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On the Welfare Cost of Inflation and the Recent Behavior of Money Demand

By Peter N. Ireland*

Inflation, brought under control in the early 1980s, remains subdued today. Still, the question remains: what cost does the Federal Reserve's well-established policy of low but positive inflation impose on the economy, when compared to the optimal monetary policy prescribed by Milton Friedman (1969), which calls for a deflation that makes the nominal interest rate equal to zero?

Robert E. Lucas, Jr. (2000), working in the tradition of Martin J. Bailey (1956) and Friedman (1969), addresses this question directly. Lucas's analysis juxtaposes two competing specifications for money demand. One, inspired by Allan H. Meltzer (1963), relates the natural logarithm of m, the ratio of nominal money balances to nominal income, to the natural logarithm of r, the short-term nominal interest rate, according to

$$\ln(m) = \ln(A) - \eta \ln(r), \tag{1}$$

where $A > 0$ is a constant and $\eta > 0$ measures the absolute value of the interest elasticity of money demand. The other, adapted from Phillip Cagan (1956), links the log of m instead to the level of r via

$$\ln(m) = \ln(B) - \xi r, \tag{2}$$

where $B > 0$ is a constant and $\xi > 0$ measures the absolute value of the interest semi-elasticity of money demand.

Figure 1 plots the log-log demand curve (1) and the semi-log demand curve (2) on the same graph, where the axes measure both m and r in levels. Lucas's (2000) preferred specifications set $\eta = 0.5$ in (1) and $\xi = 7$ in (2), then pin down the constants $A = 0.0488$ and $B = 0.3548$ so that $\ln(A)$ equals the average value of $\ln(m) + \eta \ln(r)$ and $\ln(B)$ equals the average value of $\ln(m) + \xi r$ in annual US data, 1900–1994. These same settings determine the curvature and horizontal placement of the two curves in Figure 1.

The graphs highlight how (1) and (2) describe very different money demand behavior at low interest rates: as r approaches zero, (1) implies that real balances become arbitrarily large, while (2) implies that real balances reach the finite satiation point B when expressed as a fraction of real income. Hence, as emphasized by Lucas (2000), these competing money demand specifications also have very different implications for the welfare cost of modest departures from Friedman's (1969) zero nominal interest rate rule for the optimum quantity of money.

* Boston College, Department of Economics, 140 Commonwealth Avenue, Chestnut Hill, MA 02467-3859 (e-mail: irelandp@bc.edu). I would like to thank Ed Nelson and an anonymous referee for extremely helpful comments and suggestions.

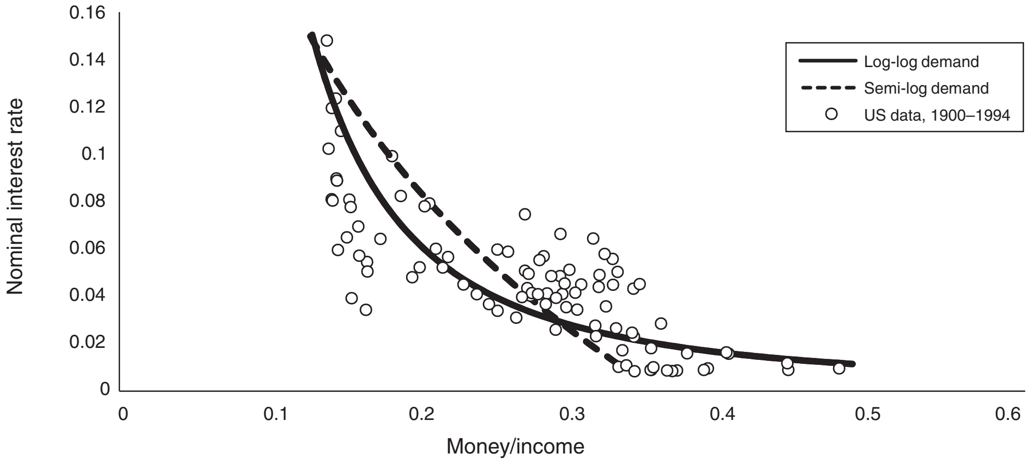

Figure 1. US Money Demand, 1900–1994

Bailey's (1956) traditional approach measures this welfare cost by integrating under the money demand curve as the interest rate rises from zero to $r > 0$ to find the lost consumer surplus, then subtracting off the seigniorage revenue rm to isolate the deadweight loss. Let $w(r)$ denote this welfare-cost measure, expressed as a function of r. Lucas (2000) shows that

$$w(r) = A\left(\frac{\eta}{1-\eta}\right)r^{1-\eta} \tag{3}$$

when money demand takes the log-log form (1) and

$$w(r) = \frac{B}{\xi}[1-(1+\xi r)e^{-\xi r}] \tag{4}$$

when money demand takes the semi-log form (2). If, as assumed by Lucas, the steady-state real interest rate equals 3 percent, so that $r = 0.03$ prevails under a policy of zero inflation or price stability, then (3) and (4) imply that this policy costs the economy the equivalent of 0.85 percent of income when money demand is log-log, but only 0.10 percent of income when money demand has the semi-log form. Likewise, an ongoing 2 percent inflation costs the economy 1.09 percent of income under (1) and (3), but only 0.25 percent of income under (2) and (4).

