underset{(2.7)}{+0.276(x_{t-1}^* - x_{t-1})}$$

for Divisia M4.[13] In both cases, the large and statistically significant coefficient on the lagged NGDP gap indicates that nominal income growth accelerates when the gap is positive and decelerates when the gap is negative, so that actual NGDP converges over time to the long-run target defined in (2). Table 1 shows, additionally, that the lagged NGDP gap retains its significance when measured using the Divisia M1 and MZM aggregates and, for all four aggregates, when the equations are reestimated across subsamples running from 1967.1 through 1979.4 and from 1980.1 through 2013.3. Thus, remarkably, a nominal income target set with reference to *any* of these Divisia monetary aggregates proves useful in forecasting future NGDP growth, even in the most recent data. Most importantly from a practical perspective, no breakdowns of the forecasting equation are observed, and no special "shift-adjustments" beyond accounting from the slow-moving trends in V_t^* using the one-sided filter are needed to maintain the stability of these empirical relationships.

TABLE 1. Estimated forecasting equations for changes in NGDP growth

Full sample: 1967.1–2013.3; Dependent variable: $\Delta^2 x_t$				
	Divisia M1		Divisia M2	
	Coefficient	t stat (p value)	Coefficient	t stat (p value)
Constant	−0.136	−0.561 (0.58)	−0.090	−0.373 (0.71)
$\Delta^2 x_{t-1}$	−0.618	−8.574 (0.00)	−0.619	−8.628 (0.00)
$\Delta^2 x_{t-2}$	−0.362	−4.456 (0.00)	−0.355	−4.390 (0.00)
$\Delta^2 x_{t-3}$	−0.283	−3.478 (0.00)	−0.271	−3.343 (0.00)
$\Delta^2 x_{t-4}$	−0.066	−0.925 (0.36)	−0.057	−0.800 (0.42)
$x^*_{t-1} - x_{t-1}$	0.346	3.858 (0.00)	0.379	4.084 (0.00)
	$\bar{R}^2 = 0.32$	DW = 2.00	$\bar{R}^2 = 0.33$	DW = 1.98
	Divisia MZM		Divisia M4	
	Coefficient	t stat (p value)	Coefficient	t stat (p value)
Constant	−0.066	−0.271 (0.79)	−0.011	−0.045 (0.96)
$\Delta^2 x_{t-1}$	−0.620	−8.513 (0.00)	−0.590	−8.002 (0.00)
$\Delta^2 x_{t-2}$	−0.358	−4.369 (0.00)	−0.324	−3.863 (0.00)
$\Delta^2 x_{t-3}$	−0.276	−3.369 (0.00)	−0.250	−2.968 (0.00)
$\Delta^2 x_{t-4}$	−0.062	−0.859 (0.39)	−0.044	−0.590 (0.56)
$x^*_{t-1} - x_{t-1}$	0.239	3.387 (0.00)	0.276	2.655 (0.01)
	$\bar{R}^2 = 0.31$	DW = 1.98	$\bar{R}^2 = 0.29$	DW = 1.99
Pre-1980 subsample: 1967.1–1979.4; Dependent variable: $\Delta^2 x_t$				
	Divisia M1		Divisia M2	
	Coefficient	t stat (p value)	Coefficient	t stat (p value)
Constant	1.882	2.681 (0.01)	0.660	1.153 (0.26)
$\Delta^2 x_{t-1}$	−0.709	−5.069 (0.00)	−0.869	−6.208 (0.00)
$\Delta^2 x_{t-2}$	−0.447	−2.685 (0.01)	−0.656	−3.903 (0.00)
$\Delta^2 x_{t-3}$	−0.362	−2.181 (0.03)	−0.579	−3.446 (0.00)
$\Delta^2 x_{t-4}$	−0.079	−0.578 (0.57)	−0.212	−1.507 (0.14)
$x^*_{t-1} - x_{t-1}$	2.563	3.543 (0.00)	0.863	2.810 (0.01)
	$\bar{R}^2 = 0.54$	DW = 1.83	$\bar{R}^2 = 0.49$	DW = 1.98
	Divisia MZM		Divisia M4	
	Coefficient	t stat (p value)	Coefficient	t stat (p value)
Constant	0.688	1.201 (0.23)	0.588	1.026 (0.31)
$\Delta^2 x_{t-1}$	−0.871	−6.239 (0.00)	−0.853	−6.050 (0.00)
$\Delta^2 x_{t-2}$	−0.657	−3.925 (0.00)	−0.630	−3.734 (0.00)
$\Delta^2 x_{t-3}$	−0.582	−3.473 (0.01)	−0.553	−3.283 (0.00)
$\Delta^2 x_{t-4}$	−0.212	−1.513 (0.14)	−0.197	−1.394 (0.17)
$x^*_{t-1} - x_{t-1}$	0.885	2.874 (0.01)	0.906	2.668 (0.01)
	$\bar{R}^2 = 0.50$	DW = 1.98	$\bar{R}^2 = 0.48$	DW = 1.97

TABLE 1. Continued.

Post-1980 Subsample: 1980.1–2013.3; Dependent variable: $\Delta^2 x_t$

	Divisia M1		Divisia M2	
	Coefficient	t stat (p value)	Coefficient	t stat (p value)
Constant	−0.281	−1.089 (0.28)	−0.201	−0.790 (0.43)
$\Delta^2 x_{t-1}$	−0.498	−5.912 (0.00)	−0.491	−5.835 (0.00)
$\Delta^2 x_{t-2}$	−0.287	−3.148 (0.00)	−0.262	−2.874 (0.00)
$\Delta^2 x_{t-3}$	−0.189	−2.079 (0.04)	−0.156	−1.700 (0.09)
$\Delta^2 x_{t-4}$	−0.072	−0.861 (0.39)	−0.048	−0.572 (0.57)
$x^*_{t-1} - x_{t-1}$	0.304	3.627 (0.00)	0.326	3.554 (0.00)
	$\bar{R}^2 = 0.23$	DW = 2.01	$\bar{R}^2 = 0.23$	DW = 1.97
	Divisia MZM		Divisia M4	
	Coefficient	t stat (p value)	Coefficient	t stat (p value)
Constant	−0.152	−0.592 (0.56)	−0.083	−0.319 (0.75)
$\Delta^2 x_{t-1}$	−0.489	−5.710 (0.00)	−0.453	−5.242 (0.00)
$\Delta^2 x_{t-2}$	−0.267	−2.886 (0.00)	−0.230	−2.406 (0.02)
$\Delta^2 x_{t-3}$	−0.166	−1.791 (0.08)	−0.141	−1.458 (0.15)
$\Delta^2 x_{t-4}$	−0.058	−0.685 (0.49)	−0.039	−0.447 (0.66)
$x^*_{t-1} - x_{t-1}$	0.194	2.890 (0.00)	0.218	2.113 (0.04)
	$\bar{R}^2 = 0.20$	DW = 1.98	$\bar{R}^2 = 0.18$	DW = 1.99

Equation (2) follows the approach in Hallman et al. (1991) by defining the long-run target for NGDP in terms of the observed value of the monetary aggregate and the trend value of velocity. It is equally useful, however, to turn the equation around and use it to identify the path for a monetary aggregate that is consistent with a desired trajectory for NGDP. Toward this end, let

$$M_t^* = PQ_t^*/V_t^* \tag{3}$$

define the target M_t^* for money that is consistent with a chosen target for nominal income PQ_t^*, given the long-run value for velocity V_t^*. In the United States between 1985 and 2007, in fact, NGDP grew at an average annual rate of almost exactly 5.5 percent. The top panel of Figure 2 plots the actual series for the logarithm of NGDP against a trend line with this slope, fitted via a least-squares regression over the 23-year period. The bottom panel, meanwhile, shows deviations of NGDP from this trend, highlighting the modest swings experienced during the "Great Moderation" as well as the much more pronounced gap that opened during the most recent recession and continues to widen today. As noted by Woodford (2012), NGDP now lies more than 15 percent below a trend line estimated with data from the period before the financial crisis.

