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Money growth and inflation in the Euro Area, UK, and USA: measurement issues and recent results

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Abstract

This paper identifies several ways in which “measurement matters” in detecting quantity-theoretic linkages between money growth and inflation in recent data from the Euro Area, United Kingdom, and USA. Elaborating on the “Barnett critique,” it uses Divisia aggregates in place of their simple-sum counterparts to gauge the effects that monetary expansion or contraction is having on inflationary pressures. It also uses one-sided time series filtering techniques to track, in real time, slowly shifting trends in velocity and real economic growth that would otherwise weaken the statistical money growth-inflation relationship. Finally, it documents how measures of inflation based on GDP were distorted severely, especially in the EA and UK, during the 2020 economic closures. Using measures based on consumption instead, estimates from the P-star model confirm that changes in money growth have strong predictive power for subsequent movements in inflation.

Keywords: Barnett critique; Divisia monetary aggregates; inflation; P-star model; quantity theory of money; velocity of money

1. Introduction

Inflation jumped higher during 2021 and 2022, reaching levels not seen in four decades. Fig. 1 shows that this jump was preceded, in the Euro Area (EA), United Kingdom (UK), and USA, by an equally striking acceleration in money growth. Both the co-movement in money growth and inflation and temporal lead of money growth over inflation are explained easily by the quantity theory of money, summarized by Milton Friedman’s (1991), p. 16) famous dictum that “inflation is always and everywhere a monetary phenomenon.”

Fig. 1 also suggests, however, that quantity-theoretic links between money growth and inflation appear strongest during episodes like that between 2020 and the present, during which both variables exhibit large movements. The connection, by contrast, seems weaker when money growth and inflation are lower and more stable.¹ Yet this fact, too, can be explained in quantity-theoretic terms, with reference to the equation of exchange $MV = PY$. This identity links changes in the money stock M to changes in the aggregate price level P , but also makes clear that money-price connections in the data may be disrupted by movements in velocity V , reflecting shifts in the demand for money relative to spending, and the transactions variable Y , usually taken to be real GDP, reflecting disturbances impacting on the economy from the supply side.

These considerations point to the need for a statistical model that controls for anticipated movements in V and Y and thereby isolates the quantity-theoretical effects that changes in money growth have on inflation at any given point in time. One such model—the “P-star” model—was developed by economists at the Federal Reserve in the late 1980s under the direction of then Chair Alan Greenspan. Hallman et al. (1991) present the P-star model to an academic audience and test the model’s implications with quarterly data from 1955:1 through 1988:4 and with annual data

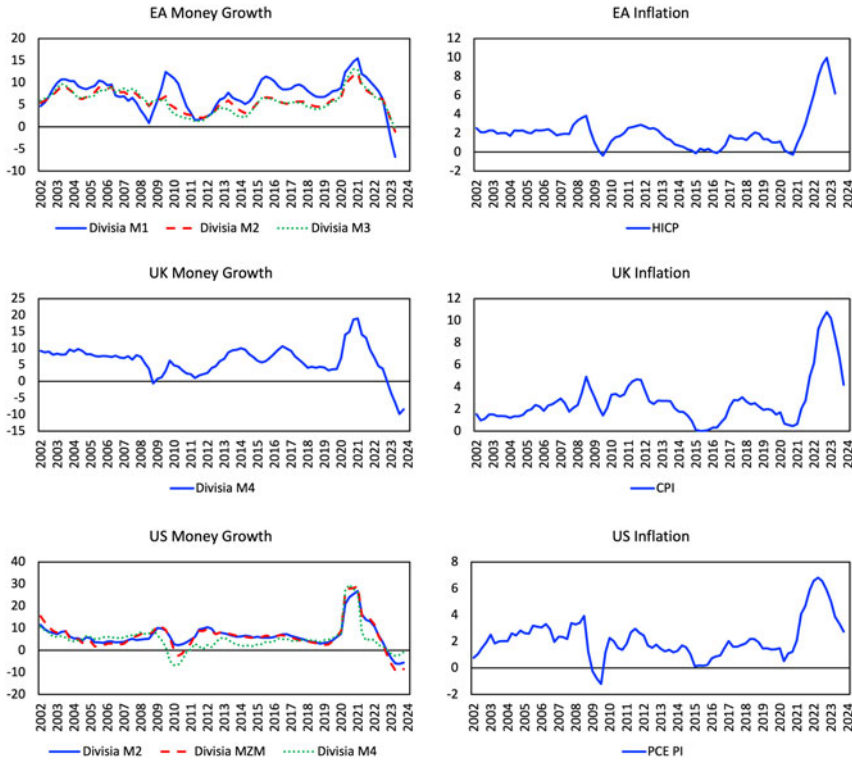


Figure 1. Money growth and inflation. Each panel shows year-over-year percentage changes in the indicated variable.

running back to 1870. This paper modifies and extends their analysis so as to apply the P-star model to the most recent data from the USA and from the EA and UK as well.

After briefly surveying previous work that has used the P-star model to interpret EA, UK, or USA data from older samples and then outlining the details of the P-star model itself, this paper turns its attention to several issues that show how “measurement matters” when it comes to detecting evidence of quantity-theoretic links between money growth and inflation that the model is designed to capture. Taken together, these issues apply to all four variables from the equation of exchange: M , V , P , and Y .

The first issue concerns the appropriate choice of monetary aggregate M . Barnett (1980) observes that conventional monetary aggregates computed as “simple sums” of the nominal values outstanding of their component assets are flawed measures of monetary or liquidity service flows received by their owners when those component assets are imperfect substitutes, a fact easily confirmed when those assets pay interest at different rates yet are all held in private agents’ portfolios. Barnett (1980) goes on to show how economic aggregation and statistical index number theory can be combined to produce Divisia aggregates of monetary services that improve, conceptually, on their simple-sum counterparts.² Chrystal and MacDonald (1994, p. 76) coin the phrase “Barnett critique” to refer to the implication that apparently fragile statistical linkages between money and other key macroeconomic variables will become much stronger when Divisia aggregates replace simple sums in econometric work. Their paper, together with Belongia (1996) and Hendrickson (2014), establish the empirical relevance of the Barnett critique by demonstrating that previously published results casting doubt on the explanatory or predictive power of money are often reversed just by replacing a simple-sum aggregate with the corresponding Divisia measure.

Also consistent with the Barnett critique, the results here provide strong evidence of quantity-theoretic links between Divisia money growth and inflation via the P-star model, regardless of the level of monetary aggregation, that is, the range of assets included in the composite. Across all three economies studied here, however, the central bank of only one—the Bank of England—provides an “official” series for Divisia money; this series for the UK is described by Fisher et al. (1993), Hancock (2005), and Berar and Owladi (2013).³ Thus, this paper relies on Divisia monetary aggregates constructed by Darvas (2014, 2015) for the EA and Barnett et al. (2013) for the USA in estimating and testing the implications of the P-star model.

The second issue involves the estimation of V^* , the long-run or equilibrium level to which the velocity of money is expected to return. Hallman et al. (1991) take V^* to be a constant, based on the stability of M2 velocity over the 1955–1988 period that is their primary focus. As discussed by Orphanides and Porter (2000), however, USA M2 velocity moved abruptly higher shortly after the publication of Hallman, Porter, and Small’s original article, throwing the model’s predictions off track.⁴ Moreover, as discussed by Bordo & Duca (2023), the velocities of Divisia monetary aggregates for the USA exhibit shifting long-run trends even in data from before 1990 and, as shown below, velocities of Divisia aggregates for the EA and UK display long-run trends as well. These considerations motivate the more flexible approach taken here, which attempts to track movements V^* , in real time, using a one-sided time series filter.

The third and final issue relates to the choice of transactions variable and the corresponding price aggregate referred to by the variables Y and P on the opposite side of the equation of exchange. In developing the P-star model, Hallman et al. (1991) use real GNP as their measure of Y and the GNP deflator as their measure of P . These are perhaps the most natural choices, as nominal GNP represents the broadest aggregate of nominal spending with which to measure PY . On the other hand, the central banks of all three economies studied here target consumer price inflation more closely. What’s more, as shown below, measures of the deflator for aggregate output appear to have been greatly distorted, especially in EA and UK, due to supply disruptions associated with the 2020 economic closures. An additional contribution of this paper is to show that, as a consequence, the P-star model fits the data much better when aggregate consumption C replaces aggregate output Y as the transactions variable and the consumption deflator replaces the output deflator in measuring the price level. Given the magnitude of the distortions—especially, again, in the EA and UK data—this same observation will likely apply to other empirical studies attempting to explain the behavior of inflation since 2020.⁵

Having discussed these issues relating to measurement, the paper goes on to test the P-star model, producing as a useful byproduct time series estimates of the P-star “price gap” that indicate, for each economy, how monetary policy has applied both upward and downward pressure on inflation in each of the three economies over sample periods extending through the present. The paper concludes, therefore, with a discussion of what the P-star model reveals about the stance of monetary policy in the EA, UK, and USA today.

