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# The welfare cost of inflation in general equilibrium

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Abstract

This paper presents a general equilibrium monetary model in which inflation distorts a variety of marginal decisions. Although individually none of the distortions is very large, they combine to yield substantial welfare cost estimates. A sustained 4 percent inflation like that experienced in the US since 1983 costs the economy the equivalent of 0.41 percent of output per year when currency is identified as the relevant definition of money and over 1 percent of output per year when M1 is defined as money. The results illustrate how the traditional, partial equilibrium approach can seriously underestimate the true cost of inflation.

*Key words*: Inflation; Growth; Welfare

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## 1. Introduction

A sound judgment regarding the desirability of price stability as the principal goal of monetary policy requires an accurate assessment of the consequences of sustained price inflation. Thus, monetary economists have devoted considerable

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effort to measuring the welfare cost of inflation. The traditional approach, developed by Bailey (1956) and Friedman (1969), treats real money balances as a consumption good and inflation as a tax on real balances. This approach measures the welfare cost by computing the appropriate area under the money demand curve.
nalysis, most notably those of Fischer (1981) and Lucas (1981), find the cost of inflation to be surprisingly low. Fischer computes the deadweight loss generated by an increase in inflation from zero to 10 percent as just 0.3 percent of GNP using the monetary base as the definition of money. Lucas places the cost of a 10 percent inflation at 0.45 percent of GNP using M1 as the measure of money. Since these estimates appear small relative to the potential cost of a disinflationary recession, they provide little support for the idea that price stability is an essential goal for monetary policy.

The inflation tax, however, may distort economic decisions along margins that the partial equilibrium approach of Bailey and Friedman ignores. This paper, therefore, takes a general equilibrium approach to assessing the welfare cost of inflation. A key feature of the model developed here is a transactions technology that gives rise to a money demand function resembling those estimated with data from the US economy. Thus, the analysis begins by accounting for Bailey–Friedman costs of inflation of the magnitude estimated by Fischer and Lucas.

The Bailey–Friedman approach, however, turns out to capture only a fraction of the total cost of inflation in this model. In fact, the model identifies several other distortions associated with the inflation tax. First, as in Cooley and Hansen (1989, 1991), inflation causes agents to inefficiently substitute out of market activity and into leisure. Second, as suggested by Karni (1974), inflation causes agents to devote productive time to activities that enable them to economize on their cash balances. When adapted to a general equilibrium setting, Karni's specification implies that inflation draws a fraction of the labor force out of goods production and into a distinct financial sector.[1] Finally, the model takes its specification for goods-producing technologies from Romer (1986), so that the allocative effects of inflation can potentially influence the growth rate, as well as the level, of aggregate output. Although none of the additional distortions is very large, they combine to yield estimates of the welfare cost of inflation that are more than three times the size of the Fischer–Lucas estimates.

Black et al. (1993), Coleman (1993), De Gregorio (1993), Gomme (1993), Jones and Manuelli (1993), Wang and Yip (1993), and Marquis and Reffett (1994) also

[1] This general equilibrium interpretation of Karni's insight is consistent with Yoshino's (1993) finding that inflation and employment in banking have been positively correlated over time in the US and other countries.

examine the effects of inflation in endogenous growth settings. Unlike previous specifications, however, the transactions technology used here generates a money demand function that is as interest-elastic as those estimated with US data.[2] Consequently, the model is ideally suited for comparing the partial equilibrium Bailey–Friedman cost to the full general equilibrium cost of inflationary policy. Indeed, the results show how the traditional, partial equilibrium approach can seriously underestimate the true cost of inflation and thereby understate the case for price stability.[3]

## 2. A general equilibrium model of the inflation tax

### 2.1. The economic environment

The economy consists of a continuum of markets, indexed by $i \in [0, 1)$, arranged on the boundary of a circle with unit circumference. In each market, a distinct, perishable consumption good is produced and traded in each period $t = 0, 1, 2, \ldots$ . Hence, the economy's consumption goods are also indexed by $i \in [0, 1)$, where good $i$ is sold in market $i$.

Large numbers of identical households, financial intermediaries, and goods-producing firms inhabit each market $i$. Enough symmetry is imposed on these agents's preferences, endowments, and technologies that the analysis considers without loss of generality the behavior of a single representative household, a single representative intermediary, and a single representative firm. The representative agents all live at market 0, so that the index $i$ measures the distance of market $i$ from their home.

The government, which otherwise plays no role in the economy, provides households with noninterest-bearing fiat money. It supplies each household with $m_0^s$ units of money at the beginning of period $t = 0$ and augments this supply by making identical lump-sum transfers $h_t$ to all households at the beginning of each period $t$. Hence, the per-household money supply $m_{t+1}^s$ at the end of date $t$ satisfies

$$m_{t+1}^s = (1 + g_t)m_t^s,$$

[2] In Cooley and Hansen's (1989) single-good cash-in-advance model, for example, the velocity of money is practically constant. Thus, money demand is highly interest-inelastic. Cooley and Hansen (1991) use a multiple-good cash-in-advance model that, in principle at least, allows velocity to vary with the inflation rate. Benabou (1991), however, demonstrates that this alternative cash-in-advance formulation also generates a very inelastic money demand function.