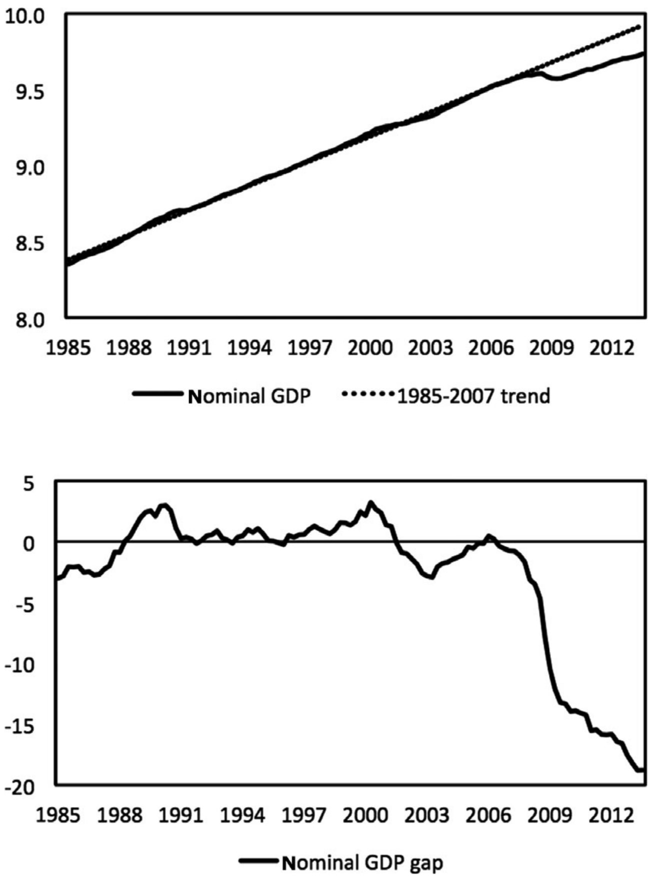

FIGURE 2. NGDP relative to trend. The top panel compares the natural logarithm of NGDP with a trend line fitted by ordinary least squares to data from 1985 through 2007. The bottom panel plots percentage-point deviations of actual NGDP from the 1985–2007 trend, extrapolated out through 2013.

Interpreting the trend line in Figure 2 as a target path for nominal GDP that extends through 2013.3, Figure 3 plots the gaps between the logs of the actual Divisia aggregates and the corresponding target values for money implied by equation (3). With the previous regression results in mind, one can view positive values for these money gaps as putting upward pressure on NGDP growth and negative values as putting downward pressure on NGDP; the gaps thereby indicate whether monetary policy was too accommodative, too restrictive, or appropriately neutral during any given period. In fact, negative values for all four money gaps are observed just before the recession of 1990–1991, and all series decline, while remaining slightly positive, before the recession of 2001. Positive gaps, meanwhile,

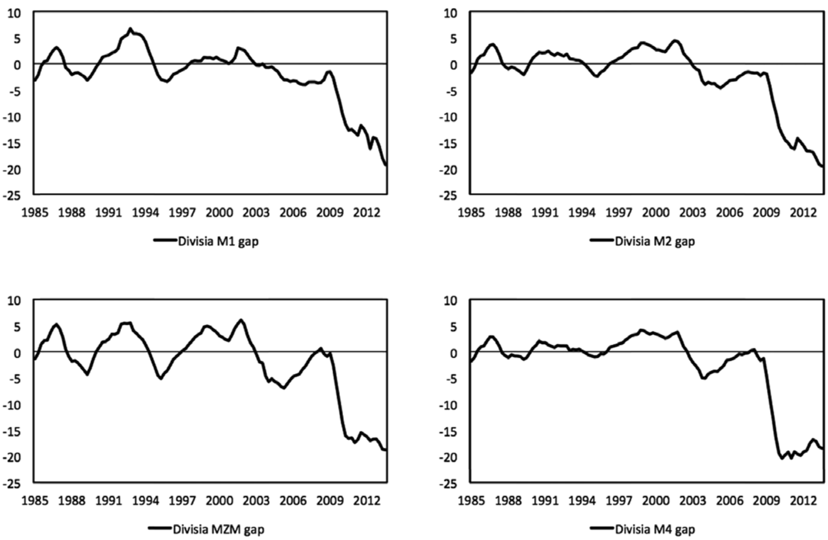

FIGURE 3. Gaps for Divisia monetary aggregates. The Divisia money gaps are measured as percentage-point differences between the actual value of each series and the desired value implied by equation (3), where the target value for NGDP is given by the trend line displayed in Figure 2.

generally appear during the economic recoveries of the middle 1980s and early 1990s. Most significantly, however, all four panels of Figure 3 suggest that the stance of monetary policy shifted gradually from ease to tightness toward the middle of the last decade and, in fact, began to exert a considerable drag on nominal income growth in 2005 and 2006, thereby supporting Hetzel's (2012) claim that Federal Reserve policy was itself a key factor in triggering the severe recession that shortly followed. The graphs also suggest that, despite the Federal Reserve's efforts to lower interest rates and increase dramatically the supply of bank reserves, insufficient growth in the monetary aggregates, particularly against the backdrop of heightened demand for safe and highly liquid assets reflected by the downward movements in trend velocity shown in Figure 1, continues to severely depress NGDP in the U.S. economy today.

Overall, the picture that emerges from Figure 3 is one of persistent volatility in the stance of monetary policy, switching from periods of ease to contraction and back again.[14] This volatility is not entirely unexpected, however, because, under a regime of interest-rate targeting, a central bank will have to change the quantity of reserves (and money) to maintain its interest-rate peg. Thus, in addition to offering a perspective on whether monetary policy has been relatively easy or restrictive at various points in time, Figure 3 also can be interpreted as offering evidence on one consequence of implementing monetary policy through an interest rate target: Judged in reference to a smooth path for NGDP, targeting the federal funds rate apparently has created an inherent instability in monetary policy. Whether because of internal divisions within the Federal Open Market Committee, outside pressure from the American political system, or the exclusion of a monetary variable from the standard macro model used for most policy analyses, the Federal Reserve seemingly has forgotten or ignored the key lessons spelled out by Friedman and Schwartz (1963) and Friedman (1968) regarding the dangers of focusing excessively on interest rates, to the exclusion of measures of the money stock, in gauging the stance of monetary policy.

In summary, the foregoing discussion has tried to establish that monetary policy has the potential to hit a long-run path for NGDP if it can control the behavior of a Divisia monetary aggregate that would keep NGDP on such a target path. It is to this question of monetary control we now turn.[15]

3. MONEY MULTIPLIERS FOR SIMPLE SUM AND DIVISIA AGGREGATES

Spindt (1983) applied Barnett's (1980) work to the derivation of general expressions for the multipliers of Divisia monetary aggregates. Here, these expressions are reproduced for the special case of aggregates formed from currency and a single type of interest-bearing deposit. The results make clear how the appearance of user-cost terms in the budget-share weights of the Divisia index can—and seemingly does—help dampen volatility in the behavior of its companion multiplier. A series of numerical examples, based on these expressions together with a model of the demand for currency and deposits drawn from Belongia and Ireland (in press), reveals that for a wide range of plausible parameterizations, the multiplier for the Divisia monetary aggregate is likely to be more stable than the multiplier for the corresponding simple sum measure. We find that this same pattern appears in the U.S. data.

Let D_t, C_t, and R_t denote the dollar values of deposits, currency, and bank reserves. The simple sum monetary aggregate M_t^s and the monetary base H_t are then defined by

$$M_t^s = D_t + C_t \tag{4}$$

and

$$H_t = R_t + C_t. \tag{5}$$

Following the usual route to obtaining an expression for the money multiplier of the simple sum aggregate, let

$$k_t = C_t/D_t \tag{6}$$

denote the currency–deposit ratio and

$$r_t = R_t/D_t \tag{7}$$

denote the reserve ratio. Using (4)–(7), the simple sum multiplier can be calculated as

$$m_t^s = \frac{M_t^s}{H_t} = \frac{D_t + C_t}{R_t + C_t} = \frac{1 + k_t}{r_t + k_t}. \tag{8}$$

Equation (8) depicts the textbook result that the money multiplier depends inversely on both the currency–deposit ratio and the reserve ratio.