2. Literature review

As discussed above, this paper uses Hallman et al.’s (1991) P-star model to detect statistical links between money growth and inflation in the EA, UK, and USA over sample periods that include the most recent episode of rapid money growth and inflation since 2020. Humphrey (1989) traces the P-star model’s intellectual origins back through the writings of David Hume, Henry Thornton, Thomas Attwood, Irving Fisher, Holbrook Working, Carl Snyder, Milton Friedman, Michael Darby, and William Poole, all of whom emphasized that the quantity-theoretic link between money growth and inflation appears with a lag, so that movements in money growth can be used to forecast future movements in inflation. Table 1 lists previous work that has applied the P-star model to data from the EA, UK, and USA. As indicated, most of these studies use conventional,

Table 1. Previous work with the P-star model

Economy	Monetary aggregation method	
	Simple sum	Divisia
EA	Groeneveld (1998)	Mäki-Frän̄ti (2007)
	Scheide and Trabandt (2000)*	Belongia and Ireland (2022)*
	Altimari (2001)*	
	Trecroci and Vega (2002)	
	Gerlach and Svensson (2003)*	
	Jansen (2004)*	
	Löchel and Polleit (2005)*	
	Czudaj (2011)*	
	Stavrev and Berger (2012)*	
UK	Hall and Milne (1994)	Bissoondeal et al. 2014
US	Moosa (1998)	Belongia and Ireland (2015, 2017)
	Lee (1999)	El-Shagi et al. (2015)*
	Orphanides and Porter (2000)	Ireland (2023)
	El-Shagi (2011)*	
	El-Shagi and Giesen (2013)*	
	Kamal (2014)*	
	Cronin (2018)	
	Moosa and Al-Nakeeb (2020)*	

Note: References marked with * use consumption-based measures of inflation; others use measures based on the GDP deflator.

simple-sum monetary aggregates, but a few emphasize that Divisia monetary aggregates provide stronger signals of the effects that money growth is having on inflation.⁶

As also noted above, this paper uses data on the Divisia monetary aggregates constructed and made publicly available by Darvas (2014, 2015) for the EA, the Bank of England for the UK, and Barnett et al. (2013) for the USA. Other work constructing Divisia monetary aggregates includes Reimers (2002), Stracca (2004), Barnett and Gaekwad (2018), and Brill et al. (2021) for the EA, Bissoondeal et al. (2010), Drake and Fleissig (2006, 2008), and Ezer (2019) for the UK, and Anderson et al. (1997a, b, c) and Anderson and Jones (2011) for the USA.

Table 1 indicates, as well, that a number of previous studies depart from Hallman et al. (1991) original analysis by testing the P-star model using consumption-based measures of inflation instead of measures based on GDP. While most of these studies use the consumer price index, Altimari (2001) and Jansen (2004) use the private consumption deflator—the same choice favored here. Replacing data on the GDP deflator with data on the CPI or the consumption deflator can be justified based on the observation that the European Central Bank, the Bank of England, and the Federal Reserve all focus on consumption-based measures of inflation. The results below demonstrate, however, that statistical considerations also play a role: the P-star model draws much stronger links between money and prices when the consumption deflator replaces the GDP deflator in measuring inflation.

Finally, in extending its application of the P-star model through the post-2020 period, this paper joins a few others in taking a quantity-theoretic view of the increase in inflation observed in all three major economies over this recent episode. Most notably, Castañeda and Congdon

(2020) provide a prescient early warning that the acceleration in money growth would lead to higher inflation through quantity-theoretic channels. As discussed here and in Greenwood and Hanke (2021), Ireland (2022, 2023), Bordo and Duca (2023), Borio et al. (2023), Congdon (2023), Hendrickson (2023), Reynard (2023), and Castañeda and Cendejas (2024), this prediction proved remarkably accurate. This work points to the usefulness of the quantity theory in general and the P-star model in particular, in gauging today whether monetary policy in the EA, UK, and USA has adjusted sufficiently to bring inflation back down to acceptable levels and in anticipating future inflationary trends in these same major economies.

3. The P-star model

The P-star model is based on the quantity theory of money and therefore takes as its starting point the equation of exchange, now written in more detail as

$$M_t V_t^y = P_t^y Y_t, \tag{1}$$

where all variables are indexed by time t so as to allow the model to be estimated and tested with quarterly data. In (1), M_t is a monetary aggregate and, updating Hallman et al. (1991), the transactions variable Y_t is taken to be aggregate output as measured by real GDP.⁷ The y superscript attached to the remaining variables is to emphasize that, with output as the transactions variable, the GDP deflator becomes the relevant measure of the aggregate price level P_t^y and monetary velocity V_t^y is computed by dividing nominal GDP $P_t^y Y_t$ by M_t .

By itself, of course, the equation of exchange holds as a identity; that is, monetary velocity is defined to make the equality exact. The P-star model, however, adds assumptions that give (1) predictive content and testable implications. It does so by defining the variable

$$P_t^{y*} = \frac{M_t V_t^{y*}}{Y_t^*} \tag{2}$$

that gives the model its name. In (2), V_t^{y*} and Y_t^* denote the natural, or equilibrium, levels to which velocity V_t^y and real GDP Y_t are expected to return in the long run. Thus, P_t^{y*} (“P-star”) represents, likewise, the long-run equilibrium nominal price level implied by the current level of the monetary aggregate M_t towards which the actual price level P_t^y measured by the GDP deflator will converge as velocity and real GDP return to their own long-run levels. Consistent with the quantity-theoretic views described by the authors surveyed by Humphrey (1989), therefore, the P-star model allows an increase in money M_t to stimulate real spending and thereby increase Y_t or to be held as excess cash balances relative to nominal spending as reflected in a decrease in V_t^y in the short run. In the long run, however, the increase in money should be reflected in a proportional increase in the aggregate nominal price level as these short-run real effects wear off.

Hallman et al. (1991) test this implication by estimating the regression

$$\Delta \pi_t^y = \alpha + \sum_{j=1}^q \beta_j \Delta \pi_{t-j}^y + \gamma (P_{t-1}^{y*} - P_{t-1}^y) + \varepsilon_t. \tag{3}$$

In (3), $\pi_t^y = 400[\ln(P_t^y) - \ln(P_{t-1}^y)]$ denotes the quarterly inflation rate, expressed in annualized, percentage-point terms, and $\Delta \pi_t^y = \pi_t^y - \pi_{t-1}^y$ the corresponding change in inflation. The lagged “price gap” variable $P_{t-1}^{y*} - P_{t-1}^y = 100[\ln(P_{t-1}^{y*}) - \ln(P_{t-1}^y)]$ is computed as the percentage-point gap between the equilibrium and actual levels P_{t-1}^{y*} and P_{t-1}^y . Once more, the y subscript attached to all of these variables emphasizes that both inflation and the price gap are measured using data on real GDP and the GDP deflator.

From the regression in (3), an estimated value of γ that is positive and statistically significant will confirm that inflation accelerates when the price gap is positive, so that P_{t-1}^y can converge to P_{t-1}^{y*} from below, and that inflation decelerates when the price gap is negative, so that P_{t-1}^y can converge to P_{t-1}^{y*} from above. In this case, the P-star price gap is a useful quantity-theoretic indicator of the effects that past money growth is expected to have on future inflation. The past changes of inflation, also included on the right-hand side of (3), allow these effects of money growth on inflation to appear more smoothly and with a longer lag than implied by the single lagged price gap alone.

4. Measurement matters

Several questions and problems arise in measuring all of the variables that appear in (1)–(3). First, the Barnett critique dictates the choice of Divisia over simple-sum monetary aggregates, here as in all other empirical studies that require measures of the money stock. For the EA, Darvas (2014, 2015) provides series on Divisia money at three levels of aggregation, corresponding to the European Central Bank’s simple-sum M1, M2, and M3 measures.⁸ In particular, M1 includes currency and overnight deposits, M2 adds selected savings and time deposits, and M3 adds repurchase agreements, money market mutual fund shares, and short-term debt securities issued by the banking sector.⁹ As discussed by Darvas (2014) and in more detail in European Central Bank (2019), the ECB adjusts its simple-sum monetary aggregates to construct “notional outstanding stocks” that correct for one-time shifts associated with “reclassifications, revaluations, and exchange rate adjustments” of financial institutions’ balance sheet items. Darvas provides similarly adjusted series for his Divisia aggregates as well; these notional stocks are used here, for the changing-composition EA economy. These quarterly data run from 2001:1 through 2023:2.

For the UK, the Bank of England provides Divisia series for the private non-financial corporations and household sector M4 aggregate, which includes notes and coin, non-interest-bearing deposits, and interest-bearing sight and time deposits. These series are described by Fisher et al. (1993), Hancock (2005), Berar and Owladi (2013), and Fleissig and Jones (2024). They run from 1977:1 through 2023:4.

For the USA, Barnett et al. (2013) construct series for Divisia money at various levels of aggregation, three of which are considered here.¹⁰ Divisia M2 includes the same assets covered by the Federal Reserve’s official, simple-sum M2 aggregate: currency, checking and savings deposits including money market deposit account balances, retail money market mutual fund shares, and small time deposits. Divisia MZM, originally referred to as “nonterm M3” by Motley (1988) and renamed “money, zero maturity” by Poole (1994, p. 104), subtracts the time deposit component from M2 but adds institutional money market mutual fund shares. Divisia M4 adds small time deposits back to MZM and then includes large time deposits, overnight and term repurchase agreements, commercial paper, and USA Treasury bills as well. Though much broader than any of the other aggregates studied here, Divisia M4 has been shown by Keating et al. (2019) and Dery and Serletis (2021) to have especially high predictive power for real economic activity and inflation in recent years, reflecting the increased importance of nonbank financial intermediaries—so-called “shadow banks”—in the US financial system. These data run from 1967:1 through 2023:4.