[3] See Imrohoroglu (1992) for a very different general equilibrium approach that also yields the result that the Bailey–Friedman analysis captures only a fraction of the total cost of inflation.

where the rate of money growth $g_t$ is given by

$$g_t = h_t/m_t^s.$$

The government announces the sequence $\{g_t\}_{t=0}^{\infty}$ of money growth rates at the beginning of period $t = 0$. From then on, all agents have perfect foresight.

### 2.2. Households and their trading opportunities

The representative household at market $i = 0$ has preferences over leisure and the continuum of consumption goods as described by the utility function

$$\sum_{t=0}^{\infty} \beta^t \left\{ \int_0^1 \ln[c_t(i)]\,\mathrm{d}i + BJ_t \right\}, \qquad \beta \in (0,1), \quad B > 0. \tag{1}$$

Thus, $c_t(i)$ denotes the household's consumption of good $i$ and $J_t$ its leisure at time $t$.

Following Lucas and Stokey (1983), the representative household is imagined to consist of two members: a worker and a shopper. During each period $t$, the representative worker rents out his household's capital stock $k_t$ at the real rate $r_t$ and supplies $l_t^g$ units of labor at the real wage $w_t$ to goods-producing firms. He also supplies $l_t^f$ units of labor to financial intermediaries. The worker makes his labor-supply decisions subject to the time constraint

$$1 \geq J_t + l_t^g + l_t^f \tag{2}$$

at each date $t$.

The representative shopper, meanwhile, travels around the circle in order to acquire goods for his household's consumption. As in Prescott (1987), Schreft (1992), and Gillman (1993), the shopper chooses between two alternative means of making purchases in each market $i$. His first alternative is to use government-issued money. Since competition equates the nominal price $p_t$ of consumption goods across markets, the shopper may acquire $c_t(i)$ units of good $i$ in exchange for $p_t c_t(i)$ units of money at time $t$.

The shopper's second alternative for purchasing good $i$ is to enlist the services of a financial intermediary. At a cost of $\gamma(i)$ units of labor, an intermediary verifies the shopper's identity and guarantees his ability to pay, so that a firm in market $i$ is willing to sell its output on credit at time $t$. The communications and record-keeping costs of facilitating a credit transaction do not depend on the size of the purchase but increase as the shopper travels farther from home. Hence, $\gamma$ is a strictly increasing function of $i$. Under the additional assumption that $\lim_{i\to 1} \gamma(i) = \infty$, some goods will always be purchased with cash, and there is a well-defined demand for money.

In exchange for its services at time $t$, the intermediary in market $i$ charges the representative household the real price $q_t(i)$. Since the intermediary's cost $\gamma(i)$ is independent of the size of the transaction but depends nontrivially on $i$, competition ensures that the function $q_t(i)$ satisfies these same properties. Thus, the representative shopper may acquire $c_t(i)$ units of good $i$ on credit at time $t$ at a total nominal cost of $p_t[c_t(i) + q_t(i)]$: $p_t c_t(i)$ to pay for the goods themselves and $p_t q_t(i)$ to compensate the intermediary.

Let the indicator function $\xi_t(i) = 0$ if the representative shopper purchases good $i$ with money at time $t$, and let $\xi_t(i) = 1$ if he uses an intermediary instead. Let $m_t$ denote the nominal cash balances carried by the shopper into time $t$; these are augmented at the beginning of the period by the government transfer $h_t$. Since the shopper must use money whenever he chooses not to hire an intermediary, he faces the cash-in-advance constraint

$$\frac{m_t + h_t}{p_t} \geq \int_0^1 [1 - \xi_t(i)] c_t(i) \, \mathrm{d}i, \tag{3}$$

in each period $t$.

After consuming its purchases at the end of time $t$, the household participates in a centralized asset market, where it receives its rental payments $r_t k_t$ and wages $w_t(l_t^g + l_t^f)$ and pays for the goods that it bought on credit earlier in time $t$. The household uses any excess funds to accumulate the cash balances $m_{t+1}$ that it will carry into period $t + 1$ and to purchase unsold output from the representative firm, which it combines with its depreciated capital stock $(1 - \delta)k_t$ in order to carry $k_{t+1}$ units of capital into period $t + 1$.

The representative household is also permitted to borrow from and lend to other households in the end-of-period asset market by issuing or purchasing one-period, nominally-denominated discount bonds. Bonds paying off $b_{t+1}$ units of money in the time $t + 1$ asset market sell for $b_{t+1}/R_t$ units of money in the time $t$ asset market, where $R_t$ is the gross nominal interest rate between $t$ and $t + 1$. Since these bonds are available in zero net supply, $b_{t+1} = 0$ must hold as an equilibrium condition in each period $t$.