Because Divisia indices are growth rate indices, however, it is useful for the sake of comparison to express the multiplier for the simple sum aggregate in its less familiar growth rate form as well. Spindt (1983) accomplishes this task using the approximations

$$\Delta \ln(M_t^s) = \left(\frac{w_t^D + w_{t-1}^D}{2}\right) \Delta \ln(D_t) + \left(\frac{w_t^C + w_{t-1}^C}{2}\right) \Delta \ln(C_t) \tag{9}$$

and

$$\Delta \ln(H_t) = \left(\frac{v_t^R + v_{t-1}^R}{2}\right) \Delta \ln(R_t) + \left(\frac{v_t^C + v_{t-1}^C}{2}\right) \Delta \ln(C_t) \tag{10}$$

for the growth rates of the simple sum aggregate and the money base, where

$$w_t^D = \frac{D_t}{M_t^s} = \frac{D_t}{D_t + C_t} = \frac{1}{1 + k_t} \tag{11}$$

and

$$w_t^C = \frac{C_t}{M_t^s} = \frac{C_t}{D_t + C_t} = \frac{k_t}{1 + k_t} \tag{12}$$

represent the quantity shares of deposits and currency in the simple sum aggregate and, analogously,

$$v_t^R = \frac{R_t}{H_t} = \frac{R_t}{R_t + C_t} = \frac{r_t}{r_t + k_t} \tag{13}$$

and

$$v_t^C = \frac{C_t}{H_t} = \frac{C_t}{R_t + C_t} = \frac{k_t}{r_t + k_t} \tag{14}$$

represent the quantity shares of reserves and currency in the monetary base. Equations (9)–(14) combine to yield

$$\begin{aligned} \Delta \ln(m_t^s) = {} & \frac{1}{2}\left(\frac{k_t}{1 + k_t} + \frac{k_{t-1}}{1 + k_{t-1}}\right) \Delta \ln(k_t) - \frac{1}{2}\left(\frac{r_t}{r_t + k_t} + \frac{r_{t-1}}{r_{t-1} + k_{t-1}}\right) \\ & \times \Delta \ln(r_t) - \frac{1}{2}\left(\frac{k_t}{r_t + k_t} + \frac{k_{t-1}}{r_{t-1} + k_{t-1}}\right) \Delta \ln(k_t), \end{aligned} \tag{15}$$

which restates (8) in growth rate form.

Meanwhile, the growth rate of the Divisia quantity aggregate M_t^d of deposits and currency is defined in discrete time by

$$\Delta \ln(M_t^d) = \left(\frac{s_t^D + s_{t-1}^D}{2}\right) \Delta \ln(D_t) + \left(\frac{s_t^C + s_{t-1}^C}{2}\right) \Delta \ln(C_t), \tag{16}$$

where (16) replaces the quantity shares that appear in (9) with expenditure shares on the monetary services provided by deposits and currency. These shares are computed using Barnett's (1978) formulas for the user costs u_t^D and u_t^C of deposits and currency,

$$u_t^D = \frac{\rho_t^B - \rho_t^D}{1 + \rho_t^B} \tag{17}$$

and

$$u_t^C = \frac{\rho_t^B}{1 + \rho_t^B}, \tag{18}$$

where ρ_t^B denotes the rate of return on a benchmark asset that provides no monetary services, ρ_t^D denotes the own rate of return of deposits, and (18) reflects the fact that currency does not pay interest. Let

$$u_t = \frac{u_t^C}{u_t^D} = \frac{\rho_t^B}{\rho_t^B - \rho_t^D} \tag{19}$$

denote the ratio of the user cost of currency to the user cost of deposits. Using this expression together with the formula (6) defining the currency–deposit ratio, the expenditure shares appearing in (16) may be computed as

$$s_t^D = \frac{u_t^D D_t}{u_t^D D_t + u_t^C C_t} = \frac{1}{1 + u_t k_t} \tag{20}$$

and

$$s_t^C = \frac{u_t^C C_t}{u_t^D D_t + u_t^C C_t} = \frac{u_t k_t}{1 + u_t k_t}. \tag{21}$$

Equations (10), (13), (14), (16), (20), and (21) combine to yield an expression for the growth rate of the money multiplier m_t^d for the Divisia aggregate:

$$\Delta \ln(m_t^d) = \frac{1}{2}\left(\frac{u_t k_t}{1 + u_t k_t} + \frac{u_{t-1} k_{t-1}}{1 + u_{t-1} k_{t-1}}\right) \Delta \ln(k_t)$$
$$- \frac{1}{2}\left(\frac{r_t}{r_t + k_t} + \frac{r_{t-1}}{r_{t-1} + k_{t-1}}\right) \Delta \ln(r_t) - \frac{1}{2}\left(\frac{k_t}{r_t + k_t} + \frac{k_{t-1}}{r_{t-1} + k_{t-1}}\right) \Delta \ln(k_t). \tag{22}$$

Spindt (1983) shows how (22) extends to the more general case, with multiple types of deposits and reserve assets.

Comparing (15) and (22) reveals that the relative user cost term u_t defined in (19) enters into the money multiplier formula for the Divisia aggregate but not for the corresponding simple sum measure. Intuitively, the two multipliers coincide when $u_t = 1$; in this case, deposits pay no interest, implying that an optimizing agent will be indifferent between the monetary services provided by an additional dollar in deposits and the monetary services provided by an additional dollar in currency and will, in that sense, view deposits and currency as perfect substitutes at the margin. Equations (15) and (22) indicate that movements in the reserve ratio r_t affect the multipliers for the Divisia and simple sum aggregates symmetrically. Because $u_t > 1$ whenever deposits do pay interest, however, the first term in parentheses on the right-hand side of (22) will typically be a larger positive number than the corresponding term in (15), suggesting that, in particular, a decrease in the currency–deposit ratio k_t that increases the money multiplier for the simple sum aggregate will tend to produce a smaller increase in the money multiplier for the Divisia aggregate.

Two observations, however, force us to stop short of using this comparison between (15) and (22) alone to claim that the money multiplier for the Divisia aggregate will surely be more stable than the money multiplier for the simple sum measure. First, although the growth rate formula (15), like the more familiar level formula (8), implies that a fall in the currency–deposit ratio will always cause the money multiplier for the simple sum aggregate to rise, sufficiently high values of the relative user cost variable u_t may cause the money multiplier to *fall* in response to the same change in k_t. Under such circumstances, m_t^d could exhibit movements that are larger, in absolute value, than the corresponding changes in m_t^s. Second, if most changes in the currency–deposit ratio reflect underlying changes in user costs brought about by exogenous shocks or monetary policy actions that change either the benchmark interest rate ρ_t^B or the spread between the benchmark rate and the own rate on deposits $\rho_t^B - \rho_t^D$, then u_t will vary together with k_t, producing movements in the money multiplier for the Divisia aggregate that are difficult to pin down from an inspection of (22) alone.