Second, although Hallman et al. (1991) took equilibrium velocity V_t^{y*} as a constant, based on the stability of fluctuations in simple-sum M2 velocity around its mean over their 1955–1988 sample period, M2 velocity moved sharply higher shortly after the publication of their article; as discussed by Orphanides and Porter (2000), this shift in velocity threw off track the predictions of the model as originally formulated. Moreover, as discussed by Bordo and Duca (2023) and as will be seen here below, the absence of a long-run trend in the velocity of simple-sum M2 before 1990 is a feature not shared by any of the Divisia monetary aggregates used here. Instead, velocities

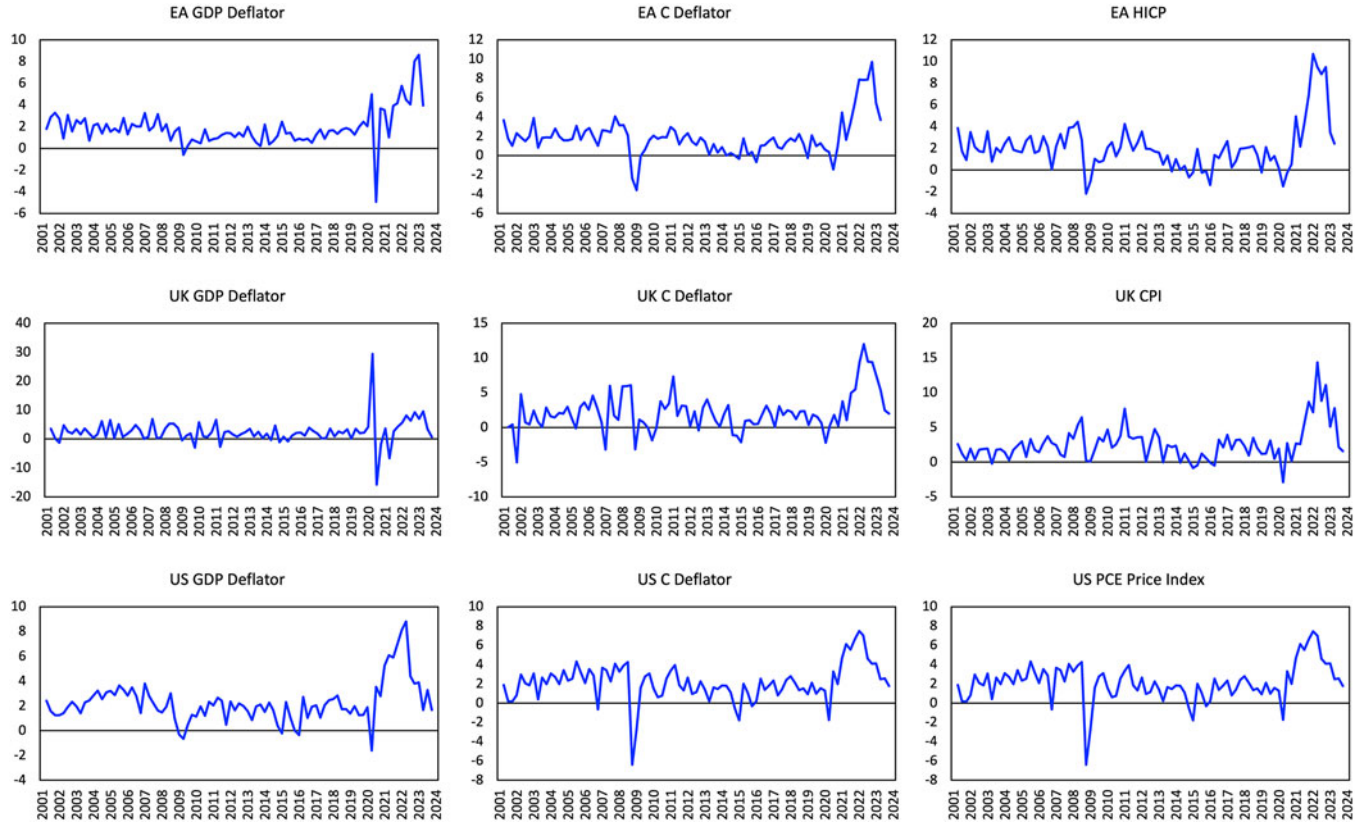


Figure 2. Inflation. Each panel shows annualized quarter-to-quarter percentage changes in the indicated price index.

Table 2. Quarter-to-quarter percentage changes in deflators for GDP and its components

	Euro area					
	Y	C	I	G	EX	IM
2020:1	0.5	0.2	0.1	1.2	-0.4	-0.8
2020:2	1.2	0.1	-1.8	4.1	-1.8	-3.5
2020:3	-1.2	-0.4	1.0	-3.8	0.3	1.1
2020:4	0.9	0.2	2.2	1.2	0.4	0.4
	UK					
	Y	C	I	G	EX	IM
2020:1	1.1	-0.5	0.8	2.9	1.0	-0.7
2020:2	7.4	0.1	0.9	36.5	-1.8	-1.3
2020:3	-3.9	0.5	-0.3	-15.4	0.4	0.3
2020:4	-0.4	0.1	-1.1	-2.6	1.1	2.3
	USA					
	Y	C	I	G	EX	IM
2020:1	0.5	0.3	0.6	0.9	-0.5	-0.3
2020:2	-0.4	-0.4	-0.1	-0.1	-4.8	-3.3
2020:3	0.9	0.8	0.7	1.0	3.1	2.1
2020:4	0.7	0.5	0.6	1.1	1.6	0.8

computed with Divisia money for the USA display slowly shifting trends: upwards through the 1990s and downwards since 2000. Velocities of EA and UK Divisia money, meanwhile, generally trend downwards.

Belongia and Ireland (2015, 2017, 2022) and Ireland (2023) show that the one-sided variant of the Hodrick-Prescott (HP) filter, described by Stock and Watson (1999, p. 301), does an adequate job of tracking movements in trend velocity for the EA and USA.¹¹ That approach is used by extension, here, for the UK as well. Application of the same one-sided HP filter also provides an estimate of the trend in the transactions variable Y_t . Importantly, the one-sided property of this filter implies that only past data are used to estimate the values of V_t^{y*} and Y_t^* in (2). This allows the corresponding lagged price gap measure $p_{t-1}^{y*} - p_{t-1}^y$ to be computed in real time and used in practice as a signal of future inflationary pressures.

Third, real GDP, as the broadest aggregate of goods and services produced and sold, usually appears to be the natural choice for the transactions variable in the equation of exchange. This conventional choice, however, raises problems when applied, especially to the EA and UK economies, over the period since 2020. To illustrate, Fig. 2 compares the behavior of three measures of inflation, measured by annualized, quarter-to-quarter changes in three aggregate price indices, for each economy. The left-hand column displays plots of inflation based on the GDP deflator—the aggregate price index implied by the choice of real GDP as the transactions variable in (1)–(3). The center column shows plots of inflation based on the deflator for the consumption component of GDP instead. Finally, the right-hand column shows plots of the inflation rate that serves as the official target for each central bank: based on the harmonized index of consumer prices for the EA, the consumer price index (CPI) for the UK, and the price index for personal consumption expenditures for the USA.

Although measures of inflation based on the consumption deflator resemble quite closely the measures targeted by each central bank, for the EA and UK especially, measures of inflation based

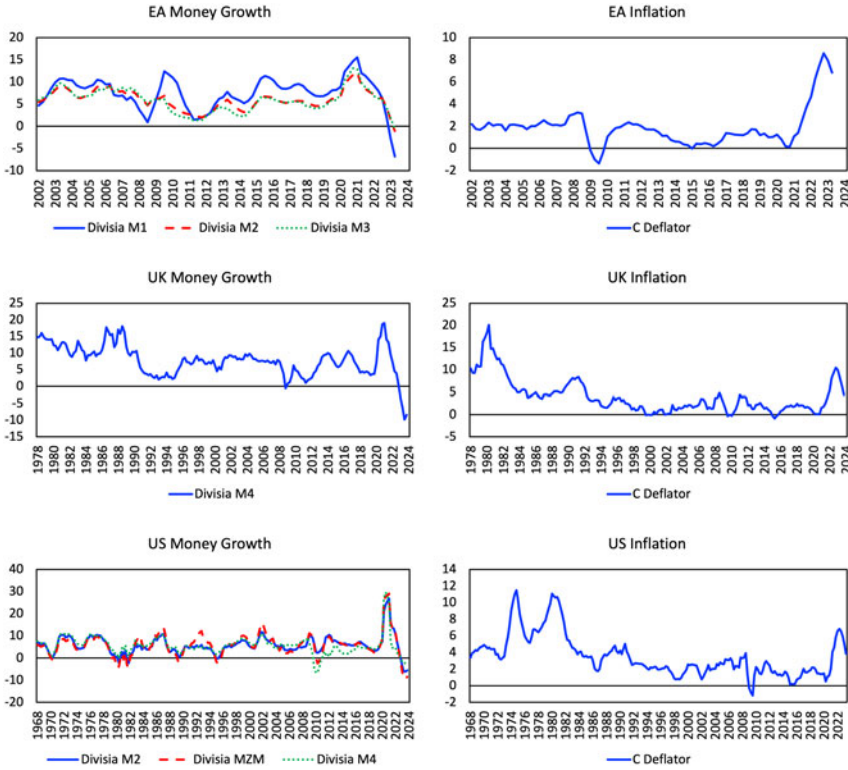


Figure 3. Money growth and inflation. Each panel shows year-over-year percentage changes in the indicated variable.

on the GDP deflator behave quite differently around the time of the 2020 economic closures. For the EA, the GDP deflator grew at an annualized rate of nearly 5 percent between the first and second quarters of 2020, even as the HIPC declined at an annualized rate of 1.5 percent. Even more striking, for the UK, the GDP deflator rose at an annualized rate of close to 30 percent, even as the CPI declined at an annualized rate of 2.9 percent. Only for the USA do all three measures of inflation behave similarly, at least over this most recent period.