These calculations underscore the importance of discerning the appropriate form of the money demand function before evaluating alternative monetary policies, including those that generate very low but positive rates of inflation. Hence, Figure 1 also plots US data on the money-income ratio and the nominal interest rate from an annual sample extending from 1900 through 1994 that is constructed, as described below in the Appendix, to resemble closely the one used by Lucas (2000). Following Lucas, m is measured by dividing the M1 money stock by nominal GDP and r is measured by the six-month commercial paper rate. Based on the same comparison between these data and the plots of (1) and (2) shown in Figure 1, Lucas concludes that the log-log specification provides a better fit and thereby argues implicitly that the Federal Reserve could secure a substantial welfare gain for American consumers by abandoning its current, low-but-positive inflation policy and adopting the Friedman rule instead.

Some doubts about Lucas's (2000) argument arise, however, once one recognizes that the log-log specification appears to deliver a substantially better fit in Figure 1 thanks in large part to its ability to track data points in two extreme clusters: one group that lies farthest out along the x -axis, representing (m, r) pairs such that m exceeds 0.4, and another group that lies highest up along the y -axis, representing (m, r) pairs such that r exceeds 0.1. The first cluster of data points, with $m > 0.4$, comes from the period 1945 through 1949. Interest rates remained low during this period, as the Federal Reserve retained its policy, first adopted during World War II, of supporting the prices of US Treasury securities. Yet, compared to the actual wartime period from 1941 through 1944, interest rates moved slightly *higher* during 1945 through 1949 and, still, the money-income ratio moved sharply higher as well. Friedman and Anna Jacobson Schwartz (1963, 580–85) attribute this anomalous behavior of money demand to widespread fears, ultimately unfounded, of a return to 1930s-style deflation and depression following the end of hostilities. Meanwhile, the second cluster of data points, with $r > 0.1$, comes from 1979–1982 and 1984, following a period of financial deregulation and innovation, as well Stephen M. Goldfeld's (1976) famous "missing money" episode of money demand instability.

Viewed in one way, these two clusters of data points might be quite informative, since they reveal how the demand for M1 in the United States changed when, first, interest rates fell to very low levels in the late 1940s and then, later, interest rates reached historical highs in the late 1970s and early 1980s. But, all the same, one might wonder if Lucas's (2000) preferred log-log money demand specification owes much of its apparent success in tracking the data from Figure 1 to its ability to link—perhaps spuriously—two disparate and unusual episodes in US monetary history. And one might also wonder, more specifically, about the relevance of the data points from 1945 through 1949—a distant period when the US financial system and indeed the US economy as a whole looked very different from the way they appear now—to an exercise that evaluates Federal Reserve policy today.

Fortunately, new data have accumulated since the mid-1990s that quite usefully complement those used in Lucas's (2000) study and offer up a chance to check on the robustness of his results and conclusions. Importantly, these new data include observations from a much more recent episode from 2002 through 2004 that also features very low nominal interest rates. Hence, Figure 2 reproduces Figure 1 after updating Lucas's sample to run through 2006. The more recent data also cover a period when the development and proliferation of retail deposit sweep programs, involving banks' efforts to reclassify their checkable deposits as money market deposits and thereby avoid statutory reserve requirements, severely distort official measures of the M1 money stock. Since, as argued by Richard G. Anderson (2003b), these sweep operations take place behind the scenes, invisible to the eyes of most account holders, Figure 2 uses data on the M1RS aggregate, defined and constructed by Donald H. Dutkowsky and Barry Z. Cynamon (2003), Cynamon, Dutkowsky, and Barry E. Jones (2006), and Dutkowsky, Cynamon, and Jones (2006) by adding the value of swept funds back into the standard M1 figures, to measure the money-income ratio since 1994.

To focus more clearly on the recent behavior of money demand, Figure 2 distinguishes between the data from 1980–2006 and the data from 1900–1979, the breakpoint coinciding with both the arrival of Paul Volcker at the Federal Reserve Board and the implementation of the Depository Institutions Deregulation and Monetary Control Act of 1980 as key events marking the start of a new chapter in US monetary history. Strikingly, the data points from the post-1980 period also trace out what looks like a stable money demand relationship, but one that seems very different from the log-log specification preferred by Lucas (2000) based on his examination of the earlier data.