As sources of funds in period $t$, the representative household has its initial money and bond holdings, its beginning-of-period government transfer, its rental and wage receipts, and its capital stock after depreciation. As uses of funds it has its purchases of consumption goods, its payments to intermediaries, and the capital, money, and bonds that it will carry into period $t + 1$. It therefore faces the budget constraint

$$\begin{aligned} &\frac{m_t + b_t + h_t}{p_t} + r_t k_t + w_t(l_t^g + l_t^f) + (1 - \delta)k_t \\ &\geq \int_0^1 c_t(i) \, \mathrm{d}i + \int_0^1 \xi_t(i) q_t(i) \, \mathrm{d}i + k_{t+1} + \frac{m_{t+1}}{p_t} + \frac{b_{t+1}}{p_t R_t}, \end{aligned} \tag{4}$$

in each period $t$. The representative household chooses sequences for $c_t(i)$, $\xi_t(i)$, $J_t$, $l_t^g$, $l_t^f$, $k_{t+1}$, $m_{t+1}$, and $b_{t+1}$ to maximize the utility function (1) subject to the constraints (2)–(4), taking the sequences for $h_t$, $r_t$, $w_t$, $p_t$, $q_t(i)$, and $R_t$ as given. It also takes its initial holdings of capital $k_0 > 0$, money $m_0 = m_0^s$, and bonds $b_0 = 0$ as given.

### *2.3. The representative intermediary's problem*

An intermediary hires $\gamma(i)$ units of labor and charges $q_t(i)$ if it helps the representative shopper purchase good $i$ on credit at time $t$. Let $\xi_t^s(i) = 1$ if the representative intermediary decides to supply these financial services to the representative household; let $\xi_t^s(i) = 0$ otherwise. Then the intermediary chooses $\xi_t^s(i)$ to maximize its profits

$$\pi_t^f = \int_0^1 \xi_t^s(i)[q_t(i) - w_t\gamma(i)]\,\mathrm{d}i, \tag{5}$$

at each date $t$, taking $w_t$ and $q_t(i)$ as given. The intermediary's total demand for labor is

$$n_t^f = \int_0^1 \xi_t^s(i)\gamma(i)\,\mathrm{d}i. \tag{6}$$

### *2.4. The representative goods-producing firm's problem*

The representative goods-producing firm in market $i = 0$ hires $k_t$ units of capital and $n_t^g$ units of labor from households in each period $t$ in order to produce output of consumption good $i = 0$. Its profits in period $t$ are

$$\pi_t^g = A(k_t)^\alpha(n_t^g)^{1-\alpha}(K_t)^\eta - r_tk_t - w_tn_t^g, \qquad \alpha \in (0,1), \quad \eta > 0. \tag{7}$$

The production function in Eq. (7) contains $K_t$, the aggregate capital stock per household at time $t$. Following Romer (1986), capital is interpreted broadly here to include stocks of human capital and disembodied knowledge in addition to physical capital. While goods production features constant returns to scale at the firm level, spillover effects associated with the accumulation of human capital generate increasing returns at the aggregate level. Increasing returns make the economy's growth rate endogenous and possibly dependent on the inflation rate. The representative firm takes the aggregate capital stock $K_t$ as well as the factor prices $r_t$ and $w_t$ as given when maximizing (7).

### *2.5. Equilibrium conditions*

A competitive equilibrium in this economy consists of sequences for prices and quantities that are consistent with the solutions to the optimization problems for households, intermediaries, and firms outlined above. Given the initial conditions $k = K_0 > 0$, $m_0 = m_0^s$, and $b_0 = 0$, equilibrium prices and quantities must also satisfy the zero profit conditions

$$\pi_t^g = \pi_t^f = 0, \tag{8}$$

the consistency condition

$$k_{t+1} = K_{t+1}, \tag{9}$$

and the market-clearing conditions for goods, labor, money, bonds, and financial services

$$A(k_t)^{\alpha+\eta}(n_t^g)^{1-\alpha} + (1-\delta)k_t = k_{t+1} + \int_0^1 c_t(i)\,\mathrm{d}i, \tag{10}$$

$$n_t^g = l_t^g, \qquad n_t^f = l_t^f, \tag{11}$$

$$m_{t+1} = m_{t+1}^s, \tag{12}$$

$$b_{t+1} = 0, \tag{13}$$

$$\xi_t(i) = \xi_t^s(i), \tag{14}$$

in each period $t$.

### 2.6. Relationship to alternative monetary models

The transactions technology used in this model is closely related to two, more conventional, money demand specifications. First, when $\gamma(i) = \infty$ for all $i$, so that making purchases on credit is infinitely costly, the representative household will choose $\xi(i) = 0$ for all $i$. In this case, Eq. (3) reduces to the same cash-in-advance constraint used by Cooley and Hansen (1989).[4]

Second, Eqs. (3), (6), (11), and (14) imply that in equilibrium the representative household can economize on real balances only by supplying more labor to the intermediary. One can, in fact, summarize this trade-off between money and labor by implicitly defining $l_t^f$ as a function of $m_t/p_t$. Substituting this function into Eqs. (1) and (2) then reveals that the transactions technology is formally equivalent to a money-in-the-utility-function specification.[5] The explicit description of households' trading opportunities provided here, however, yields a precise statement of exactly how and why money enters into the utility function.

[4] Indeed, using their parameter values, we were able to exactly reproduce Cooley and Hansen's results in this case.

[5] We would like to thank the referee for alerting us to this fact.