To reveal the source of these divergent price movements, Table 2 reports (unannualized) quarter-to-quarter percentage changes in the deflators for real GDP and its main components over the four quarters of 2020. Clearly, for both the EA and UK, unusual behavior in the deflator for government purchases accounts for the differences. Jessop (2020) elaborates, explaining that the very large increase in the UK GDP deflator reflects estimates that the real quantities of goods and services delivered in the health and education sectors of the economy fell sharply in the second quarter of 2020 even as nominal spending of those services increased.

The highly unusual dynamics displayed by the EA and UK GDP deflators should be noted in all work studying inflation over sample periods including 2020. In particular, they dictate an alternative choice made here, to re-formulate the equation of exchange (1) as

$$M_t V_t^c = P_t^c C_t, \tag{4}$$

using real consumption C_t as the scale variable, the deflator for consumption as the corresponding price index P_t^c , and velocity V_t^c measured by dividing nominal consumption spending $P_t^c C_t$ by M_t . The formula for the P-star variable then becomes

$$P_t^{c*} = \frac{M_t V_t^{c*}}{C_t^*}, \tag{5}$$

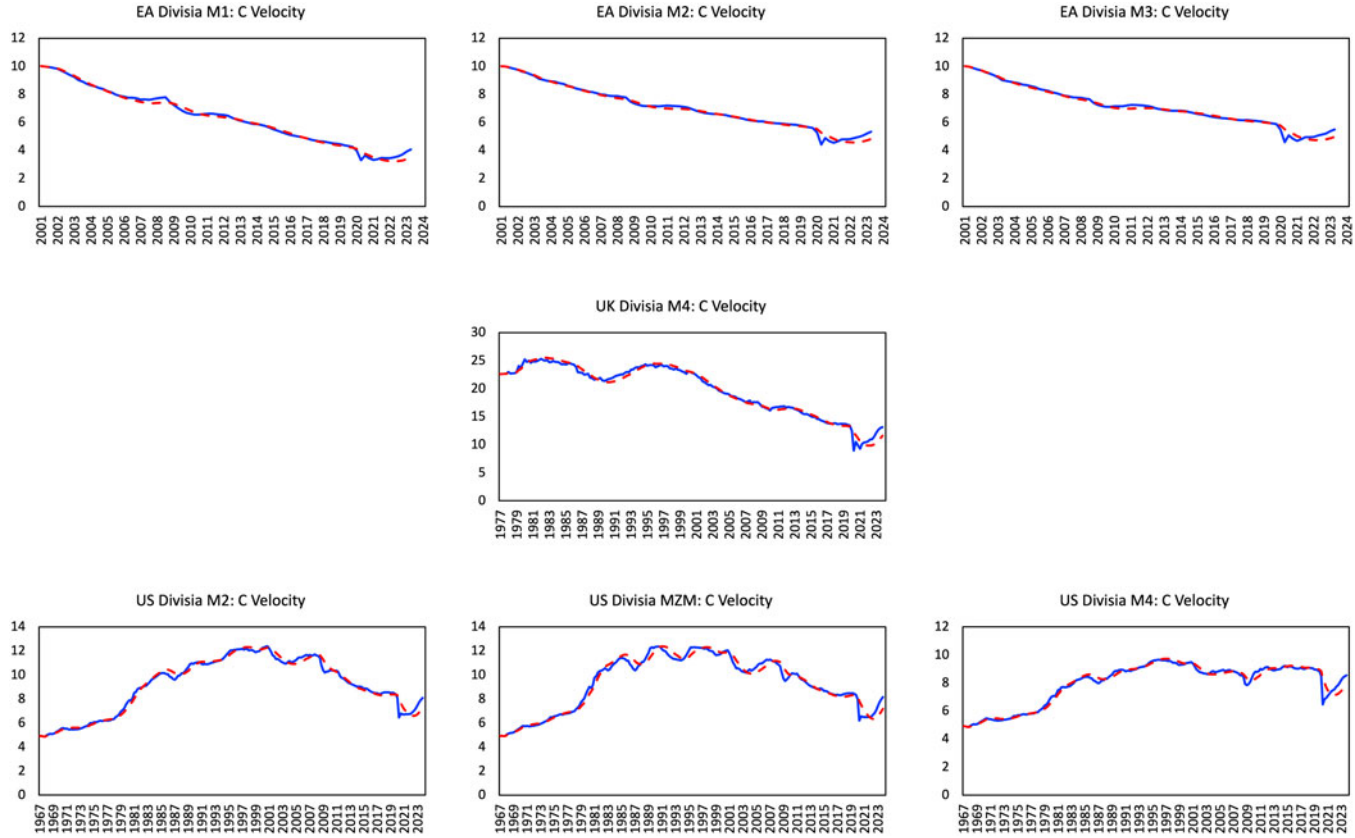


Figure 4. Monetary velocity based on consumption. Each panel shows the consumption velocity of the indicated monetary aggregate (solid blue line) together with its long-run trend (dashed red line), computed using the one-sided Hodrick-Prescott filter.

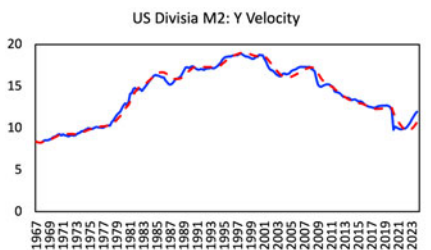
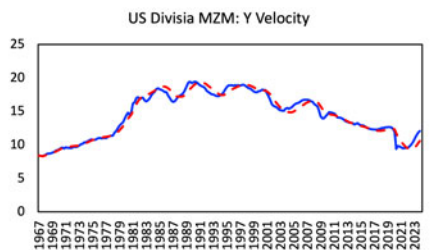
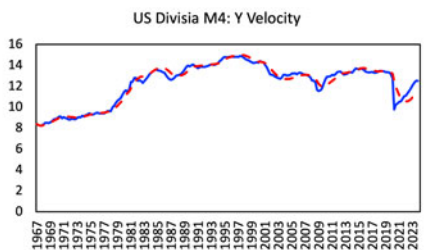
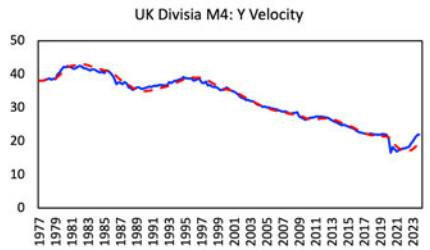
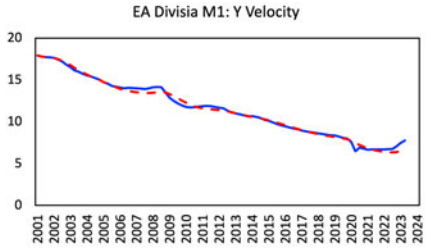
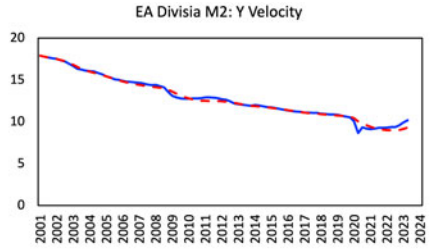
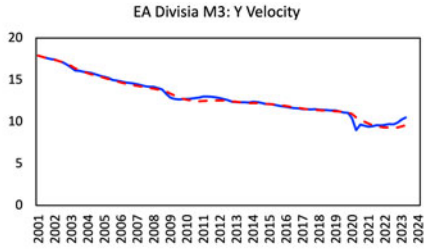


Figure 5. Monetary velocity based on output. Each panel shows the income velocity of the indicated monetary aggregate (solid blue line) together with its long-run trend (dashed red line), computed using the one-sided Hodrick-Prescott filter.