To resolve these ambiguities, we combine (15) and (22), which are, by themselves, simply accounting formulas that identify the more fundamental determinants of the money multipliers, with elements drawn from the more detailed general equilibrium model of the demand for monetary assets presented in Belongia and Ireland (in press). In this model, a representative household economizes on shopping time using an aggregate M_t^a of monetary services obtained from currency C_t and deposits D_t, where the monetary aggregator takes the constant-elasticity-of-substitution form

$$M_t^a = [\nu^{1/\omega} C_t^{(\omega-1)/\omega} + (1-\nu)^{1/\omega} D_t^{(1-\omega)/\omega}]^{\omega/(\omega-1)}, \qquad \textbf{(23)}$$

and the parameters satisfy $0 < \nu < 1$ and $\omega > 0$. With this specification, the household optimally chooses the currency–deposit ratio k_t as a function of the same opportunity cost variable u_t defined earlier, in (19). In particular,

$$k_t = \left(\frac{\nu}{1-\nu}\right)\left(\frac{1}{u_t}\right)^{\omega}, \tag{24}$$

a relation that associates an increase in the opportunity cost of currency relative to deposits with a decline in the currency–deposit ratio. Equation (24) can be combined with either (15) or (22) to obtain a model of how the money multiplier for either the simple sum or the Divisia aggregate changes in response to movements in the currency–deposit ratio that are ultimately driven by changes in the user cost variable u_t.

Monthly data covering the period from 1980.01 through 2007.12 guide us in calibrating this model. The sample's starting date marks the beginning of the era in which consumers had access to a wide range of interest-earning deposits, whereas the terminal date ensures that the figures are not influenced unduly by the extreme fluctuations in monetary variables witnessed (and shown, for instance, in our own Figure 3) during and since the financial crisis. Over this period, the average ratio of Federal Reserve Bank of St. Louis adjusted reserves to deposits was 0.09 for M1, 0.02 for M2, 0.025 for MZM, and 0.01 for M4; hence, in evaluating (15) and (22), the reserve ratio is fixed at $r = 0.09$, $r = 0.02$, $r = 0.025$, or $r = 0.01$ in separate sets of examples corresponding to each these four monetary aggregates. The average ratio of currency to deposits, meanwhile, was 0.45 for M1, 0.10 for M2, 0.125 for MZM, and 0.055 for M4; hence, in (24), the parameter ν is chosen to match the value of $k = 0.45$, $k = 0.10$, $k = 0.125$, or $k = 0.055$ across the different sets of examples.[16]

As noted in Belongia and Ireland (in press), the price aggregator,

$$\rho_t^B - \rho_t^a = [\nu(\rho_t^B)^{1-\omega} + (1-\nu)(\rho_t^B - \rho_t^D)^{1-\omega}]^{1/(1-\omega)}, \tag{25}$$

is dual to the quantity aggregator (23), where ρ_t^a denotes the own rate of return on the true monetary aggregate M_t^a andρ_t^B and ρ_t^D are, as in (17)–(19), the benchmark return and the own rate on deposits. Data provided through the Center for Financial Stability and also described by Barnett et al. (2013) include readings on benchmark rates of return ρ_t^B as well as on interest rate aggregates for Divisia M1, M2, MZM, and M4 that can serve as measures of ρ_t^a. Average values over the period 1980.01—2007.12 in these data are $\rho^B = 0.078$ for the benchmark rate, $\rho^B - \rho^a = 0.070$ for the M1 aggregate, $\rho^B - \rho^a = 0.040$ for M2, $\rho^B - \rho^a = 0.048$ for MZM, and $\rho^B - \rho^a = 0.030$ for M4. We use these figures, together with (25), to back out implied values for ρ^D, the average own rate on deposits in each monetary aggregate, and then substitute the average benchmark and deposit rates into (19) to obtain an initial setting for u_t when evaluating (15), (22), and (24) numerically.

With the model thereby calibrated to cases corresponding to each of the four Divisia measures of money, Table 2 shows values of the derivatives $\partial \ln(m_t^s)/\partial u_t$ and $\partial \ln(m_t^d)/\partial u_t$ computed numerically using (15), (22), and (24) for various values of the parameter ω measuring the elasticity of substitution between currency

TABLE 2. Responsiveness of simple sum and Divisia money multipliers to changes in user costs and the currency–deposit ratio

	M1 calibration $\partial \ln(m)/\partial u$		M2 calibration $\partial \ln(m)/\partial u$	
ω	Simple sum	Divisia	Simple sum	Divisia
0.1	0.0445	0.0415	0.0343	0.0303
0.5	0.2221	0.2068	0.1686	0.1483
0.9	0.3990	0.3712	0.2966	0.2594
1.1	0.4872	0.4531	0.3572	0.3114
1.5	0.6630	0.6159	0.4685	0.4047
2.0	0.8814	0.8177	0.5745	0.4859
	MZM calibration $\partial \ln(m)/\partial u$		M4 calibration $\partial \ln(m)/\partial u$	
ω	Simple sum	Divisia	Simple sum	Divisia
0.1	0.0409	0.0369	0.0277	0.0248
0.5	0.2023	0.1823	0.1356	0.1208
0.9	0.3596	0.3231	0.2365	0.2096
1.1	0.4363	0.3914	0.2829	0.2498
1.5	0.5845	0.5222	0.3622	0.3162
2.0	0.7562	0.6708	0.3943	0.3302

and deposits. Thus, each entry in the table quantifies the response of the money multiplier for either the simple sum or the Divisia aggregate to a shock or monetary policy action that increases the relative user cost of currency and thereby leads, through (24), to a decrease in the currency–deposit ratio. In every case, the results confirm the intuition suggested earlier by a direct comparison of (15) and (22). The positive values reported for $\partial \ln(m_t^s)/\partial u_t$ indicate that a shock that causes the currency–deposit ratio to fall causes the simple sum multiplier to rise; but the values reported for $\partial \ln(m_t^d)/\partial u_t$—still positive, yet distinctly smaller in magnitude—show that the Divisia multiplier rises as well, but by a smaller amount. Thus, although it is possible to concoct examples in which the opposite is true, this realistically calibrated model consistently suggests that the money multiplier for a Divisia aggregate is likely to be more stable than the money multiplier for the corresponding simple sum measure.

Table 3 shows that the relationship predicted by the model also holds true in the U.S. data. For the same period used in the calibration exercise, and for three additional sample periods considered in the following forecasting exercises, the money multiplier for each Divisia monetary aggregate has a standard deviation that is smaller than that of the multiplier for the corresponding simple sum measure, with only one exception—for the M2 aggregate in the sample period

TABLE 3. Standard deviations of money multipliers for simple sum and Divisia aggregates

	M1		M2	
Sample period	Simple sum	Divisia	Simple sum	Divisia
1980.01–2007.12	0.0052	0.0051	0.0058	0.0055
1967.01–1979.09	0.0039	0.0038	0.0042	0.0043
1984.01–1999.12	0.0049	0.0046	0.0051	0.0046
1967.01–2007.12	0.0048	0.0047	0.0055	0.0053
	MZM		M4	
Sample period	Simple sum	Divisia	Simple sum	Divisia
1980.01–2007.12	0.0094	0.0064	0.0066	0.0056
1967.01–1979.09	0.0048	0.0044	0.0044	0.0041
1984.01–1999.12	0.0062	0.0052	0.0051	0.0043
1967.01–2007.12	0.0083	0.0059	0.0060	0.0052

1984.01–1999.12—appearing across a total of 16 comparisons. Whether these smaller month-to-month movements in the Divisia money multiplier are also forecastable is the subject of the next section.

4. FORECASTING EXPERIMENTS

4.1. Overview

The multiplier relationships explored in the preceding section suggest several hypotheses and related experiments that would update the results reported by Spindt (1984). Because one of the potential errors that could move NGDP off a targeted path would be control errors that result from an inability to forecast movements in the Divisia money multiplier out of sample, our specific goal here is to evaluate, within the context of a NGDP targeting framework, which instrument and monetary aggregate would be most likely to keep NGDP on a target path. With four Divisia aggregates—M1, M2, MZM, and M4—as the basis for calculating a future path for money, the central bank must decide which instrument is most closely linked to the behavior of these measures.[17] In the forecasting exercise that follows, we will consider multipliers derived from four potential instruments of control: Adjusted reserves, total reserves, nonborrowed reserves, and the adjusted monetary base.