## 3. The general equilibrium effects of the inflation tax

The appendix demonstrates that in equilibrium, there exists a borderline index $s_t$ for each date $t$ such that the representative household purchases all goods with indices $i \leq s_t$ on credit and all goods $i > s_t$ with cash. This borderline index is determined by the solution to

$$\gamma(s_t) = [\ln(\lambda_t + \mu_t) - \ln(\lambda_t)]/w_t\lambda_t, \tag{15}$$

where $\lambda_t$ is the nonnegative multiplier on the budget constraint (4) and $\mu_t$ is the nonnegative multiplier on the cash-in-advance constraint (3) from the household's optimization problem. As in Schreft (1992) and Gillman (1993), the shopper uses credit close to home and cash far from home, since intermediation costs increase with distance.

The representative household's optimal $c_t(i)$ is a step function at each date $t$:

$$c_t(i) = \begin{cases} c_t^1 = 1/\lambda_t & \text{for} \quad i \leq s_t, \\ c_t^0 = 1/(\lambda_t + \mu_t) & \text{for} \quad i > s_t, \end{cases} \tag{16}$$

where, since $\mu_t \geq 0$, $c_t^1 \geq c_t^0$. Eq. (16) and the cash-in-advance constraint (3) determine equilibrium money demand as

$$(m_t + h_t)/p_t = (1 - s_t)c_t^0. \tag{17}$$

Eqs. (6), (11), and (14) determine employment in the financial sector as

$$l_t^f = \int_0^{s_t} \gamma(i)\,\mathrm{d}i. \tag{18}$$

The inflation tax causes the household's cash-in-advance constraint to bind, so that higher rates of inflation tend to be associated with larger values of the multiplier $\mu_t$. Since $\gamma$ is increasing as a function of $i$, Eq. (15) suggests that higher inflation rates are also associated with higher values of $s_t$. That is, under higher rates of inflation the household purchases a wider range of goods with the help of intermediaries.

Eq. (16) then indicates that the inflation tax distorts consumption and production decisions in two ways. First, since $c_t^1 > c_t^0$, the representative household purchases different consumption goods in different quantities; its marginal rate of substitution between cash and credit goods deviates from the corresponding marginal rate of transformation. Second, since $c_t^0$ is decreasing as a function of $\mu_t$, the representative household purchases cash goods in smaller quantities so that overall, market activity is reduced. These are the marginal effects of the inflation tax studied by Cooley and Hansen (1989, 1991). Here, however, the production technology described in Eq. (7) allows these allocative effects of inflation to change the growth rate, as well as the level, of aggregate output.

Eq. (17) suggests that the representative household economizes on its cash balances in the face of a positive inflation tax both by purchasing a wider range of goods without money (i.e., by increasing $s_t$) and by consuming less of those goods that it purchases with money (i.e., by decreasing $c_t^0$). Thus, the demand for money is interest-elastic and gives rise to the Bailey–Friedman cost of the inflation tax.

Finally, Eq. (18) indicates that as the household increases $s_t$ in response to a higher inflation tax, the size of the labor force employed in the financial sector increases. The diversion of labor resources out of productive activity and into finance also contributes to the welfare cost of inflation. Again, the goods-producing technology in (7) provides a channel through which this allocative effect can influence the long-run growth rate.

Thus, the model associates a number of distortions with the inflation tax. It is not possible, however, to assess the magnitude of any of these distortions analytically. Hence, the following sections apply numerical methods to measure the effects of inflation in general equilibrium.

## 4. Model parameterization

In order to apply numerical methods, specific values must be assigned to the model's parameters. The household's discount rate is set at $\beta = 0.99$ and the depreciation rate at $\delta = 0.025$ so that each period in the model corresponds to one quarter year. Sustained, balanced growth occurs when the aggregate production function is linear in the capital stock, so $\alpha = 0.4$ and $\eta = 0.6$. With $A = 0.265$, the economy grows at a constant annual rate of 2 percent (the US average since 1959) under a constant annual inflation rate of 5 percent (again, the US average since 1959). The representative worker devotes one-fifth of his time to labor (the figure used by King and Rebelo 1993) under 5 percent inflation when $B = 4.25$.

The magnitude of the Bailey–Friedman cost of inflation hinges on two numbers: the size of the tax base and the interest elasticity of money demand. When the intermediary's cost function is specialized to

$$\gamma(i) = \gamma[i/(1-i)]^{\theta}, \qquad \gamma > 0, \qquad \theta > 0, \tag{19}$$

the parameters $\gamma$ and $\theta$ can be chosen so that the size of the tax base and the interest elasticity of money demand in the model match corresponding figures in the US economy.[6] The next section constructs equilibria for two specifications,

[6] Cooley and Hansen (1991) choose one parameter to match the size of the inflation tax base in their model with the analogous figure from the US data. Similarly, Lacker and Schreft (1993) choose one parameter to match the interest elasticity in their model and the US data. Thus, the approach taken here combines the methods of these earlier studies by setting two parameters in order to match both the size of the tax base and the interest elasticity in the model and data.

one in which money is defined as currency and the other in which money is defined as M1. The alternative definitions of money require different sets of values for $\gamma$ and $\theta$.

Following Cooley and Hansen (1991), the size of the inflation tax base in the US economy is measured by the fraction of all purchases that are made using money. Avery et al. (1987) report that in 1984, when inflation was about 4 percent, US households made 30 percent of their transactions with currency and 82 percent of their transactions with M1. These fractions correspond to the value of $1 - s_t$ under 4 percent inflation in the model.