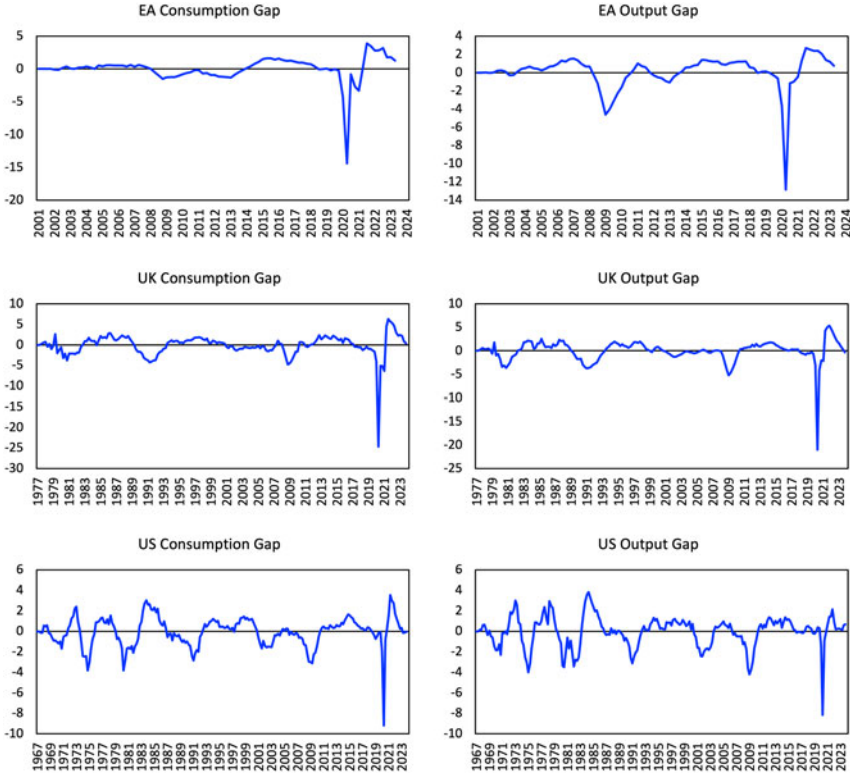


Figure 6. Consumption and output gaps. Each panel shows the percentage-point deviation of the indicated variable from its long-run trend, computed using the one-sided Hodrick-Prescott filter.

where V_t^{c*} and C_t^* are the trend components of consumption velocity and real consumption, computed with the one-sided HP filter. In terms of these new choices, the P-star regression (3) gets reformulated as

$$\Delta \pi_t^c = \alpha + \sum_{j=1}^q \beta_j \Delta \pi_{t-j}^c + \gamma (p_{t-1}^{c*} - p_{t-1}^c) + \varepsilon_t, \tag{6}$$

where $\pi_t^c = 400[\ln(P_t^c) - \ln(P_{t-1}^c)]$, $\Delta \pi_t^c = \pi_t^c - \pi_{t-1}^c$, and $p_{t-1}^{c*} - p_{t-1}^c = 100[\ln(P_{t-1}^{c*}) - \ln(P_{t-1}^c)]$. Now, the c subscript attached to all of these variables emphasizes that both inflation and the price gap are measured using data on real consumption and the consumption deflator.

Fig. 3 plots series on money growth and consumption price inflation over the longest sample available for each of the three economies: extending back to 2001 for the EA, 1977 for the UK, and 1967 for the USA.¹² Figs. 4 and 5 show that for each economy and each monetary aggregate, both consumption and income velocities display considerable long-run drift. In each case however, these movements are tracked quite closely, in real time, by the estimate of trend provided by the one-sided HP filter. Finally, Fig. 6 shows that the consumption and output gaps, defined as the percentage-point deviation of each variable from its estimated trend component, behave similarly in all three economies. Taken together, therefore, these figures imply that the primary reason for replacing output with consumption in moving from the model described by (1)–(3) to that described by (4)–(6) is to minimize distortions associated with the measurement of prices and quantities of government services during the 2020 economic closures, which make GDP price

Table 3. Estimated P-star models: common sample period, consumption as transactions variable

Dependent variable: Change in inflation $\Delta\pi_t^c$ based on the consumption deflator									
Euro Area, 2001:1 - 2023:2									
	Divisia M1			Divisia M2			Divisia M3		
	Estimate	t stat	p value	Estimate	t stat	p value	Estimate	t stat	p value
Constant	0.13	0.88	0.38	0.10	0.72	0.47	0.09	0.69	0.56
$\Delta\pi_{t-1}^c$	-0.21	-2.01	0.05	-0.19	-1.82	0.07	-0.18	-1.72	0.09
$\rho_{t-1}^{c*} - \rho_{t-1}^c$	0.18	3.56	0.00	0.24	3.31	0.00	0.20	2.97	0.00
UK, 2001:1-2023:2									
	Divisia M4								
	Estimate	t stat	p value	Estimate	t stat	p value	Estimate	t stat	p value
Constant				0.23	0.92	0.36			
$\Delta\pi_{t-1}^c$				-0.52	-5.03	0.00			
$\Delta\pi_{t-2}^c$				-0.28	-2.54	0.01			
$\Delta\pi_{t-3}^c$				-0.06	-0.57	0.57			
$\rho_{t-1}^{c*} - \rho_{t-1}^c$				0.20	2.73	0.01			
USA, 2001:1-2023:2									
	Divisia M2			Divisia MZM			Divisia M4		
	Estimate	t stat	p value	Estimate	t stat	p value	Estimate	t stat	p value
Constant	0.08	0.42	0.67	0.08	0.45	0.66	0.06	0.31	0.76
$\Delta\pi_{t-1}^c$	-0.39	-3.98	0.00	-0.39	-3.89	0.00	-0.36	-3.58	0.00
$\Delta\pi_{t-2}^c$	-0.32	-3.30	0.00	-0.32	-3.24	0.00	-0.30	-3.00	0.00
$\rho_{t-1}^{c*} - \rho_{t-1}^c$	0.20	3.59	0.00	0.15	3.21	0.00	0.14	2.82	0.01

Note: Maximal lag length for changes in inflation selected based on the Schwarz Information Criterion.

inflation a misleading measure of inflation compared to those most relevant to consumers and central bankers.

Sources for the non-monetary data used here are standard. For the EA, price and quantity data on GDP and its components are from the Eurostat database and apply, as do Darvas (2014, 2015) Divisia monetary aggregates, to the changing-composition Euro Area. Data on the HIPC are from the European Central Bank’s data portal. For the UK, all non-monetary series come from the Office of National Statistics. Finally, for the USA, all non-monetary data are from the Federal Reserve Bank of St. Louis’ FRED database. All of the series available from these sources are seasonally adjusted, except for the UK CPI. Thus, the measures of inflation for the UK CPI shown in Figs. 1 and 2 are de-seasonalized via a regression using quarterly dummy variables.

Finally, the P-star regressions (3) and (6), whether cast in terms of GDP or consumption, take the form of the more general autoregressive distributed lag models described and discussed by Pesaran (2015, pp. 121-3). Hence, following Pesaran’s suggestion, the Schwarz (1978) information criterion is used to select the optional value for q , the number of lagged changes in inflation included in each specification. Values ranging from $q = 1$ through $q = 8$ are considered. Across all of the results reported below, the Schwarz criterion typically selects models with just one or two

Table 4. Estimated P-star models: longest sample periods, consumption as transactions variable

Dependent variable: Change in inflation $\Delta\pi_t^c$ based on the consumption deflator									
Euro Area, 2001:1–2023:2									
	Divisia M1			Divisia M2			Divisia M3		
	Estimate	t stat	p value	Estimate	t stat	p value	Estimate	t stat	p value
Constant	0.13	0.88	0.38	0.10	0.72	0.47	0.09	0.69	0.56
$\Delta\pi_{t-1}^c$	-0.21	-2.01	0.05	-0.19	-1.82	0.07	-0.18	-1.72	0.09
$p_{t-1}^{c*} - p_{t-1}^c$	0.18	3.56	0.00	0.24	3.31	0.00	0.20	2.97	0.00
UK, 1977:1–2023:4									
	Divisia M4								
	Estimate	t stat	p value						
Constant				-0.02	-0.12	0.91			
$\Delta\pi_{t-1}^c$				-0.61	-8.86	0.00			
$\Delta\pi_{t-2}^c$				-0.31	-4.50	0.00			
$p_{t-1}^{c*} - p_{t-1}^c$				0.17	3.23	0.00			
USA, 1967:1–2023:4									
	Divisia M2			Divisia MZM			Divisia M4		
	Estimate	t stat	p value	Estimate	t stat	p value	Estimate	t stat	p value
Constant	0.00	0.01	0.99	-0.00	-0.04	0.97	-0.00	-0.05	0.96
$\Delta\pi_{t-1}^c$	-0.36	-5.67	0.00	-0.35	-5.56	0.00	-0.34	-5.45	0.00
$\Delta\pi_{t-2}^c$	-0.26	-4.22	0.00	-0.26	-4.14	0.00	-0.25	-4.04	0.00
$p_{t-1}^{c*} - p_{t-1}^c$	0.12	4.36	0.00	0.09	3.84	0.00	0.12	3.98	0.00

Note: Maximal lag length for changes in inflation selected based on the Schwarz Information Criterion.

lags and never favors a model with more than $q = 4$, the value originally used by Hallman et al. (1991) when estimating (3) with quarterly data from 1955:1 through 1988:4.