Even after correcting for the effects of retail sweep programs, money balances displayed only modest growth relative to income during the 2002–2004 episode of very low interest rates,

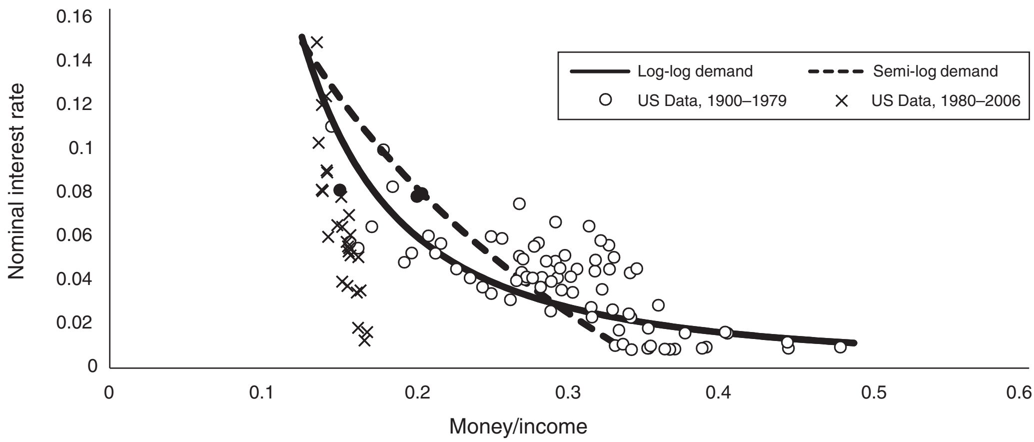

FIGURE 2. US MONEY DEMAND, 1900–2006

suggesting that the semi-log specification (2) with its finite satiation point may now provide a more accurate description of money demand. Furthermore, the new data points appear to trace out a demand curve that is far less interest-elastic than either of the two curves drawn in to track the earlier data from Figure 1. Both of these shifts, in functional form and toward a smaller (in absolute value) elasticity or semi-elasticity, work to reduce Lucas's (2000) estimate of the welfare cost of inflation. But, to make sure that the patterns appearing in Figure 2 are real and not optical illusions and to sharpen the quantitative estimate of the welfare cost of inflation implied by the recent behavior of money demand, the next section presents some more formal statistical results.

II. ... and the Recent Behavior of Money Demand

While Lucas's (2000) focus on a long historical time series extending back to the start of the previous century requires the use of annual data, the focus here on the post-1980 period allows for the use of readily available quarterly figures, again as described in the Appendix. Running from 1980:I through 2006:IV, the money-income ratio is measured by dividing the sweep-adjusted M1 money stock, the M1RS aggregate referred to above, by nominal GDP. And since the Federal Reserve discontinued its reported series for the six-month commercial paper rate in 1997, the three-month US Treasury bill rate serves instead as the measure of r; in any case, US Treasury bills come closer to matching the risk-free, nominally denominated bonds that serve as an alternative store of value in theoretical models of money demand.

Following most of the empirical literature on US money demand since R. W. Hafer and Dennis W. Jansen (1991) and Dennis L. Hoffman and Robert H. Rasche (1991), the econometric analysis of these data revolves around the ideas of nonstationarity and cointegration introduced by Robert F. Engle and C. W. J. Granger (1987). Specifically, a finding that the semi-log specification (2) describes a cointegrating relationship linking two nonstationary variables, the money-income ratio and the nominal interest rate, coupled with a finding that the log-log specification (1) fails to describe the same sort of relationship, provides formal statistical evidence supporting the more casual impressions gleaned from visual inspection of Figure 2 that the semi-log form offers a better fit to the post-1980 data.

Note that these statistical tests, which check first for nonstationarity in, and then cointegration between, the variables $\ln(m)$ and $\ln(r)$ in (1) and the variables $\ln(m)$ and r in (2), require one to adopt a somewhat schizophrenic view of the data since, in a linear statistical framework, the analysis of (1) requires $\ln(r)$ to follow an autoregressive process with a unit root, while the same analysis of (2) requires r to follow an autoregressive process with a unit root. Youngsoo Bae (2005) helps to cure this schizophrenia by providing a more detailed discussion of the case in which both (1) and (2) can be estimated under the common assumption that r follows an autoregressive process with a unit root, with (1) viewed as a nonlinear relationship between $\ln(m)$ and r and (2) viewed as a linear relationship between the same two variables. The analysis here, by contrast, follows Anderson and Rasche (2001) by putting the two competing specifications on equal footing ex ante, treating both as linear relationships linking $\ln(m)$ and $\ln(r)$ in one case and $\ln(m)$ and r in the other.