Annual data from 1959–1991 yield estimates of the money demand equations

$$\ln(vc) = 2.88 + 2.73R,$$

$$\ln(v1) = 1.24 + 5.95R,$$

where $vc$ is the income velocity of currency, $v1$ is the income velocity of M1, and $R$ is the six-month commercial paper rate.[7] The OLS coefficients on $R$ in these equations measure the long-run interest semi-elasticity of money demand. An analogous statistic in the model economy is

$$[\ln(v_{10}) - \ln(v_0)]/(R_{10} - R_0),$$

where $v_{10}$ and $v_0$ are the constant annual velocities of money and $R_{10}$ and $R_0$ are the constant annual nominal interest rates that prevail under constant annual inflation rates of 10 percent and zero.

To match the tax base and the elasticity figures in the data and model, $\gamma = 0.00075$ and $\theta = 2.45$ for the currency specification and $\gamma = 0.00933$ and $\theta = 0.333$ for the M1 specification. With these combinations of $\gamma$ and $\theta$, the annual velocity of money under 5 percent inflation in the model economy is 19.9 for currency and 7.6 for M1. Annualized velocity in the model varies inversely with the assumed length of each period; a shorter period length implies a higher annual velocity. The fact that these figures for velocity are similar in magnitude to the averages of 21.6 for currency and 5.4 for M1 found in US data since 1959 indicates that they are also consistent with the identification of one model period as one quarter year.

## 5. The quantitative effects of inflation in general equilibrium

This section considers the effects of monetary policies that call for constant rates of money growth. These policies give rise to steady-state equilibria in which all variables grow at constant rates.

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[7] All data are taken from the Economic Report of the President (1993). The series for velocity are constructed using GDP as the measure of income.

Table 1
The welfare cost of inflation

| | Annual inflation rate | | | |
|---|---|---|---|---|
| | Friedman rule | 0 percent | 4 percent | 10 percent |
| *Currency specification* | | | | |
| Annual rate of money growth | – 0.0394 | 0.0212 | 0.0618 | 0.1228 |
| Fraction of transactions using cash | 1.0000 | 0.3452 | 0.3012 | 0.2646 |
| Annual velocity of money | 5.7129 | 16.7262 | 19.3199 | 22.2562 |
| Total labor supply | 0.2016 | 0.2008 | 0.2005 | 0.2002 |
| Fraction of labor force in finance | 0.0000 | 0.0017 | 0.0028 | 0.0041 |
| Annual rate of output growth | 0.0218 | 0.0212 | 0.0210 | 0.0207 |
| Welfare cost (percentage of output) | – 0.9142 | 0.0000 | 0.4087 | 0.9155 |
| *M1 specification* | | | | |
| Annual rate of money growth | – 0.0394 | 0.0203 | 0.0605 | 0.1216 |
| Fraction of transactions using cash | 1.0000 | 0.9482 | 0.8051 | 0.5173 |
| Annual velocity money | 5.7129 | 6.0247 | 7.1356 | 11.3350 |
| Total labor supply | 0.2016 | 0.1987 | 0.1981 | 0.2003 |
| Fraction of labor force in finance | 0.0000 | 0.0007 | 0.0042 | 0.0148 |
| Annual rate of output growth | 0.0218 | 0.0203 | 0.0197 | 0.0197 |
| Welfare cost (percentage of output) | – 2.2159 | 0.0000 | 1.0767 | 1.7273 |

### *5.1. The effects of inflation on velocity, labor supply, and growth*

Table 1 describes steady-state equilibria under the benchmark policy that yields a constant zero inflation rate.[8] It compares these equilibria to those obtaining under constant 4 (the US average since 1983) and 10 (the alternative policy considered by Fischer, 1981, and Lucas, 1981) percent annual rates of inflation. It also reports results from adopting the Friedman (1969) rule, under which the money supply is contracted at the rate of time preference so as to make the nominal interest rate equal to zero.

Under a constant rate of inflation, the representative shopper makes a constant fraction of his purchases with cash. The model is parameterized so that with 4 percent annual inflation, this constant fraction is about 30 percent if money is defined as currency and about 80 percent if money is defined as M1.

[8] Zero inflation serves as a benchmark here since it is also used as a benchmark by Fischer (1981) and Lucas (1981) and since, as noted by Carlstrom and Gavin (1993), price stability is the most widely-cited objective for monetary policy in the US economy.

Table 1 indicates that for either specification, the shopper uses money in a smaller range of transactions when inflation is higher. Thus, the steady-state velocity of money rises with the inflation rate.

The model is parameterized so that the representative worker devotes approximately 20 percent of his time to labor. Table 1 shows that as the inflation rate rises, the household tends to substitute out of market activity, which requires either money or costly financial services, and into leisure, which can be enjoyed without the use of a means of exchange. In addition to this substitution effect, however, there is a negative wealth effect associated with an increase in the inflation tax. While the substitution effect always dominates in Cooley and Hansen's (1989, 1991) models, Cole and Stockman (1992) find that the wealth effect can easily dominate in their version of the cash-in-advance model in which the use of money can be circumvented at a cost in terms of real resources. The wealth effect can dominate here as well, so that an increase from 4 to 10 percent inflation increases the household's labor supply under the M1 specification.