5. Results

Tables 3 (Tables 4, 5) through 6 present the main results, which are easily described. Tables 3 and 4 report estimates of (6), using consumption as the transactions variable. The results in Table 3 come from the common sample of data, running from 2001:1 through 2023:2, available for all three economies. Those in Table 4 use data from the longest sample of data available for each economy: 2001:1–2023:2 for the EA, 1977:1–2023:4 for the UK, and 1967:1–2023:4 for the USA. In both tables, the lagged price gap always enters the regression with a positive coefficient that is highly significant. In every single case, the p value rejecting the null hypothesis that $\gamma = 0$ is 0.01 or smaller. The range of estimates for γ , from 0.09 to 0.24, cleanly encompasses the value of 0.148 first reported by Hallman et al. (1991, p. 847).

Table 5 shows how, for the common sample period 2001:2–2023:2, these positive and statistically significant estimates of γ disappear for the EA and UK when output replaces consumption as

Table 5. Estimated P-star models: common sample period, output as transactions variable

Dependent variable: Change in inflation $\Delta\pi_t^y$ based on the output deflator									
Euro Area, 2001:1–2023:2									
	Divisia M1			Divisia M2			Divisia M3		
	Estimate	t stat	p value	Estimate	t stat	p value	Estimate	t stat	p value
Constant	0.10	0.67	0.51	0.11	0.72	0.48	0.11	0.73	0.47
$\Delta\pi_{t-1}^y$	-0.89	-8.70	0.00	-0.88	-8.62	0.00	-0.88	-8.69	0.00
$\Delta\pi_{t-2}^y$	-0.53	-5.52	0.00	-0.53	-5.15	0.00	-0.53	-5.19	0.00
$\rho_{t-1}^{y*} - \rho_{t-1}^y$	-0.01	-0.16	0.87	0.01	0.14	0.89	0.02	0.27	0.79
UK, 2001:1–2023:2									
	Divisia M4								
	Estimate	t stat	p value	Estimate	t stat	p value	Estimate	t stat	p value
Constant				0.24	0.47	0.64			
$\Delta\pi_{t-1}^y$				-0.87	-9.07	0.00			
$\Delta\pi_{t-2}^y$				-0.49	-5.14	0.00			
$\rho_{t-1}^{y*} - \rho_{t-1}^y$				0.05	0.29	0.77			
USA, 2001:1–2023:2									
	Divisia M2			Divisia MZM			Divisia M4		
	Estimate	t stat	p value	Estimate	t stat	p value	Estimate	t stat	p value
Constant	0.03	0.27	0.79	0.04	0.30	0.76	0.02	0.14	0.89
$\Delta\pi_{t-1}^y$	-0.41	-4.30	0.00	-0.39	-4.08	0.00	-0.37	-3.83	0.00
$\rho_{t-1}^{y*} - \rho_{t-1}^y$	0.17	4.67	0.00	0.13	4.25	0.00	0.12	3.95	0.00

Note: Maximal lag length for changes in inflation selected based on the Schwarz Information Criterion.

the transactions variable. Only for the USA do the estimates of γ remain positive and significant. The contrasting results in Tables 3 and 5 underscore the importance of measurement in assessing the quantity-theoretic links between money and prices. Together with the close look at the data themselves in Table 2, they serve more generally to warn future researchers that distortions in the measurement of price indices for government purchases in the EA and UK around the time of the 2020 economic closures are large enough to affect statistical estimates of and hypothesis tests about the magnitudes of key model coefficients.

In Table 6, also reporting results with output as the scale variable but from the longest sample period available for each economy, the coefficient on the lagged price gap regains its size and significance when, in particular, the UK sample used to estimate it is extended back to 1977. This raises the question of how important the 2020 data are in shaping the results: does the choice of scale variable for the P-star model matter for any other periods as well?

To answer this question, Tables 7 and 8 compare estimates based on consumption to those based on GDP over the longest pre-2020 samples available for each economy. Table 7 shows that, using consumption as the transactions variable, the lagged price gap enters positively and significantly in all of the estimated equations, except when Divisia M3 is used as the measure of money

Table 6. Estimated P-star models: longest sample periods, output as transactions variable

Dependent variable: Change in inflation $\Delta\pi_t^y$ based on the output deflator									
Euro Area, 2001:1–2023:2									
	Divisia M1			Divisia M2			Divisia M3		
	Estimate	t stat	p value	Estimate	t stat	p value	Estimate	t stat	p value
Constant	0.10	0.67	0.51	0.11	0.72	0.48	0.11	0.73	0.47
$\Delta\pi_{t-1}^y$	-0.89	-8.70	0.00	-0.88	-8.62	0.00	-0.88	-8.69	0.00
$\Delta\pi_{t-2}^y$	-0.53	-5.52	0.00	-0.53	-5.15	0.00	-0.53	-5.19	0.00
$p_{t-1}^{y*} - p_{t-1}^y$	-0.01	-0.16	0.87	0.01	0.14	0.89	0.02	0.27	0.79
UK, 1977:1–2023:4									
	Divisia M4								
	Estimate	t stat	p value						
Constant				-0.08	-0.24	0.81			
$\Delta\pi_{t-1}^y$				-0.80	-11.99	0.00			
$\Delta\pi_{t-2}^y$				-0.43	-6.55	0.00			
$p_{t-1}^{y*} - p_{t-1}^y$				0.21	2.17	0.03			
USA, 1967:1–2023:4									
	Divisia M2			Divisia MZM			Divisia M4		
	Estimate	t stat	p value	Estimate	t stat	p value	Estimate	t stat	p value
Constant	-0.01	-0.11	0.91	-0.01	-0.07	0.94	-0.01	-0.08	0.93
$\Delta\pi_{t-1}^y$	-0.37	-5.75	0.00	-0.31	-5.01	0.00	-0.31	-5.02	0.00
$\Delta\pi_{t-2}^y$	-0.15	-2.39	0.02						
$p_{t-1}^{y*} - p_{t-1}^y$	0.10	4.32	0.00	0.07	3.64	0.00	0.09	3.93	0.00

Note: Maximal lag length for changes in inflation selected based on the Schwarz Information Criterion.

for the EA. Table 8 reveals, by contrast, that the lagged price gap in most cases loses its significance when output is used as the transactions variable instead. Notably, this time, the results weaken considerably for the USA in addition to the EA. Once more, these results highlight the importance of measurement—particularly in the measurement of inflation—when looking for quantity-theoretic links between money and prices.

It is important to ask, as well, whether the stronger relationships between money and consumption-based measures of prices drawn by the P-star model may have changed over time. The short sample of available data makes it difficult to do this for the EA, but the longer samples available for the UK and USA can be split and used to re-estimate the models for two distinct subsamples. For the UK, 1993 presents itself as the natural date for a sample split, since the Bank of England began targeting inflation at that time. And while the Federal Reserve did not officially announce an explicit inflation target until much later, a look back at future 3 shows that, by 1993, inflation in the USA had also returned to the low and stable levels that characterize other economies' experiences under inflation targeting.

Accordingly, Tables 9 and 10 compare results when the consumption-based P-star regressions are estimated over pre- and post-1993 data for both the UK and USA. Although the estimated

Table 7. Estimated P-star models: Pre-2020 sample periods, consumption as transactions variable

Dependent variable: Change in inflation $\Delta\pi_t^c$ based on the consumption deflator									
Euro Area, 2001:1–2019:4									
	Divisia M1			Divisia M2			Divisia M3		
	Estimate	t stat	p value	Estimate	t stat	p value	Estimate	t stat	p value
Constant	0.04	0.27	0.78	0.04	0.27	0.97	0.03	0.20	0.84
$\Delta\pi_{t-1}^c$	-0.34	-3.02	0.00	-0.22	-1.95	0.05	-0.21	-1.87	0.07
$\Delta\pi_{t-2}^y$	-0.25	-2.33	0.02						
$\rho_{t-1}^{c*} - \rho_{t-1}^c$	0.19	3.15	0.00	0.20	2.00	0.05	0.13	1.55	0.13
UK, 1977:1–2019:4									
	Divisia M4								
	Estimate	tstat	pvalue						
Constant				-0.12	-0.60	0.55			
$\Delta\pi_{t-1}^c$				-0.70	-9.20	0.00			
$\Delta\pi_{t-2}^c$				-0.45	-5.16	0.00			
$\Delta\pi_{t-3}^c$				-0.17	-2.29	0.02			
$\rho_{t-1}^{c*} - \rho_{t-1}^c$				0.13	1.86	0.06			
USA, 1967:1–2019:4									
	Divisia M2			Divisia MZM			Divisia M4		
	Estimate	t stat	p value	Estimate	t stat	p value	Estimate	t stat	p value
Constant	-0.00	-0.04	0.97	-0.01	-0.08	0.94	-0.01	-0.10	0.92
$\Delta\pi_{t-1}^c$	-0.33	-5.03	0.00	-0.33	-4.98	0.00	-0.32	-4.90	0.00
$\Delta\pi_{t-2}^c$	-0.28	-4.29	0.00	-0.28	-4.28	0.00	-0.28	-4.25	0.00
$\rho_{t-1}^{c*} - \rho_{t-1}^c$	0.10	2.69	0.01	0.06	2.15	0.03	0.09	2.21	0.03

Note: Maximal lag length for changes in inflation selected based on the Schwarz Information Criterion.

coefficient on the lagged price gap remains sizable when estimated with pre-1993 data from the UK, a large standard error prevents this estimate from achieving statistical significance. The UK coefficient increases further in magnitude and regains its significance for the post-1993 period. For the USA, estimated values of γ are positive and significant across both subsamples, but as for the UK, these values also become larger and more significant when estimated with post-1993 data. Thus, the results in Tables 9 and 10 consistently indicate that the predictive power of money for inflation has actually *strengthened* in the post-1993 period.