Table 1 displays results from applying the Phillips-Perron unit root test described by Peter C. B. Phillips and Pierre Perron (1988) and James D. Hamilton (1994, ch. 17) to each of the three variables: $\ln(m)$, $\ln(r)$, and r. The table reports values for $\hat{\mu}$ and $\hat{\rho}$, the intercept and slope coefficients from an ordinary least squares regression of each variable on a constant and its own lagged value, together with the Phillips-Perron test statistic Z_t, which corrects the conventional t-statistic for testing the null hypothesis of a unit root, $\rho = 1$, for serial correlation in the regression error using Whitney K. Newey and Kenneth D. West's (1987) estimator of the error variance. In particular, Table 1 reports Z_t as computed for values of the lag truncation parameter q, that is, the bound on the number of sample autocovariances used in computing the Newey-West estimate, ranging from 0 (imposing no serial correlation, in which case Z_t coincides with the more familiar t-statistic) to 8 (allowing for positive autocorrelations running out to eight quarters or two years). Critical values for Z_t appear under the heading "Case 2" in Hamilton's (1994, 763) Table B.6. None of these test statistics allows the null hypothesis of a unit root to be rejected, paving the way for tests of cointegration between pairs of these apparently nonstationary variables.

Intuitively, the Phillips-Ouliaris test for cointegration described by Phillips and S. Ouliaris (1990) and Hamilton (1994, ch. 19) uses ordinary least squares to estimate the intercept and slope coefficient in the linear relationship (1) linking the nonstationary variables $\ln(m)$ and $\ln(r)$ or (2) linking the nonstationary variables $\ln(m)$ and r, then applies a Phillips-Perron (1988) test to determine whether the regression error from the equation is stationary or nonstationary. In the case where the null hypothesis of a unit root in the error can be rejected, then either (1) or (2) represents a cointegrating relationship: a stationary linear combination of two nonstationary variables. Table 2 displays results associated with these Phillips-Ouliaris tests: the intercept and slope coefficients $\hat{\alpha}$ and $\hat{\beta}$ from a linear regression of the form (1) or (2), the slope coefficient $\hat{\rho}$ from a regression of the error term from (1) or (2) on its own lagged value (without a constant, since the error has mean zero), and the Phillips-Ouliaris statistic Z_t for values of the Newey-West (1987) lag truncation parameter q ranging again between 0 and 8. Critical values for Z_t so constructed appear under the heading "Case 2" in Hamilton's (1994, 766) Table B.9.

Confirming the apparent breakdown from Figure 2 of Lucas's (2000) preferred log-log specification in the post-1980 data, none of the tests summarized in Table 2's top panel rejects the null hypothesis of no cointegration between $\ln(m)$ and $\ln(r)$. On the other hand, all of the tests in the table's bottom panel reject their null of no cointegration between $\ln(m)$ and r at the 90 or 95 percent confidence level. Taken together, these results provide statistical evidence of a tighter money demand relationship of the semi-log form for the post-1980 period. And again confirming the visual impressions from Figure 2, the estimated semi-elasticity of 1.79 (in absolute value) for 1980–2006 stands far below Lucas's choice of 7 made to fit the data from 1900–1994.

Both of Lucas's (2000) specifications (1) and (2) impose a unitary income elasticity of money demand by relating the interest rate terms $\ln(r)$ and r to the log of the money-income ratio $\ln(m)$. To make sure that the failure of the Phillips-Ouliaris (1990) tests summarized in Table 2 to reject their null hypothesis of no cointegration between $\ln(m)$ and $\ln(r)$ does not stem directly from

TABLE 1—PHILLIPS-PERRON UNIT ROOT TEST RESULTS

$\ln(m)$	$\hat{\mu}$	$\hat{\rho}$	q	Z_t
	−0.0410	0.9778	0	−1.1923
			1	−1.3826
			2	−1.5262
			3	−1.6488
			4	−1.7269
			5	−1.7792
			6	−1.8165
			7	−1.8313
			8	−1.8256
$\ln(r)$	$\hat{\mu}$	$\hat{\rho}$	q	Z_t
	−0.1049	0.9682	0	−1.6449
			1	−1.7672
			2	−1.8595
			3	−1.9385
			4	−1.9943
			5	−2.0366
			6	−2.0697
			7	−2.0836
			8	−2.0863
r	$\hat{\mu}$	$\hat{\rho}$	q	Z_t
	0.0029	0.9361	0	−2.4602
			1	−2.5036
			2	−2.5028
			3	−2.5100
			4	−2.4994
			5	−2.4939
			6	−2.4932
			7	−2.4757
			8	−2.4604

Notes: Each panel reports $\hat{\mu}$ and $\hat{\rho}$, the intercept and slope coefficient from an ordinary least squares regression of the variable on a constant and its own lag, together with Z_t, the Phillips-Perron statistic corrected for autocorrelation in the regression error, computed using the Newey-West estimate of the error variance for various values of the lag truncation parameter q. The critical values for Z_t are reported by Hamilton (1994, Table B.6, 763): −2.58 (10 percent), −2.89 (5 percent), and −3.51 (1 percent).