Higher rates of inflation shift the allocation of the labor force in addition to changing the total labor supply. Table 1 shows that while the fraction of the labor force working for intermediaries is always less than 1.5 percent, this share rises with the inflation rate. The substitution of labor out of goods production and into leisure and finance tends to reduce the growth rate of output via the spillover effects of aggregate activity. But in general, the effects of inflation on growth are small: 10 percent inflation reduces the growth rate from 2.12 to 2.07 percent under the currency specification and from 2.03 to 1.97 percent under the M1 specification.

### 5.2. *The effects of inflation on welfare*

As in Cooley and Hansen (1989, 1991), the welfare cost of inflation is measured here by the permanent percentage increase in the consumption of all goods that makes that representative household as well off under a positive rate of inflation as it is under zero inflation. This figure is converted from a fraction of consumption to a fraction of output by multiplying it by the ratio of consumption to output. Table 1 shows that when money is defined as currency, a sustained 4 percent inflation like that experienced in the US since 1983 costs the equivalent of a permanent 0.41 percent decrease in output. A 10 percent inflation costs almost 0.92 percent of output. When money is defined as M1, a 4 percent inflation costs 1.08 percent of output and a 10 percent inflation costs 1.73 percent of output. Table 1 also shows that the welfare gain from adopting the Friedman rule is equivalent to a 0.91 percent increase in output under the currency specification and a 2.22 percent increase in output under the M1 specification.

Table 2 reports the Bailey–Friedman cost of inflation in the model economy, computed as the area under the money demand curve that is lost as the

Table 2
The Bailey–Friedman cost of inflation

| | Annual inflation rate | | | |
|---|---|---|---|---|
| | Friedman rule | 0 percent | 4 percent | 10 percent |
| *Currency specification* | | | | |
| Welfare cost (percentage of output) | – 0.5876 | 0.0000 | 0.0147 | 0.0605 |
| *M1 specification* | | | | |
| Welfare cost (percentage of output) | – 0.0137 | 0.0000 | 0.0573 | 0.4220 |

steady-state inflation rate increases. Since by construction the model gives rise to a money demand curve that resembles those estimated with US data, the Bailey–Friedman costs are quite similar to those reported by Fischer (1981) and Lucas (1981). With money defined as currency, the Bailey–Friedman analysis puts the cost of a 10 percent inflation at about 0.06 percent of output. Both the tax base and the elasticity of demand are larger when money is defined as M1. Hence, the Bailey–Friedman cost of inflation is higher as well: a 10 percent inflation costs about 0.42 percent of income.

The Bailey–Friedman approach also indicates that the welfare gain from adopting the Friedman rule is substantial, equal to 0.59 percent of output, under the currency specification. Recall that for currency, the parameters of the transaction technology (19) are set so that under 4 percent inflation the representative household makes only 30 percent of its purchases with money. The representative household makes all of its purchases with cash under the Friedman rule, since the zero nominal interest rate eliminates the opportunity cost of holding real balances. In order to reduce the fraction of cash transactions from 100 percent under the Friedman rule to 30 percent under 4 percent inflation, the model must give rise to a money demand function that is extremely elastic at low nominal rates of interest. Here, as in Lucas (1993), the high elasticity of money demand at interest rates close to zero implies that there are large welfare gains from moving to the Friedman rule.

Comparing the welfare cost estimates in Tables 1 and 2 illustrates that the partial equilibrium analysis of Bailey and Friedman generally captures only a fraction of the total cost of the inflation tax. In addition to its effects on velocity, the inflation tax causes agents to inefficiently allocate productive labor across its various uses. The labor-supply effects may seem small, but they contribute to estimates of the total welfare cost of inflation that are much larger than those obtained using the Bailey–Friedman approach.

Table 3
The welfare cost of inflation with exogenous growth

| | Annual inflation rate | | | |
|---|---|---|---|---|
| | Friedman rule | 0 percent | 4 percent | 10 percent |
| *Currency specification* | | | | |
| Annual rate of money growth | – 0.0394 | 0.0200 | 0.0608 | 0.1220 |
| Fraction of transactions using cash | 1.0000 | 0.3470 | 0.3021 | 0.2650 |
| Annual velocity of money | 5.7043 | 16.6176 | 19.2472 | 22.2107 |
| Total labor supply | 0.2013 | 0.2006 | 0.2004 | 0.2001 |
| Fraction of labor force in finance | 0.0000 | 0.0017 | 0.0027 | 0.0041 |
| Annual rate of output growth | 0.0200 | 0.0200 | 0.0200 | 0.0200 |
| Welfare cost (percentage of output) | – 0.1569 | 0.0000 | 0.0875 | 0.2032 |
| *M1 specification* | | | | |
| Annual rate of money growth | – 0.0394 | 0.0200 | 0.0608 | 0.1220 |
| Fraction of transactions using cash | 1.0000 | 0.9488 | 0.8035 | 0.5158 |
| Annual velocity money | 5.7043 | 6.0194 | 7.1520 | 11.3744 |
| Total labor supply | 0.2013 | 0.1986 | 0.1982 | 0.2004 |
| Fraction of labor force in finance | 0.0000 | 0.0007 | 0.0042 | 0.0149 |
| Annual rate of output growth | 0.0200 | 0.0200 | 0.0200 | 0.0200 |
| Welfare cost (percentage of output) | – 0.1903 | 0.0000 | 0.2739 | 0.9231 |

### *5.3. Decomposing the welfare effects*

In order to see how the various labor-supply effects contribute to the welfare cost estimates reported in Table 1, Table 3 considers a version of the model with exogenous growth. This version of the model replaces the production function shown in Eq. (7) with the Cobb–Douglas specification

$$(k_t)^{\alpha}(x_t n_t^g)^{1-\alpha},$$

where sustained growth is now driven by exogenous technological change:

$$x_t = \rho x_{t-1}.$$

As before, $\alpha = 0.4$; the economy grows at the annual rate of 2 percent in steady state when $\rho = (1.02)^{1/4}$.