Across the interval 1967.01–2007.12, we first estimate univariate ARMA models for each multiplier series over three subsamples. The results of those estimations then are used to calculate errors from static, out-of-sample forecasts over horizons of three years following the terminal data point of the estimation interval. The subsamples were chosen to evaluate the effects of notable institutional changes, thereby confronting the models with their greatest challenge. The first forecast period covers the period of the Fed's experiment with monetary targeting (1979–1982). The second estimation period ends at the time of the Y2K injection of reserves so that the forecast period covers a sample period when the Fed was draining reserves from the system and then dealing with a recession that may have been caused by its excessively restrictive actions post-Y2K.[18] The third estimation period spans the Great Moderation and ends just prior to the onset of the most recent downturn; most notably, however, this is a period in which any emphasis on money and monetary control had disappeared from discussions of monetary policy.

Before proceeding with the forecasting experiment, it is instructive to present the data in broad overview. Also, because of the wholesale changes in financial markets that occurred in the early 1980s, these summary statistics are reported for three sample periods: 1967.01–1979.09, 1984.01–1999.12, and the entire 1967.01–2007.12 period under study. Although the data in Table 4 reveal very broad similarities across alternative money multipliers and over time, the multiplier derived from nonborrowed reserves exhibits a standard deviation that is substantially larger than that of the base or adjusted reserves; somewhat surprisingly, this result prevails even in the sample period prior to the advent of financial innovations. On its face this does not mean that nonborrowed reserves cannot be used as the central bank's instrument of control or that movements in this multiplier cannot be forecasted out-of-sample, but its consistently larger standard deviation is something to note as the forecasting exercises are undertaken.[19]

The results of the static forecasts are reported in Table 5. Again, we conduct the forecasting experiment over three different periods of time to minimize the chances that any particular result is due to happenstance.[20] Variables chosen to represent the central bank's policy instrument (H) in each table include adjusted reserves (ADJ RES), nonborrowed reserves (NBR), total reserves (TOT RES), and the adjusted monetary base (BASE) as reported by the Federal Reserve Bank of St. Louis. The cell entries include two error statistics: Root-mean-squared error (RMSE) and mean absolute error (MAE). We first discuss results for each monetary aggregate in turn, then attempt to draw more general conclusions by reviewing the results as a group, and conclude with a final set of experiments that speak directly to the possibility of using a Divisia monetary aggregate as an intermediate target against the backdrop of the financial crisis of 2008 and the institutional disruptions and changes that followed.

4.2. The Divisia M1 Aggregate

The results for the Divisia measure of M1 and the four variables used to represent the Fed's policy instrument indicate that, in all cases and across all sample periods, the monetary base multiplier is associated with the smallest MAE and RMSE. Moreover, in many cases, the error statistics for the base multiplier are an order of magnitude smaller than those of the next closest competitor. Thus, if the Fed were to implement this particular approach to NGDP targeting with Divisia M1 as

TABLE 4. Descriptive statistics for Divisia money multipliers

	Monetary instrument			
	Base	NBR	Total reserves	Adj. reserves
		A. 1967.01–1979.09		
		Divisia M1		
Mean	−0.000826	0.002128	0.001895	0.000422
Std. dev.	0.003829	0.017671	0.008794	0.008848
		Divisia M2		
Mean	−0.000611	0.002343	0.002109	0.000636
Std. dev.	0.004323	0.017550	0.008821	0.009017
		Divisia MZM		
Mean	−0.001378	0.001576	0.001342	−0.000131
Std. dev.	0.004254	0.017510	0.008746	0.008956
		Divisia M4		
Mean	0.000009	0.002963	0.002729	0.001256
Std. dev.	0.004132	0.017580	0.008714	0.008866
		B. 1984.01–1999.12		
		Divisia M1		
Mean	−0.001473	0.002262	0.002371	−0.001017
Std. dev.	0.004621	0.020722	0.008952	0.018064
		Divisia M2		
Mean	−0.002424	0.001319	0.001440	−0.002087
Std. dev.	0.004625	0.021049	0.010089	0.018700
		Divisia MZM		
Mean	−0.001586	0.002148	0.002257	−0.001131
Std. dev.	0.005310	0.020750	0.009776	0.043356
		Divisia M4		
Mean	−0.001889	0.001854	0.001976	−0.001551
Std. dev.	0.004338	0.021412	0.010439	0.018589
		C. 1967.01–2007.12		
		Divisia M1		
Mean	−0.000941	0.003006	0.002174	0.000652
Std. dev.	0.004732	0.031682	0.022221	0.020273
		Divisia M2		
Mean	−0.001302	0.002646	0.001813	0.000291
Std. dev.	0.005260	0.032112	0.022987	0.021071
		Divisia MZM		
Mean	−0.001049	0.002898	0.002066	0.000544
Std. dev.	0.005961	0.032246	0.023205	0.021200
		Divisia M4		
Mean	−0.000633	0.003315	0.002482	0.000960
Std. dev.	0.005237	0.032358	0.023416	0.021217

TABLE 5. Results for forecasting experiments with money multipliers

Instrument	Model	MAE	RMSE	Model	MAE	RMSE
A. Estimation period: 1967.01–1979.09; out-of-sample period: 1979.10–1982.09						
	Divisia M1			Divisia MZM		
Base	ARMA(1,2)	0.004419	0.005239	ARMA(1,1)	0.006263	0.007945
NBR	ARMA(2,2)	0.024783	0.030728	ARMA(4,4)	0.020427	0.025344
Tot res	ARMA(2,3)	0.006940	0.010241	ARMA(3,2)	0.009999	0.013780
Adj res	ARMA(1,2)	0.008876	0.011062	ARMA(1,2)	0.010545	0.014031
	Divisia M2			Divisia M4		
Base	ARMA(2,3)	0.015637	0.017310	ARMA(6,5)	0.032790	0.045822
NBR	ARMA(5,4)	0.044617	0.055799	ARMA(3,3)	0.020827	0.025869
Tot res	ARMA(3,2)	0.009312	0.012963	ARMA(3,2)	0.008400	0.012096
Adj res	ARMA(5,6)	0.009508	0.012102	ARMA(1,2)	0.009490	0.012180
B. Estimation period: 1985.01–1999.12; out-of-sample period: 2000.01–2002.12						
	Divisia M1			Divisia MZM		
Base	ARMA(1,1)	0.004842	0.007828	ARMA(7,7)	0.009557	0.012701
NBR	ARMA(3,3)	0.026168	0.062926	ARMA(3,3)	0.028746	0.067466
Tot res	ARMA(1,3)	0.029462	0.075954	ARMA(1,3)	0.030866	0.080885
Adj res	ARMA(5,2)	0.035735	0.062042	ARMA(4,2)	0.033044	0.061030
	Divisia M2			Divisia M4		
Base	ARMA(3,3)	0.006023	0.009882	ARMA(3,3)	0.007008	0.010612
NBR	ARMA(3,3)	0.028232	0.066725	ARMA(2,2)	0.079774	0.118379
Tot res	ARMA(2,2)	0.039584	0.083957	ARMA(2,2)	0.034567	0.084109
Adj res	ARMA(5,2)	0.037751	0.065781	ARMA(3,4)	0.035146	0.064114
C. Estimation period: 1989.01–2004.12; out-of-sample period: 2005.01–2007.12						
	Divisia M1			Divisia MZM		
Base	ARMA(3,3)	0.003943	0.005058	MA(2)	0.003834	0.005130
NBR	ARMA(6,4)	0.027529	0.074589	ARMA(3,5)	0.029071	0.074203
Tot res	AR(2)	0.015820	0.020533	AR(2)	0.015743	0.019947
Adj res	ARMA(2,1)	0.013571	0.017504	ARMA(2,2)	0.017504	0.017167
	Divisia M2			Divisia M4		
Base	ARMA(3,1)	0.004787	0.006522	ARMA(6,4)	0.010442	0.013152
NBR	ARMA(2,4)	0.028372	0.073952	ARMA(1,3)	0.029727	0.074820
Tot res	ARMA(6,4)	0.040134	0.050778	ARMA(6,4)	0.041092	0.055687
Adj res	ARMA(2,2)	0.013285	0.016887	ARMA(3,3)	0.013537	0.017679

TABLE 5. Continued.