the imposition of this additional constraint, Table 3 displays results of Phillips-Ouliaris tests applied to the more flexible specification that links the log of real money balances $\ln(M/P)$ to the log of real GDP $\ln(Y/P)$ and the log of the nominal interest rate $\ln(r)$ with the GDP deflator P used to convert both series for money M and income Y from nominal to real. Hence, the table shows the ordinary least squares estimates of the intercept $\hat{\alpha}$ together with the slope coefficients $\hat{\beta}_y$ and $\hat{\beta}_r$ that measure the income and interest elasticities of money demand. And, as before, the table shows values of the Phillips-Ouliaris statistic Z_t for values of the Newey-West (1987) lag truncation parameter q ranging between 0 and 8. In Table 3, however, the critical values for Z_t differ from those in Table 2, partly because the regression includes two right-hand-side variables instead of one, but also because the upward trend in the new right-hand-side variable $\ln(Y/P)$ requires that the entries from the "Case 3" panel of Hamilton's (1994, 766) Table B.9 be used in place of those from "Case 2" from before.

In Table 3, the point estimate $\hat{\beta}_y = 1.10$ of the income elasticity parameter exceeds unity, and estimate $\hat{\beta}_r = 0.057$ of the interest elasticity parameter declines when compared to the case shown in Table 2 where a unitary income elasticity is imposed. Nevertheless, the basic result

Table 2—Phillips-Ouliaris Cointegration Test Results

	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\rho}$	q	Z_t
$\ln(m) = \alpha - \beta \ln(r)$					
	−2.1474	0.0873	0.9351	0	−1.8768
				1	−2.0501
				2	−2.1720
				3	−2.3457
				4	−2.4447
				5	−2.5277
				6	−2.6090
				7	−2.6401
				8	−2.6430
$\ln(m) = \alpha - \beta r$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\rho}$	q	Z_t
	−1.7800	1.7944	0.8575	0	−3.1065*
				1	−3.1926*
				2	−3.1612*
				3	−3.2526*
				4	−3.2694*
				5	−3.3238*
				6	−3.4075**
				7	−3.4134**
				8	−3.4083**

Notes: Each panel reports $\hat{\alpha}$ and $\hat{\beta}$, the intercept and slope coefficient from the ordinary least squares regression of $\ln(m)$ on $\ln(r)$ or r; $\hat{\rho}$, the slope coefficient from an ordinary least squares regression of the corresponding regression error on its own lagged value; and Z_t, the Phillips-Ouliaris statistic for $\rho = 1$, corrected for autocorrelation in the residual, computed using the Newey-West estimate of the error variance for various values of the lag truncation parameter q. The critical values for Z_t are reported by Hamilton (1994, Table B.9, 766): −3.07 (10 percent), −3.37 (5 percent), and −3.96 (1 percent). Hence * and ** indicate that the null hypothesis of no cointegration can be rejected at the 90 and 95 percent confidence levels.

from Table 2—that none of the Phillips-Ouliaris (1990) tests rejects its null hypothesis of no cointegration—carries over to Table 3, confirming the robustness of that basic result and casting further doubt on the relevance of the log-log specification (1).

Table 4, meanwhile, builds on the success of the semi-log specification (2) by presenting "dynamic OLS" (DOLS) estimates of the parameters of the cointegrating relationship linking $\ln(m)$ and r. Each of the parameter estimates in this table comes from an ordinary least squares regression of $\ln(m)$ on a constant, the level of the nominal interest rate r, and p leads and lags of Δr, the quarter-to-quarter change in the nominal interest rate. Importantly, these dynamic regressions *assume* that the nonstationary variables $\ln(m)$ and r are cointegrated; unlike the static regressions from Table 2, they cannot be used to *test* the hypotheses of cointegration or no cointegration. On the other hand, James H. Stock and Mark W. Watson (1993) and Hamilton (1994, ch. 19) demonstrate that under the assumption of cointegration, the dynamic OLS estimates are asymptotically efficient and asymptotically equivalent to maximum likelihood estimates obtained, for example, through Søren Johansen's (1988) methods. In addition, conventional Wald test statistics formed from these dynamic OLS estimates have conventional normal or chi-squared asymptotic distributions, making it possible to draw familiar comparisons between the parameter estimates and their standard errors. Again as explained by Stock and Watson (1993) and Hamilton (1994), adding leads and lags of Δr to the estimated equations controls for possible correlation between the interest rate r and the residual from the cointegrating relationship linking $\ln(m)$ and r; however, any serial correlation that remains in the error term from the dynamic equation must still be accounted for when constructing standard errors for the DOLS estimates. Therefore, Table 4 reports DOLS estimates of the intercept and slope coefficients $\hat{\alpha}$ and $\hat{\beta}$ from the cointegrating relationship, together with standard errors s.e. $(\hat{\beta})$ for $\hat{\beta}$ computed using Newey