Table 3 demonstrates that, for the most part, the effects of inflation do not depend on whether growth is endogenous or exogenous. The changes in velocity, total labor supply, and the share of the labor force in finance shown in Table 3 are almost identical to those in Table 1. The welfare cost estimates, however, are much smaller under exogenous growth: 10 percent inflation costs

Table 4
The welfare cost of inflation with fixed capital

| | Annual inflation rate | | | |
|---|---|---|---|---|
| | Friedman rule | 0 percent | 4 percent | 10 percent |
| *Currency specification* | | | | |
| Annual rate of money growth | – 0.0394 | 0.0200 | 0.0608 | 0.1220 |
| Fraction of transactions using cash | 1.0000 | 0.3470 | 0.3021 | 0.2650 |
| Annual velocity of money | 5.6983 | 16.6176 | 19.2560 | 22.2332 |
| Total labor supply | 0.2011 | 0.2006 | 0.2005 | 0.2003 |
| Fraction of labor force in finance | 0.0000 | 0.0017 | 0.0027 | 0.0041 |
| Annual rate of output growth | 0.0200 | 0.0200 | 0.0200 | 0.0200 |
| Welfare cost (percentage of output) | – 0.1046 | 0.0000 | 0.0646 | 0.1524 |
| *M1 specification* | | | | |
| Annual rate of money growth | – 0.0394 | 0.0200 | 0.0608 | 0.1220 |
| Fraction of transactions using cash | 1.0000 | 0.9488 | 0.8035 | 0.5158 |
| Annual velocity money | 5.6879 | 6.0194 | 7.1603 | 11.3873 |
| Total labor supply | 0.2007 | 0.1986 | 0.1984 | 0.2006 |
| Fraction of labor force in finance | 0.0000 | 0.0007 | 0.0042 | 0.0149 |
| Annual rate of output growth | 0.0200 | 0.0200 | 0.0200 | 0.0200 |
| Welfare cost (percentage of output) | – 0.0454 | 0.0000 | 0.2152 | 0.8640 |

0.20 percent, rather than 0.92 percent, of output under the currency specification and 0.92 percent, rather than 1.73 percent, of output under the M1 specification. As emphasized by Lucas (1987), policies that induce even small changes in an economy's growth rate have substantial welfare consequences.

Since these results indicate that growth effects play a large role in generating the welfare cost estimates reported in Table 1, it is worth noting that empirically, Kormendi and Meguire (1985), Fischer (1991), and De Gregorio (1993) find that differences in inflation do contribute significantly to explaining cross-country differences in growth. Levine and Renelt (1992) argue that results from cross-country studies such as these are not generally robust. However, the results in Table 1 also explain why the inflation-growth rate link may be difficult to detect in the data: the changes in growth rates are extremely small.

The welfare costs that remain after eliminating endogenous growth are decomposed still further in Tables 4 and 5. The labor-supply effects shown in Table 3 act to lower the marginal product of capital; this reduction in marginal product implies that the steady state capital stock typically decreases as inflation rises. To isolate the cost of this effect, Table 4 reports results for the exogenous growth model when the path for the capital stock is held fixed across steady

Table 5
The welfare cost of inflation with fixed output

| | Annual inflation rate | | | |
|---|---|---|---|---|
| | Friedman rule | 0 percent | 4 percent | 10 percent |
| *Currency specification* | | | | |
| Annual rate of money growth | – 0.0394 | 0.0200 | 0.0608 | 0.1220 |
| Fraction of transactions using cash | 1.0000 | 0.3470 | 0.3021 | 0.2650 |
| Annual velocity of money | 5.7043 | 16.6176 | 19.2472 | 22.2107 |
| Total labor supply | 0.2003 | 0.2006 | 0.2008 | 0.2011 |
| Fraction of labor force in finance | 0.0000 | 0.0017 | 0.0027 | 0.0041 |
| Annual rate of output growth | 0.0200 | 0.0200 | 0.0200 | 0.0200 |
| Welfare cost (percentage of output) | – 0.1038 | 0.0000 | 0.0639 | 0.1507 |
| *M1 specification* | | | | |
| Annual rate of money growth | – 0.0394 | 0.0200 | 0.0608 | 0.1220 |
| Fraction of transactions using cash | 1.0000 | 0.9488 | 0.8035 | 0.5158 |
| Annual velocity money | 5.7043 | 6.0194 | 7.1520 | 11.3744 |
| Total labor supply | 0.1985 | 0.1986 | 0.1993 | 0.2015 |
| Fraction of labor force in finance | 0.0000 | 0.0007 | 0.0042 | 0.0148 |
| Annual rate of output growth | 0.0200 | 0.0200 | 0.0200 | 0.0200 |
| Welfare cost (percentage of output) | – 0.0405 | 0.0000 | 0.2109 | 0.8604 |

states. In the currency specification, the cost of 10 percent inflation falls from 0.20 to 0.15 percent of output, indicating that the capital stock effect accounts for about one-quarter of the welfare cost in the exogenous growth case. For the M1 specification, the cost falls from 0.92 to 0.86 percent of output when capital is held fixed.