Tables 11 and 12 revisit the Barnett critique, by re-running the benchmark regressions from Tables 3 and 4 using simple sums in place of Divisia monetary aggregates. For the changing-composition EA, the simple-sum series are for notional outstanding stocks of M1, M2, and M3, also used by Darvas (2014) and supplied in the same dataset, available through Bruegel, cited above. The Bank of England provides the series for simple-sum M4 for the non-financial and household sector for the UK, and the FRED database provides a long sample of data on simple-sum M2 for the USA.¹³ Somewhat surprisingly, the same strong statistical relationships between

Table 8. Estimated P-star models: Pre-2020 sample periods, output as transactions variable

Dependent variable: Change in inflation $\Delta\pi_t^y$ based on the output deflator									
Euro Area, 2001:1–2019:4									
	Divisia M1			Divisia M2			Divisia M3		
	Estimate	t stat	p value	Estimate	t stat	p value	Estimate	t stat	p value
Constant	−0.00	−0.00	1.00	0.00	0.00	1.00	−0.00	−0.02	0.98
$\Delta\pi_{t-1}^y$	−0.90	−7.56	0.00	−0.90	−7.48	0.00	−0.90	−7.50	0.00
$\Delta\pi_{t-2}^y$	−0.57	−3.61	0.00	−0.58	−3.58	0.00	−0.58	−3.61	0.00
$\Delta\pi_{t-3}^y$	−0.22	−1.42	0.16	−0.23	−1.45	0.15	−0.23	−1.47	0.15
$\Delta\pi_{t-4}^y$	0.02	0.19	0.85	0.02	0.13	0.89	0.01	0.12	0.91
$p_{t-1}^{y*} - p_{t-1}^y$	0.04	1.00	0.32	0.03	0.43	0.67	0.02	0.33	0.74
UK, 1977:1–2019:4									
	Divisia M4								
	Estimate	t stat	p value						
Constant	−0.17	−0.67	0.50						
$\Delta\pi_{t-1}^y$	−0.82	−10.85	0.00						
$\Delta\pi_{t-2}^y$	−0.52	−5.78	0.00						
$\Delta\pi_{t-3}^y$	−0.20	−2.71	0.01						
$p_{t-1}^{y*} - p_{t-1}^y$	0.21	2.30	0.02						
USA, 1967:1–2019:4									
	Divisia M2			Divisia MZM			Divisia M4		
	Estimate	t stat	p value	Estimate	t stat	p value	Estimate	t stat	p value
Constant	−0.02	−0.23	0.82	−0.02	−0.25	0.80	−0.02	−0.27	0.79
$\Delta\pi_{t-1}^y$	−0.33	−4.86	0.00	−0.33	−4.82	0.00	−0.33	−4.80	0.00
$\Delta\pi_{t-2}^y$	−0.19	−2.84	0.00	−0.19	−2.83	0.01	−0.19	−2.84	0.01
$p_{t-1}^{y*} - p_{t-1}^y$	0.05	1.75	0.08	0.03	1.30	0.19	0.03	1.09	0.28

Note: Maximal lag length for changes in inflation selected based on the Schwarz Information Criterion.

the P-star price gap and future changes in inflation evident in Tables 3 and 4 with Divisia money also appear in Tables 11 and 12 with the corresponding simple sums.

Fig. 7 sheds some light on the source of these results. The left-hand panels confirm that, in all three economies, Divisia and simple-sum monetary aggregates have sometimes behaved quite differently. And for many purposes, these differences *do* matter. For the USA, in particular, simple-sum M2 growth greatly and persistently exceeded the rate of Divisia M2 growth from 1979 through 1984. Barnett (2012, pp. 102–11) describes this episode, during which prominent monetarists, including Milton Friedman, were misled by high rates of simple-sum money growth and incorrectly predicted the return of higher rates of inflation. The right-hand panels of Fig. 7 show, however, that P-star price gaps behave quite similarly, whether based on Divisia or simple-sum monetary aggregates. Thus, the results from Table 11 and 12 can be interpreted as showing that, after accounting for slow-moving trends in velocity using the one-sided Hodrick-Prescott

Table 9. Estimated P-star models: Pre-1993 sample periods, consumption as transactions variable

Dependent variable: Change in inflation $\Delta\pi_t^c$ based on the consumption deflator									
UK, 1977:1–1992:4									
Divisia M4									
	Estimate	tstat	pvalue						
Constant	-0.14	-0.34	0.74						
$\Delta\pi_{t-1}^c$	-0.68	-5.56	0.00						
$\Delta\pi_{t-2}^c$	-0.36	-2.96	0.00						
$\rho_{t-1}^{c*} - \rho_{t-1}^c$	0.14	1.21	0.23						
USA, 1967:1–1992:4									
Divisia M2			Divisia MZM			Divisia M4			
	Estimate	t stat	p value	Estimate	t stat	p value	Estimate	t stat	p value
Constant	0.02	0.15	0.88	0.00	0.03	0.97	0.02	0.12	0.91
$\Delta\pi_{t-1}^c$	-0.26	-2.73	0.01	-0.26	-2.68	0.01	-0.26	-2.73	0.01
$\rho_{t-1}^{c*} - \rho_{t-1}^c$	0.08	2.03	0.04	0.06	1.81	0.07	0.09	2.04	0.04

Note: Maximal lag length for changes in inflation selected based on the Schwarz Information Criterion.

Table 10. Estimated P-star models: Post-1993 sample periods, consumption as transactions variable

Dependent variable: Change in inflation $\Delta\pi_t^c$ based on the consumption deflator									
UK, 1993:1–2023:4									
Divisia M4									
	Estimate	tstat	pvalue						
Constant	0.05	0.21	0.83						
$\Delta\pi_{t-1}^c$	-0.56	-6.47	0.00						
$\Delta\pi_{t-2}^c$	-0.27	-3.15	0.00						
$\rho_{t-1}^{c*} - \rho_{t-1}^c$	0.21	3.37	0.00						
USA, 1993:1–2023:4									
Divisia M2			Divisia MZM			Divisia M4			
	Estimate	t stat	p value	Estimate	t stat	p value	Estimate	t stat	p value
Constant	-0.02	-0.12	0.91	-0.02	-0.11	0.91	-0.01	-0.11	0.92
$\Delta\pi_{t-1}^c$	-0.43	-4.98	0.00	-0.39	-4.55	0.00	-0.36	-4.28	0.00
$\Delta\pi_{t-2}^c$	-0.43	-4.79	0.00	-0.32	-3.71	0.00	-0.30	-3.48	0.00
$\Delta\pi_{t-3}^c$	-0.17	-1.93	0.06						
$\Delta\pi_{t-3}^c$	-0.27	-3.24	0.00						
$\rho_{t-1}^{c*} - \rho_{t-1}^c$	0.17	4.27	0.00	0.12	3.55	0.00	0.13	3.34	0.00

Note: Maximal lag length for changes in inflation selected based on the Schwarz Information Criterion.

Table 11. Estimated P-star models: common sample period, simple-sum money

Dependent variable: Change in inflation $\Delta\pi_t^c$ based on the consumption deflator									
Euro Area, 2001:1–2023:2									
	Simple-Sum M1			Simple-Sum M2			Simple-Sum M3		
	Estimate	t stat	p value	Estimate	t stat	p value	Estimate	t stat	p value
Constant	0.13	0.90	0.37	0.07	0.45	0.65	0.06	0.42	0.67
$\Delta\pi_{t-1}^c$	-0.21	-2.03	0.05	-0.16	-1.48	0.14	-0.15	-1.42	0.16
$p_{t-1}^{c*} - p_{t-1}^c$	0.18	3.59	0.00	0.15	2.16	0.03	0.11	1.75	0.08
UK, 2001:1–2023:2									
	Simple-Sum M4								
	Estimate	t stat	p value						
Constant				0.25	0.99	0.33			
$\Delta\pi_{t-1}^c$				-0.51	-4.95	0.00			
$\Delta\pi_{t-2}^c$				-0.28	-2.47	0.02			
$\Delta\pi_{t-3}^c$				-0.05	-0.49	0.62			
$p_{t-1}^{c*} - p_{t-1}^c$				0.26	2.84	0.01			
USA, 2001:1–2023:2									
	Simple-Sum M2								
	Estimate	t stat	p value						
Constant				0.05	0.29	0.77			
$\Delta\pi_{t-1}^c$				-0.39	-3.97	0.00			
$\Delta\pi_{t-2}^c$				-0.32	-3.27	0.00			
$p_{t-1}^{c*} - p_{t-1}^c$				0.23	3.68	0.00			

Note: Maximal lag length for changes in inflation selected based on the Schwarz Information Criterion.

filter, the P-star model can glean the “right” monetary signal even from the “wrong” monetary aggregate.