Table 3—Phillips-Ouliaris Cointegration Test Results

$\ln(M/P) = \alpha + \beta_y \ln(Y/P) - \beta_r \ln(r)$	$\hat{\alpha}$	$\hat{\beta}_y$	$\hat{\beta}_r$	$\hat{\rho}$	q	Z_t
	−2.9778	1.1030	0.0572	0.9258	0	−2.0663
					1	−2.2775
					2	−2.4447
					3	−2.6419
					4	−2.7549
					5	−2.8369
					6	−2.9096
					7	−2.9344
					8	−2.9224

Notes: The table reports $\hat{\alpha}$, $\hat{\beta}_y$, and $\hat{\beta}_r$, the intercept and slope coefficients from the ordinary least squares regression of $\ln(M/P)$ on $\ln(Y/P)$ and $\ln(r)$; $\hat{\rho}$, the slope coefficient from an ordinary least squares regression of the regression error on its own lagged value; and Z_t, the Phillips-Ouliaris statistic for $\rho = 1$, corrected for autocorrelation in the residual, computed using the Newey-West estimate of the error variance for various values of the lag truncation parameter q. The critical values for Z_t are reported by Hamilton (1994, Table B.9, 766): −3.52 (10 percent), −3.80 (5 percent), and −4.36(1 percent).

and West's (1987) estimator of the regression error variance for various values of the lag truncation parameter q.

The dynamic OLS estimates from Table 4 resemble quite closely their static counterparts shown in Table 2. In addition, for each value of p, the number of leads and lags of Δr included in the dynamic regressions, and for each value of q, the number of regression-error autocorrelations allowed for in the Newey-West (1987) variance estimator, a conventional comparison between a point estimate and its standard error can be made. In particular, the t-statistic $\hat{\beta}$/s.e. $(\hat{\beta})$ has the usual, normal asymptotic distribution, confirming that the estimated interest elasticity of money demand differs significantly from zero. The tight standard errors around the point estimates of 1.8 to 1.9 also confirm that interest semi-elasticity is significantly smaller in absolute value than Lucas's (2000) setting of 7.

Finally, Table 5 follows Table 3 by relaxing the assumption of a unitary income elasticity of money demand, but this time for the semi-log specification and using the dynamic OLS approach justified by the previous finding of cointegration between the money-income ratio and the level of the nominal interest rate. The table reports point estimates $\hat{\alpha}$, $\hat{\beta}_y$, and $\hat{\beta}_r$ of the intercept and slope coefficients from the cointegrating relationship linking the log of real money balances $\ln(M/P)$ to the log of real income $\ln(Y/P)$ and the level of the nominal interest rate r, when p leads and lags of the changes $\Delta \ln(Y/P)$ and Δr in real income and the interest rate are also included in the ordinary least squares regression. The table shows standard errors s.e. $(\hat{\beta}_y)$ and s.e. $(\hat{\beta}_r)$ for $\hat{\beta}_y$ and $\hat{\beta}_r$ as well, corrected for serial correlation using Newey and West's (1987) estimator of the regression error variance for various values of the lag truncation parameter q, and corresponding Wald (F) statistics for testing the null hypothesis of a unitary income elasticity maintained in the previous regressions.

Notably, while the point estimates of the income elasticity in Table 5 come in slightly higher than one, none of the Wald tests rejects its null hypothesis of a unitary income elasticity. And, as in Table 4, comparisons of $\hat{\beta}_r$ and s.e. $(\hat{\beta}_r)$ reveal that the estimates of the interest semi-elasticity are not only significantly different from zero, but also significantly smaller in absolute value than Lucas's (2000) setting of 7.

All of these results point to the semi-log specification (2) with a unitary income elasticity of money demand as providing the best description of the post-1980 data. Accordingly, Table 6 presents estimates of the welfare cost of inflation implied by the corresponding formula (4), based on the regression results shown previously in Tables 2 and 4. Since, as noted above, the

Table 4—Dynamic OLS Estimates

$\ln(m) = \alpha - \beta r$	$\hat{\alpha}$	$\hat{\beta}$	s.e. $(\hat{\beta})$	p	q
	−1.7731	1.8939	0.1387	1	2
			0.1697		4
			0.1887		6
			0.1996		8
	−1.7719	1.9013	0.1497	2	2
			0.1829		4
			0.2027		6
			0.2136		8
	−1.7732	1.8639	0.1660	3	2
			0.2025		4
			0.2248		6
			0.2372		8
	−1.7738	1.8261	0.1619	4	2
			0.1949		4
			0.2166		6
			0.2300		8

Notes: The table shows $\hat{\alpha}$ and $\hat{\beta}$, the constant and slope coefficients from the cointegrating vector linking $\ln(m)$ and r, obtained from a dynamic ordinary least squares regression of $\ln(m)$ on a constant, r, and p leads and lags of Δr, together with the standard error s.e. $(\hat{\beta})$ for $\hat{\beta}$, corrected for autocorrelation in the residual and computed using the Newey-West estimate of the error variance for various values of the lag trucation parameter q.