Instrument	Model	MAE	RMSE	Model	MAE	RMSE
D. Estimation period: 1993.01–2008.12; out-of-sample period: 2009.01–2011.12						
	Divisia M1			Divisia MZM		
Adj base	ARMA(4,2)	0.014501	0.017471	ARMA(1,1)	0.011935	0.014888
	Divisia M2			Divisia M4		
Adj base	ARMA(2,4)	0.024957	0.033344	ARMA(1,2)	0.004979	0.006724

its guide, the monetary base would appear to be the policy instrument that would generate the smallest control error. With respect to the general results over sample periods, it is interesting to note that, for the most part, the forecast errors are not markedly different across time. This result is surprising because the introduction of new bank liabilities not subject to reserve requirements, the increasing use of "sweep" activities by banks, and the reduction in reserve requirements more generally should have made monetary control subject to *larger* errors.

4.3. Intermediate Levels of Aggregation: Divisia M2 and MZM

The results for the Divisia measures of M2 and MZM, although conforming broadly to those for Divisia M1, produce a few exceptions to the rule that the base multiplier will generate the smallest control errors. In the earliest (1967–1979) sample period, which uses the initial era of financial innovations for its out-of-sample forecast period, total reserves and adjusted reserves are associated with error statistics that are somewhat smaller for the Divisia M2 multiplier; the smallest errors for the Divisia MZM aggregate, however, are those associated with the base, as in the case of Divisia M1. And, as in the previous case, errors associated with nonborrowed reserves are the largest and there is little variation in the forecast errors over time. If the Federal Reserve were to implement this particular approach to NGDP targeting with either Divisia M2 or MZM as its guide, the monetary base would appear to be the policy instrument that would generate the smallest control error.

4.4. The Divisia M4 Aggregate

Results for the M4 multipliers indicate that, as in the case of Divisia M2, the multiplier derived from the monetary base produces the lowest forecast errors, except in the first sample period, when total reserves and adjusted reserves produce smaller forecast error statistics. Also, as in all other cases, the nonborrowed-reserves instrument produces the highest MAE and RMSE values. Finally, it is interesting to note that the control errors for the much broader M4 liabilities grouping are similar to those for more narrow groupings. Thus, although one reason to choose between a narrow and broad intermediate target often is how closely it is associated with the central bank's instrument of control, there is nothing in Table 5 that would lead one to prefer one Divisia measure strongly to the other; it seems as if the central bank could use the monetary base to influence the path of any with comparable success.

4.5. The Recent Financial Crisis and Monetary Control

The foregoing experiments all were conducted over sample periods prior to the recent financial crisis and responses to it by the Federal Reserve that have made, in the minds of many observers, reserves and the monetary base uninformative indicators of central bank actions. Moreover, the introduction of payment of interest on reserves would have weakened, if not severed, any link between traditional measures of central bank liabilities and money to the degree that discussions of monetary control would be all but a moot point, post-2008. Taken at face value, these points might seem correct.

For practical purposes, however, the question facing a central bank always becomes one of what it wishes to accomplish. For example, there is little doubt that sweep accounts represent an effort by banks to evade reserve requirements and this evasion complicates measurement of the money supply. As a bank regulator, however, the Federal Reserve has a number of options to control or eliminate this behavior if, in fact, greater control and more accurate measurement of the money supply were a policy objective.[21]

With regard to the post-2008 environment, similar logic applies. Although it is true that the Federal Reserve has added a large volume of assets to its portfolio and begun to pay interest on reserves, these actions have not necessarily distorted all linkages between the Fed's balance sheet and the aggregate quantity of money. Tatom (2011), for example, has derived both balance sheet and multiplier relationships in the aftermath of the financial crisis and found that a relatively straightforward adjustment—subtracting excess reserves from the monetary base—would provide an accurate representation of monetary policy actions that would affect the money supply. Plots of this adjusted series, both in log-levels and in growth rates, are shown in Figure 4. The data indicate that, popular discussions to the contrary, this series is relatively stable even across the turbulent period of 2008–2012.

Because the other measures that had been used to represent the monetary policy instrument have been distorted by recent events, we turn finally to this adjusted measure to examine whether the Federal Reserve still could have controlled the behavior of money through the period of the financial crisis. The bottom portion (Section D) of Table 5 reports these results using an estimation period of 1993.01–2008.12 and an out-of-sample forecasting interval that spans 2009.01–2011.12. The results for the Divisia multipliers indicate that the mean absolute error and RMSE statistics are about three times as large as the those for the monetary base over the period immediately preceding the financial crisis. The same error statistics, however, remain comparable to those for the other three monetary instruments

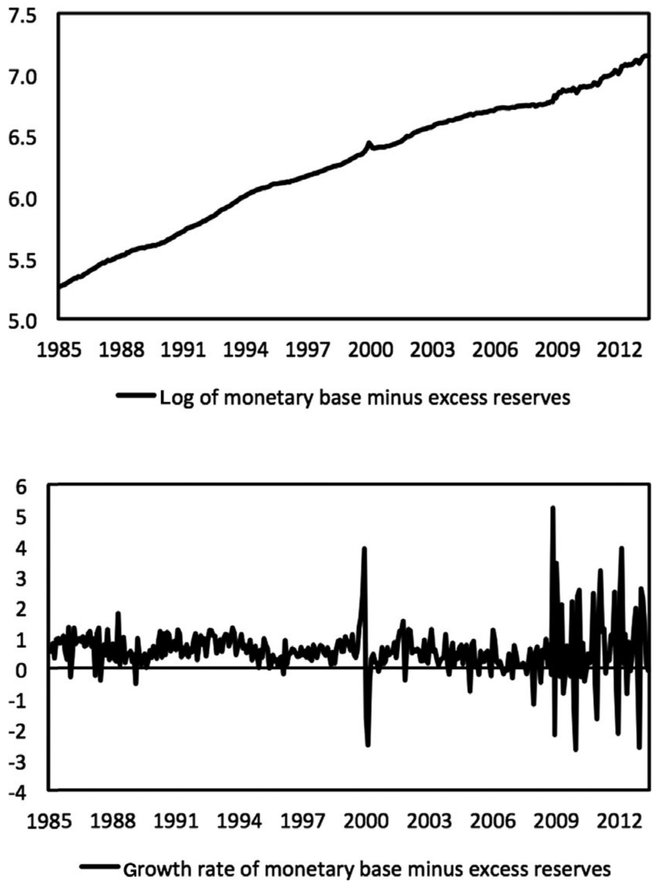

FIGURE 4. The recent behavior of the monetary base adjusted for excess reserves.

(nonborrowed reserves, total reserves, and adjusted reserves) computed for earlier sample periods. In this context, a disinterested observer could conclude that the financial crisis and the introduction of innovations such as the payment of interest on reserves has weakened the link between the monetary base and both Divisia aggregates, making forecasting errors in its multiplier more like those of other potential instruments of control. On the other hand, because the Fed has been directing its efforts to targeting the federal funds rate, which, by construction, has allowed money to vary freely, the same observer could conclude that much could be done to tighten the link between the adjusted monetary base and the monetary aggregates *if* the Fed wished to control their behavior rather than the funds rate.

An exception to these results is Divisia M4, where, for the period of the financial crisis, the out-of-sample error statistics for its money multiplier not only are lower than those for earlier periods but are among the lowest reported for any multiplier in any sample period. Because a superlative index does not change unless an income effect is present and financial crises may induce agents to reallocate funds to broad maturities not included in the more narrow aggregates, it is possible that this small forecast error for Divisia M4 is associated with the financial crisis. Some additional evidence in support of this conjecture can be found in Figure 1, where, in the plots of trend velocities, that for Divisia M4 appears to be the most stable, and in Figure 3, where the money gap for Divisia M4, unlike the gaps for Divisia M1 and MZM, does not give a false signal of more rapidly rising NGDP growth in the mid-1990s. Overall, although all four aggregates demonstrate broadly similar results over all three sample periods, the experiment with data for the recent financial crisis offers some evidence to distinguish Divisia M4 from the other three monetary aggregates.