Finally, Table 5 shows the results when labor in goods production is held fixed as well. With both capital and labor held fixed, the supply of output is also fixed; the only remaining labor-supply effect involves the absorption of labor by the financial sector. Table 5 reveals that, in fact, holding labor in goods production fixed does very little to lower the welfare costs, indicating that the direct effect of the shift out of goods production and into leisure contributes little to the overall cost of inflation.

Thus, to summarize the results: a large fraction of the welfare cost estimates in Table 1 can be attributed to growth effects. The effects of inflation on the capital stock, holding growth fixed, also account for a good portion of the costs. The reduction in labor supply to the goods-producing sector, however, turns out to be only a minor component, implying that the absorption of labor by the financial sector accounts for the bulk of the remaining costs.

## 6. Conclusion

In the general equilibrium model developed here, the inflation tax distorts a variety of marginal decisions. Agents inefficiently economize on their holdings of real cash balances. They substitute out of market activity by taking more leisure. They divert productive resources out of goods production and into finance.

The model shows that individually, none of these distortions is very large. By construction, the model's money demand function matches those estimated with US data. Hence, the Bailey–Friedman cost of inflation in the model is similar in magnitude to the figures obtained with US data by Fischer (1981) and Lucas (1981), which are too small to justify the expense of a disinflationary recession. Similarly, the effects of inflation on the total labor supply and its sectoral allocation are small. These labor-supply effects are allowed to influence the economy's long-run growth rate, but a 10 percent inflation turns out to reduce the growth rate by only 0.05 percent compared to a regime of price stability.

The various small distortions, however, combine to yield substantial estimates of the total cost of inflation. A 4 percent inflation like that experienced in the US since 1983 costs the economy 0.41 percent of output per year when currency is identified as the relevant definition of money and over 1 percent of output per year when M1 is defined as money. These higher estimates strengthen the case for making price stability the principal objective for monetary policy.

More generally, the findings demonstrate the usefulness of general equilibrium models for policy evaluation. In this case, a partial equilibrium approach to measuring the welfare cost of suboptimal policy grossly underestimates the true welfare effects. Only when all of the policy-induced distortions are considered together can reliable estimates be obtained.

## Appendix

This appendix derives the equilibrium conditions presented as Eqs. (15)–(18) in the text.

In the representative household's optimization problem, let $\mu_t$ and $\lambda_t$ be nonnegative multipliers on the constraints (3) and (4). Let $c_t^0(i)$ be the household's consumption of good $i$ at time $t$ if it purchases this good with money; let $c_t^1(i)$ be the household's consumption of good $i$ at time $t$ if it purchases this good on credit. The first-order conditions from the household's problem include

$$c_t^0(i) = c_t^0 = 1/(\lambda_t + \mu_t), \tag{A.1}$$

$$c_t^1(i) = c_t^1 = 1/\lambda_t, \tag{A.2}$$

$$c_t(i) = [1 - \xi_t(i)]c_t^0 + \xi_t(i)c_t^1, \tag{A.3}$$

$$\xi_t(i) = \begin{cases} 1 & \text{if} \quad \ln(c_t^1) - \lambda_t[c_t^1 + q_t(i)] \geq \ln(c_t^0) - (\lambda_t + \mu_t)c_t^0, \\ 0 & \text{otherwise,} \end{cases} \tag{A.4}$$

while the first-order condition from the intermediary's problem is

$$\xi_t^s(i) = \begin{cases} 1 & \text{if} \quad q_t(i) \geq w_t\gamma(i), \\ 0 & \text{otherwise.} \end{cases} \tag{A.5}$$

Eq. (A.5), along with (5), (8), and (14), implies that

$$q_t(i) = w_t\gamma(i), \tag{A.6}$$

for all $i$ such that $\xi(i) = 1$. Eqs. (A.1)–(A.4) and (A.6) then imply the existence of the borderline index $s_t$ satisfying

$$\gamma(s_t) = [\ln(\lambda_t + \mu_t) - \ln(\lambda_t)]/w_t\lambda_t, \tag{A.7}$$

such that

$$\xi_t(i) = 1, \quad c_t(i) = c_t^1 \quad \text{for} \quad i \leq s_t, \tag{A.8a}$$

$$\xi_t(i) = 0, \quad c_t(i) = c_t^0 \quad \text{for} \quad i > s_t. \tag{A.8b}$$

Eqs. (A.7) and (A.8) correspond to (15) and (16) in the text. Eq. (A.8) and the cash-in-advance constraint (3) imply Eq. (17) in the text. In light of (A.8) and the market-clearing conditions (11) and (14), Eq. (6) can be rewritten as Eq. (18) in the text.

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