static and dynamic OLS estimates look quite similar, so do the implied welfare costs. Assuming, as before, that the steady-state real interest rate equals 3 percent, so that $r = 0.03$ corresponds to zero inflation, $r = 0.05$ corresponds to 2 percent annual inflation, and $r = 0.13$ corresponds to 10 percent annual inflation, the regression coefficients put the welfare cost of pursuing a policy of price stability as opposed to the Friedman (1969) rule at less than 0.015 percent of income, the cost of 2 percent inflation at less than 0.04 percent of income, and the cost of 10 percent inflation at less than 0.25 percent of income. These welfare cost estimates lie far below those computed by Lucas (2000) and bring the analysis full circle, back to Figures 1 and 2 and the apparent steepening and leftward shift of the money demand function in the years since 1980. Interestingly, these figures also provide estimates of the cost of 10 percent inflation compared to price stability, $w(0.13) - w(0.03)$, that lie between 0.20 and 0.22 percent of income, numbers that are still smaller than, but resemble more closely, Stanley Fischer's (1981) estimate of 0.30 percent of income and Lucas's (1981) estimate of 0.45 percent of income.

These results suggest that the Federal Reserve's current policy, which generates low but still positive rates of inflation, provides an adequate approximation in welfare terms to the alternative of moving all the way to Friedman's (1969) deflationary rule for a zero nominal interest rate. Before closing, however, it should be emphasized that these welfare cost estimates account for only the money demand distortion brought about by positive nominal interest rates. Michael Dotsey and Ireland (1996) demonstrate that, in general equilibrium, other marginal decisions can also be distorted when inflation rises, having an impact on both the level and growth rate of aggregate output, while Martin Feldstein (1997) argues that the interactions between inflation and a tax code that is not completely indexed can add substantially to the welfare cost of inflation. To the extent that these additional sources of inefficiency remain present in the post-1980 US economy, there will of course be larger gains to reducing inflation below its current low level.

Two extensions to the analysis here and in Lucas (2000) immediately suggest themselves and, indeed, were pointed to originally by Lucas himself. First, this study follows Lucas (2000) by

Table 5—Dynamic OLS Estimates

$\ln(M/P) = \alpha + \beta_y \ln(Y/P) - \beta_r r$	$\hat{\alpha}$	$\hat{\beta}_y$	s.e. $(\hat{\beta}_y)$	$\hat{\beta}_r$	s.e. $(\hat{\beta}_r)$	p	q	$W(\beta_y = 1)$
	−2.0668	1.0329	0.0325	1.7066	0.2555	1	2	1.0194
			0.0398		0.3122		4	0.6827
			0.0443		0.3476		6	0.5509
			0.0468		0.3673		8	0.4934
	−1.9947	1.0254	0.0329	1.6952	0.2677	2	2	0.5965
			0.0400		0.3260		4	0.4023
			0.0444		0.3618		6	0.3267
			0.0470		0.3828		8	0.2918
	−2.0964	1.0361	0.0348	1.5159	0.3100	3	2	1.0716
			0.0423		0.3760		4	0.7281
			0.0468		0.4162		6	0.5943
			0.0496		0.4411		8	0.5292
	−2.1753	1.0440	0.0336	1.4736	0.3101	4	2	1.7126
			0.0398		0.3676		4	1.2189
			0.0439		0.4051		6	1.0033
			0.0466		0.4298		8	0.8915

Notes: Each panel shows $\hat{\alpha}$, $\hat{\beta}_y$, and $\hat{\beta}_r$, the constant and slope coefficients from the cointegrating vector linking $\ln(M/P)$, $\ln(Y/P)$, and r, obtained from a dynamic ordinary least squares regression of $\ln(M/P)$ on a constant, $\ln(Y/P)$, r, and p leads and lags of $\Delta\ln(Y/P)$ and Δr. The standard errors s.e. $(\hat{\beta}_y)$ and s.e. $(\hat{\beta}_r)$ for $\hat{\beta}_y$ and $\hat{\beta}_r$ are corrected for autocorrelation in the residual and computed using the Newey-West estimate of the error variance for various values of the lag trucation parameter q, and the Wald (F) statistic $W(\beta_y = 1)$ can be used to test the null hypothesis of a unitary long-run income elasticity of money demand. The critical values for $W(\beta_y = 1)$ are reported by Hamilton (1994, Table B.2, 754): 2.71 (10 percent), 3.84 (5 percent), and 6.63 (1 percent).