5. CONCLUSION

Because monetary policy, when implemented by manipulation of the federal funds rate, has been viewed as impotent when the funds rate reaches its zero bound, some observers have suggested that the Fed attempt to meet its dual mandate by setting a target for NGDP. Although taking no position on the merits of NGDP targeting relative to alternatives that stabilize the price level or the money supply instead, this paper modifies a framework suggested by Working (1923) and similar to that of the P-Star model and illustrates how it might be used to target nominal income.[22] It shows that the central bank can use the monetary base to control the path for either a broad or narrow Divisia monetary aggregate and, through this device, it can keep NGDP on a long-run path that is consistent with potential income. The framework is built on traditional quantity-theoretic foundations and draws directly from Barnett's (1980) economic approach to monetary aggregation. Its procedures are transparent, convenient to implement and monitor in real time, and therefore easy to communicate to the public as well. If stabilizing nominal income is to be recognized as an objective for monetary policy in the United States, our results pave a clear path toward achieving that goal.

NOTES

1. An exception is Sumner (1989, 1995) and the suggestion of implementing monetary policy through the use of a nominal GDP futures market.
2. See Ireland (in press) for a theoretical analysis of policies that pay interest on reserves and how those policies change the way that monetary policy actions propagate through the economy.
3. Because the United States switched from reporting nominal GNP to NGDP after some of this initial work had been completed, it may be convenient to couch the discussion in generic terms as "nominal income targeting." For a survey of issues regarding nominal income targeting, see Bean (1983). Clark (1994) offers some evidence on lagged adjustment v. forecast adjustment rules when NGDP targeting is implemented.
4. In this context, it is interesting to note that Working, nearly ninety years ago, devoted an appendix of his paper to an attempt to create an "Index for a Medium of Exchange." Even though he was writing

long before the era of financial innovations and the payment of interest on checkable deposits, he intuited that different components of a monetary aggregate should be weighted differently and in this appendix he made an early attempt to do just that.

5. The results in West (1986) are one reason we take no position on the desirability of NGDP targeting. Using a model presented in Bean (1983), West demonstrated that the preference of nominal income targeting over, say, money supply targeting depends on values of certain parameters and, a priori, there is no clear reason to believe those should take a value that would lead a policy maker to prefer one option over the other. We also take no position on whether targeting the level of nominal GDP is to be preferred to targeting the growth rate of NGDP. Throughout, our purpose is derive a practical approach to targeting the level of NGDP if that is to become the central bank's adopted goal.

6. For more background, see the surveys in Humphrey (1973) and Laidler (2013).

7. Although Taylor (1993) published his famous paper on a rule for the implementation of monetary policy after the P-star paper was published, he did not cite it. Nonetheless, he had this to say about an alternative rule in that paper (pp. 209–210): "Since the mid-1970s monetary targets have been used in many countries to state targets for inflation. If money velocity were stable, then, given an estimate of potential output growth, money targets would imply a target for the price level; given velocity and a real output target, the target price level would obviously fall out algebraically from the money supply target. Even though the 1980s have shown that money velocity is not stable in the short run, the long-run stability of the velocity of some monetary measures allows one to state targets for the price level. For example, with an estimated secular growth of real output of 2.5 percent and a steady velocity, a money growth range of 2.5 percent to 6.5 percent—the Fed's targets for 1992—would imply that the price level target grows at 0 to 4 percent per year. Given biases such as index number problems in measuring prices, the 2-percent per year implicit target inflation rate is probably very close to price stability or 'zero' inflation."

8. For a discussion of issues related to estimating potential output in real time, see, e.g., Orphanides and van Norden (2002).

9. For a more recent discussion and survey of the extensive literature on the Divisia monetary aggregates, see Barnett (2012).

10. Orphanides and Porter (2000) revisited the earlier P-star model by replacing the original measure of trend velocity (its average value over the sample) with values derived from a forecasting equation for velocity. They conclude that the essential properties of the original P-star model are restored when this method is employed. Dueker (1993) addresses this same problem of variations in velocity by reestimating McCallum's original model with a time-varying parameter estimation technique.

11. Similar Divisia M1, M2, and MZM aggregates are available from the Federal Reserve Bank of St. Louis' FRED database; Anderson and Jones (2011) describe their construction in detail. El-Shagi et al. (in press) and Hendrickson (in press) are two other recent studies that use the CFS Divisia aggregates in empirical analyses, whereas Serletis and Rahman (in press) use the series from FRED.

12. Along the same lines, it is interesting to note once again that Working (1923) himself found it necessary to control for slow-moving shifts in trends by including time squared and cubed in addition to time itself in his regression equations. Because Working's regression-based approach might well be considered an early version of the modern, though only slightly more elaborate, filtering procedures used here, we find it especially useful to trace the origins of our own approach back to his as well as to the more familiar P-star model.

13. Again, following Hallman et al. (1991), we multiply quarterly changes in nominal GDP growth by 400 so that they are expressed in annualized percentage points. The nominal GDP gap in these regressions is multiplied by 100 so that it is measured in percentage points. A constant term, shown in Table 1 but not in the equations as displayed here, is also included in each regression.

14. Hetzel (2008, Chap. 23, and 2012, Chap. 8) characterizes these variations as "stop–go" monetary policy and offers a detailed explanation for why it may have evolved in this manner over the past five decades. Belongia and Ireland (2013) incorporate the Divisia monetary aggregates into a structural vector autoregressive time series model to draw even tighter links between monetary and real instability in data running from 1967 through the present.

15. The sequence of steps taken in constructing our monetary framework for targeting nominal GDP is similar in certain respects to the approach taken in McCallum (1990).

16. Equation (24) implies that the average currency–deposit ratio also depends on the elasticity of substitution parameter. The setting for ν is adjusted as ω varies across the range of examples considered here to maintain these constant values of k.

17. Barnett (1982) notes that selection of a monetary aggregate for use in the conduct of monetary policy depends, in part, on the role that aggregate is to serve; Barnett (in press) provides a more recent discussion along similar lines. In this context, the ability of the central bank to control the aggregate's behavior by changing the value of the instrument is an important consideration.

18. In the middle of this estimation period, the Fed eliminated reserve requirements on nonpersonal time deposits in December 1990 and reduced reserve requirements on demand deposits from 12 to 10 percent in April 1992.

19. Nonborrowed reserves pose another issue in the aftermath of the recent financial crisis and the episodes of quantitative easing. Because the Federal Reserve has chosen to include in certain types of borrowed reserves some new forms of bank borrowing not held as reserves, the accounting result has been to generate negative values for nonborrowed reserves; this is clearly a non sequitur but, nonetheless, this is what the nonborrowed reserves data show. The results in this paper are not affected by this phenomenon because of the sample periods used, but it would be an issue for any monetary control strategy based on a nonborrowed reserves instrument.

20. For example, the relatively low and stable rates of base/reserves/money growth over the last decade may introduce an "illusion" of more precise monetary control. Stability in inflation and interest rates coupled with generally stable real growth also could contribute to this illusion. Or these results may suggest that the standard money multiplier model be reexamined in the context of modern institutional arrangements with special attention to changes that would tend to enhance monetary control.

21. Feldstein and Stock (1994, pp. 51–54) make a similar point with respect to their NGDP targeting framework based on simple sum M2. They argue that the Federal Reserve could exercise tighter control over sum M2 by reextending reserve requirements to the non-M1 components of the broader monetary aggregate and paying interest on reserves as well.