using assumptions about the functional form of the money demand curve, justified by observations on the behavior of money demand at low, but still positive, interest rates, to draw inferences about the behavior of money demand as those interest rates approach zero. The key issue is not so much whether the demand for money depends on the logarithm or the level of the nominal interest rate but, instead, whether there exists some finite satiation point that places a limit on money demand under the Friedman (1969) rule. This empirical strategy makes the data from the most recent episode, from 2002 through 2004, of low nominal interest rates in the United States particularly important here in the same way that, as argued above, data from the earlier episode from 1945 through 1949 are for the conclusions in Lucas (2000). Finding additional sources of information about the limiting behavior of money demand as interest rates approach zero, whether from time-series data from other economies or from cross-sectional data as suggested by Casey B. Mulligan and Xavier Sala-i-Martin (2000), remains a critical task for sharpening existing estimates of the welfare cost of modest rates of inflation. Second, the analysis here and in Lucas (2000) uses M1 as the measure of money, based on the idea that this narrow aggregate reflects most closely the medium of exchange role that money plays in theory. Broadening the empirical focus by examining how the demand for other liquid assets behaves under very low nominal interest rates, and the theoretical focus by deriving the implications of this behavior for estimates of the welfare cost of inflation, perhaps through the Divisia approach to monetary aggregation pioneered by William A. Barnett (1980), remains another critical task for future research.

Appendix: Data Sources

The annual data displayed in Figures 1 and 2 come from sources identical or very closely comparable to those used by Lucas (2000). To measure money, figures on M1 for 1900–1914 are taken from the US Bureau of the Census (1960, Series X-267). Figures on M1 for 1915–1958 are

Table 6—Welfare Cost Estimates

Regression	$B = \exp(\hat{\alpha})$	$\xi = \hat{\beta}$	Welfare cost (percent of income) Zero percent inflation $w(0.03)$	Two percent inflation $w(0.05)$	Ten percent inflation $w(0.13)$
Static OLS (Table 2)	0.1686	1.7944	0.0131	0.0356	0.2192
Dynamic OLS, $p = 1$ (Table 4)	0.1698	1.8939	0.0139	0.0377	0.2310
Dynamic OLS, $p = 2$ (Table 4)	0.1700	1.9013	0.0140	0.0379	0.2320
Dynamic OLS, $p = 3$ (Table 4)	0.1698	1.8639	0.0137	0.0372	0.2279
Dynamic OLS, $p = 4$ (Table 4)	0.1697	1.8261	0.0134	0.0365	0.2239

Notes: Each row presents estimates of the welfare costs of 0, 2 percent, and 10 percent annual inflation based on the regression results from Tables 2 and 4, using the formula (4) for the money demand specification (2) with intercept $\ln(B) = \hat{\alpha}$ and semi-elasticity $\xi = \hat{\beta}$ (in absolute value). The welfare cost calculations assume that the steady-state annual real interest rate equals 3 percent, so that $r = 0.03$ under price stability, $r = 0.05$ under 2 percent annual inflation, and $r = 0.13$ under 10 percent annual inflation.

taken from Anderson (2003a, Table 3, columns 3 and 10) and come, originally, from Friedman and Schwartz (1970) for 1915–1946 and Rasche (1987, 1990) for 1947–1958. Figures on M1 for 1959–2006 are taken from the Federal Reserve Bank of St. Louis FRED database and are adjusted from 1994 onward by adding back into M1 the funds removed by retail deposit sweep programs using estimates described by Cynamon, Dutkowsky, and Jones (2006).

To measure nominal income, figures on nominal GDP for 1900–1928 are constructed by taking John W. Kendrick's (1961, Table A-III, column 5) series for real GDP and multiplying it by a series for the deflator constructed by dividing nominal GNP (Table A-IIb, column 11) by real GNP (Table A-III, column 1). Lucas (2000), too, uses the deflator for GNP to translate Kendrick's figures for real GDP into a corresponding series for nominal GDP. Although his source for the deflator is the US Bureau of the Census (1960, Series F-5), the numbers from that table resemble quite closely those that come directly from Kendrick's (1961) monograph. Figures on nominal GDP for 1929–2006 come from the FRED database.

Finally, to measure the nominal interest rate, data on the six-month commercial paper rate are taken from Friedman and Schwartz (1982, Table 4.8, column 6) for 1900–1975 and from the *Economic Report of the President* (2003, Table B-73) for 1976–1997. The Federal Reserve stopped publishing the interest rate series reported in this last source in 1997; hence, the interest rate for 1998–2006 is the three-month AA nonfinancial commercial paper rate, drawn from the FRED database.

The quarterly, post-1980 data used in the econometric analysis summarized in Tables 1–6 all come from the Federal Reserve Bank of St. Louis FRED database, but the series for M1 is adjusted by adding back the funds removed by retail deposit sweep programs using estimates described by Cynamon, Dutkowsky, and Jones (2006): the money stock is therefore measured by their M1RS aggregate. Nominal GDP again measures income, and the three-month US Treasury bill rate measures the nominal interest rate. Finally, in the regressions that use real money balances and real GDP independently instead of together in the form of the money-income ratio, the nominal series for money and income are both divided by the GDP deflator.

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