22. In addition to the theoretical issues about NGDP targeting mentioned in endnote 5, another practical matter arises from the data: NGDP is not reported monthly and thus the viability of within-quarter policy decisions could be compromised. One possible way to circumvent this problem is the adoption of nowcasting methods to generate estimates for the behavior of nominal GDP between the quarterly official data announcements. To this end, Giannone et al. (2008) and Chauvet and Potter (2013) both offer some guidance on this issue for real GDP. Because money is related to nominal variables in the long run, these nowcasting techniques imply that money, if only as an indicator variable, will be of interest. Barnett et al. (2014) apply nowcasting techniques to nominal GDP data.

REFERENCES

Anderson, Richard G. and Barry E. Jones (2011) A comprehensive revision of the monetary services (Divisia) indexes. *Federal Reserve Bank of St. Louis Review* 93, 325–359.

Barnett, William A. (1978) The user cost of money. *Economics Letters* 1, 145–149. Reprinted in William A. Barnett and Apostolos Serletis, *The Theory of Monetary Aggregation*, pp. 6–10. Amsterdam: Elsevier, 2012.

Barnett, William A. (1980) Economic monetary aggregates: An application of index number and aggregation theory. *Journal of Econometrics* 14, 11–48. Reprinted in William Barnett and Apostolos Serletis, *The Theory of Monetary Aggregation*, pp. 11–48. Amsterdam: Elsevier, 2013.

Barnett, William A. (1982) The optimal level of monetary aggregation. *Journal of Money, Credit and Banking* 14, 687–710. Reprinted in William Barnett and Apostolos Serletis, *The Theory of Monetary Aggregation*, pp. 125–149. Amsterdam: Elsevier, 2012.

Barnett, William A. (2012) *Getting It Wrong: How Faulty Monetary Statistics Undermine the Fed, the Financial System, and the Economy*. Cambridge, MA: MIT Press.

Barnett, William A. (in press) Friedman and Divisia monetary measures. In Robert Cord and Dan Hammond (eds.), *Milton Friedman: Contributions to Economics and Public Policy*. Oxford, UK: Oxford University Press.

Barnett, William A., Marcelle Chauvet, and Danilo Leiva Leon (2014) Real-Time Nowcasting of Nominal GDP. MPRA Paper 53699, University Library of Munich, Germany.

Barnett, William A., Jia Liu, Ryan Mattson, and Jeff van den Noort (2013) The new CFS Divisia monetary aggregates: Design, construction, and data sources. *Open Economies Review* 24, 101–124.

Bean, Charles R. (1983) Targeting nominal income: An appraisal. *Economic Journal* 93, 806–819.

Belongia, Michael T. and Peter N. Ireland (2013). Instability: Monetary and Real. Working paper 830, Department of Economics, Boston College, Chestnut Hill.

Belongia, Michael T. and Peter N. Ireland (in press) The Barnett critique after three decades: A New Keynesian analysis. *Journal of Econometrics*.

Chauvet, Marcelle and Simon Potter (2013) Forecasting output. In Graham Elliott and Allan Timmermann (eds.), *Handbook of Economic Forecasting,* Vol. 2A, pp. 141–194. Amsterdam: Elsevier.

Clark, Todd E. (1994) Nominal GDP targeting rules: Can they stabilize the economy? *Federal Reserve Bank of Kansas City Economic Review* (Third Quarter), 11–25.

Dueker, Michael J. (1993) Can nominal GDP targeting rules stabilize the economy? *Federal Reserve Bank of St. Louis Review* (May/June), 15–29.

El-Shagi, Makram, Sebastian Giesen, and Logan J. Kelly (in press). The quantity theory revisited: A new structural approach. *Macroeconomic Dynamics*.

Feldstein, Martin and James H. Stock (1994). The use of a monetary aggregate to target nominal GDP. In N. Gregory Mankiw (ed.), *Monetary Policy*, pp. 7–69. Chicago: University of Chicago Press.

Fisher, Irving (1920) *Stabilizing the Dollar*. New York: Macmillan.

Friedman, Milton (1968) The role of monetary policy. *American Economic Review* 58, 1–17.

Friedman, Milton and Anna Jacobson Schwartz (1963). *A Monetary History of the United States, 1867–1960*. Princeton, NJ: Princeton University Press.

Giannnone, Domenico, Lucrezia Reichlin and David Small (2008) Nowcasting: The real-time informational content of macroeconomic data. *Journal of Monetary Economics* 24, 665–676.

Hallman, Jeffrey J., Richard D. Porter and David H. Small (1991) Is the price level tied to the M2 monetary aggregate in the long run? *American Economic Review* 81, 841–858.

Hendrickson, Joshua R. (in press) Redundancy or mismeasurement? A reappraisal of money. *Macroeconomic Dynamics*.

Hetzel, Robert L. (2008) *The Monetary Policy of the Federal Reserve: A History*. New York: Cambridge University Press.

Hetzel, Robert L. (2012) *The Great Recession: Market Failure or Policy Failure?* New York: Cambridge University Press.

Hodrick, Robert J. and Edward C. Prescott (1997) Postwar U.S. business cycles: An empirical investigation. *Journal of Money, Credit and Banking* 29, 1–16.

Humphrey, Thomas M. (1973) Empirical tests of the quantity theory of money in the United States, 1900–1930. *History of Political Economy* 5, 285–316.

Ireland, Peter N. (in press) The macroeconomic effects of interest on reserves. *Macroeconomic Dynamics*.

Laidler, David E. W. (2013) Professor Fisher and the quantity theory: A significant encounter. *European Journal of the History of Economic Thought* 20, 174–205.

McCallum, Bennett T. (1988) Robustness properties of a rule for monetary policy. *Carnegie-Rochester Conference Series on Public Policy* 29, 173–204.

McCallum, Bennett T. (1990) Could a monetary base rule have prevented the Great Depression? *Journal of Monetary Economics* 26, 3–26.

Meltzer, Allan H. (1987) Limits of short-run stabilization policy. *Economic Inquiry* 25, 1–14.

Motley, Brian (1988) Should M2 be redefined? *Federal Reserve Bank of San Francisco Economic Review* (Winter), 33–51.

Orphanides, Athanasios and Richard Porter (2000) P* revisited: Money-based inflation forecasts with a changing equilibrium velocity. *Journal of Economics and Business* 52, 87–100.

Orphanides, Athanasios and Simon van Norden (2002) The unreliability of output gap estimates in real time. *Review of Economics and Statistics* 84, 569–583.

Reynard, Samuel (2007) Maintaining low inflation: Money, interest rates, and policy stance. *Journal of Monetary Economics* 54, 1441–1471.

Serletis, Apostolos and Sajjadur Rahman (in press) The case for Divisia money targeting. *Macroeconomic Dynamics*.

Spindt, Paul A. (1983) The money multiplier when money is measured as a Divisia quantity index. *Economics Letters* 13, 219–222.

Spindt, Paul A. (1984) Modelling the money multiplier and the controllability of the Divisia monetary quantity aggregates. *Review of Economics and Statistics* 66, 314–319.

Stock, James H. and Mark W. Watson (1999) Forcasting inflation, *Journal of Monetary Economics* 44, 293–335.

Sumner, Scott (1989) Using futures instrument prices to target nominal income. *Bulletin of Economic Research* 41, 157–162.

Sumner, Scott (1995) The impact of futures price targeting on the precision and credibility of monetary policy, *Journal of Money, Credit and Banking* 27, 89–106.

Tatom, John A. (2011) U.S. Monetary Policy in Disarray. Networks Financial Institute working paper 2011-WP-21. Terre Haute: Indiana State University.

Taylor, John B. (1993) Discretion versus policy rules in practice. *Carnegie-Rochester Series on Public Policy* 39, 195–214.

West, Kenneth D. (1986) Targeting nominal income: A note. *Economic Journal* 96, 1077–1083.

Woodford, Michael (2012) Methods of policy accommodation at the interest-rate lower bound. In *The Changing Policy Landscape*, pp. 185–288. Kansas City, MO: Federal Reserve Bank of Kansas City.

Working, Holbrook (1923) Prices and the quantity of circulating medium, 1890–1921. *Quarterly Journal of Economics* 37, 229–253.