Finally, (4) and (5) imply that the consumption-based P-star price gap can be decomposed into two components,

$$\begin{aligned}
 \hat{p}_t^{c*} - p_t^c &= 100[\ln(P_t^{c*}) - \ln(P_t^c)] \\
 &= 100[\ln(V_t^{c*}) - \ln(V_t^c)] + 100[\ln(C_t) - \ln(C_t^*)] \\
 &= (v_t^{c*} - v_t^c) + (c_t - c_t^*),
 \end{aligned}
 \tag{7}$$

the first measuring the percentage-point deviation of equilibrium velocity V_t^{c*} from its actual level V_t^c and the second measuring the percentage-point deviation of consumption C_t from its equilibrium level C_t^* .¹⁴ This decomposition highlights the quantity-theoretic foundations of the P-star model, which allow an increase in the money stock M_t to be held temporarily as excess real balances, thereby reducing V_t^c relative to V_t^{c*} . or to yield a transitory increase in real spending, thereby increasing C_t relative to C_t^* . Then, in the long run, the price level P_t will rise to meet P_t^* as these velocity and consumption gaps are closed. Using (7), the P-star regression (6) can be

Table 12. Estimated P-star models; longest sample periods, simple-sum money

Dependent variable: Change in inflation $\Delta\pi_t^c$ based on the consumption deflator									
Euro Area, 2001:1–2023:2									
	Simple-Sum M1			Simple-Sum M2			Simple-Sum M3		
	Estimate	<i>t</i> stat	<i>p</i> value	Estimate	<i>t</i> stat	<i>p</i> value	Estimate	<i>t</i> stat	<i>p</i> value
Constant	0.13	0.90	0.37	0.07	0.45	0.65	0.06	0.42	0.67
$\Delta\pi_{t-1}^c$	−0.21	−2.03	0.05	−0.16	−1.48	0.14	−0.15	−1.42	0.16
$\rho_{t-1}^{c*} - \rho_{t-1}^c$	0.18	3.59	0.00	0.15	2.16	0.03	0.11	1.75	0.08
UK, 1977:1–2023:4									
	Simple-Sum M4								
	Estimate	<i>t</i> stat	<i>p</i> value						
Constant	−0.01	−0.03	0.98						
$\Delta\pi_{t-1}^c$	−0.60	−8.71	0.00						
$\Delta\pi_{t-2}^c$	−0.30	−4.36	0.00						
$\rho_{t-1}^{c*} - \rho_{t-1}^c$	0.22	3.05	0.00						
USA, 1967:1–2023:4									
	Simple-Sum M2								
	Estimate	<i>t</i> stat	<i>p</i> value						
Constant	−0.00	−0.03	0.97						
$\Delta\pi_{t-1}^c$	−0.35	−5.61	0.00						
$\Delta\pi_{t-2}^c$	−0.26	−4.12	0.00						
$\rho_{t-1}^{c*} - \rho_{t-1}^c$	0.15	4.37	0.00						

Note: Maximal lag length for changes in inflation selected based on the Schwarz Information Criterion.

expanded to include both the velocity and consumption gaps instead of the price gap alone,

$$\Delta\pi_t^c = \alpha + \sum_{j=1}^q \beta_j \Delta\pi_{t-j}^c + \gamma_1 (v_{t-1}^{c*} - v_{t-1}^c) + \gamma_2 (c_{t-1} - c_{t-1}^*) + \varepsilon_t, \tag{8}$$

and thereby used to gauge the importance of each term separately.

Tables 13 and 14 confirm that, for all three economies, lagged velocity gaps always enter positively and significantly into the expanded regression (8). Lagged consumption gaps generally do so as well, except for the USA in cases where Divisia MZM or Divisia M4 is used to measure money. Thus, both margins of adjustment—but especially that measured by the velocity gap—appear important in accounting for the delayed response of prices that then give changes in money their predictive power for inflation.

6. Conclusion

Taken together, the results presented here highlight several ways in which “measurement matters” in assessing the quantity-theoretic links between money and prices using the P-star model.

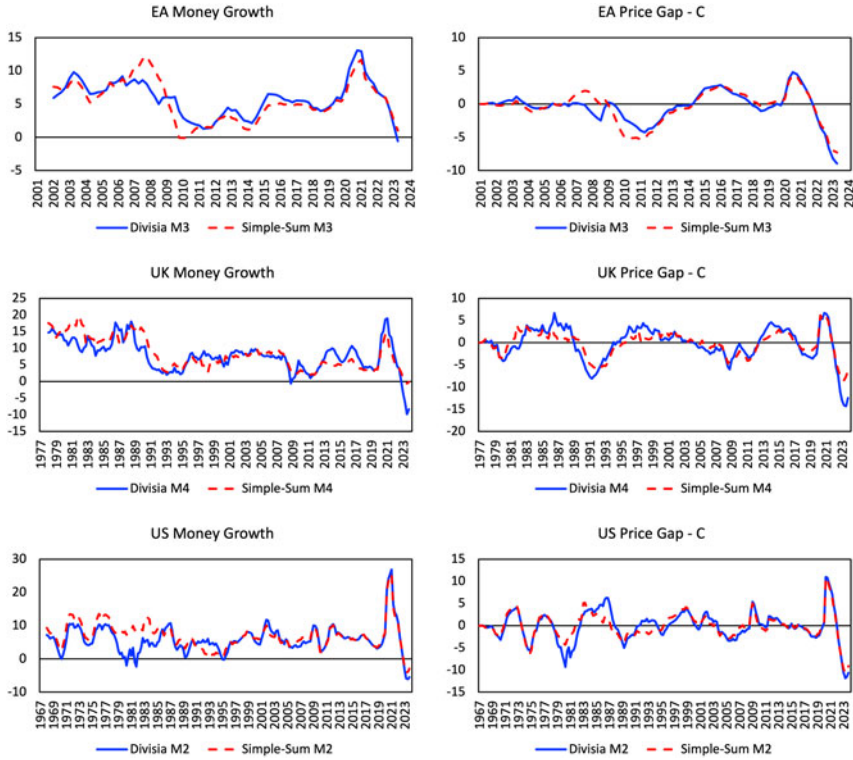


Figure 7. Money growth and price gaps. Left-hand panels compare the year-over-year percentage changes in a Divisia monetary aggregate to those in the corresponding simple-sum aggregate. Right-hand panels compare the P-star price gaps computed using the same Divisia and simple-sum aggregates, using consumption as the transactions variable. Positive price gaps indicate periods when monetary policy puts upward pressure on inflation; negative values indicate periods when monetary policy puts downward pressure on inflation.

Most importantly, distortions in the measurement of the GDP price deflator, especially for the EA and UK economies during 2020 but for all three economies and at other times as well, make consumption-based measures of transactions and inflation preferable. This finding should be noted by researchers, even those working with alternative models, attempting to explain the dramatic movements in inflation seen across many of the world’s economies since 2020.

The results show, in addition, that adjusting measures of monetary ease or restraint by accounting for slowly shifting long-run trends in velocity and real economic growth is crucial when using those measures to forecast changes in inflation. In fact, the approach taken here, using the one-side Hodrick-Prescott filter, makes these adjustments satisfactorily even when, ignoring the importance of the Barnett critique, simple-sum measures are used instead of Divisia monetary aggregates.

Returning to Fig. 7, plotting the P-star price gap variables for each economy using the consumption-based measure of prices as implied by (4)–(6), several more conclusions emerge. First, price gaps near zero in the USA and below zero in the EA and UK signal that, despite central banks’ efforts to provide additional stimulus through “forward guidance” promising to hold short-term interest rates near zero for extended periods of time and multiple rounds of “quantitative

Table 13. Estimated P-star models: common sample periods, consumption and velocity gaps

Dependent variable: Change in inflation $\Delta\pi_t^c$ based on the consumption deflator									
Euro Area, 2001:1–2023:2									
	Divisia M1			Divisia M2			Divisia M3		
	Estimate	t stat	p value	Estimate	t stat	p value	Estimate	t stat	p value
Constant	0.17	1.24	0.22	0.12	0.83	0.41	0.09	0.67	0.51
$\Delta\pi_{t-1}^c$	-0.28	-2.85	0.01	-0.21	-2.09	0.04	-0.20	-1.95	0.05
$\Delta\pi_{t-2}^c$	-0.23	-2.26	0.03						
$v_{t-1}^{c*} - v_{t-1}^c$	0.20	4.11	0.00	0.29	3.95	0.00	0.23	3.52	0.00
$c_{t-1} - c_{t-1}^*$	0.39	4.39	0.00	0.48	4.30	0.00	0.41	3.89	0.00
UK, 2001:1–2023:2									
	Divisia M4								
	Estimate	tstat	pvalue						
Constant				0.27	1.06	0.29			
$\Delta\pi_{t-1}^c$				-0.54	-5.16	0.00			
$\Delta\pi_{t-2}^c$				-0.32	-2.76	0.01			
$\Delta\pi_{t-3}^c$				-0.08	-0.78	0.44			
$v_{t-1}^{c*} - v_{t-1}^c$				0.21	2.88	0.01			
$c_{t-1} - c_{t-1}^*$				0.30	2.68	0.01			
USA, 2001:1–2023:2									
	Divisia M2			Divisia MZM			Divisia M4		
	Estimate	t stat	p value	Estimate	t stat	p value	Estimate	t stat	p value
Constant	0.09	0.51	0.61	0.09	0.45	0.65	0.06	0.31	0.76
$\Delta\pi_{t-1}^c$	-0.48	-4.55	0.00	-0.40	-3.84	0.00	-0.36	-3.51	0.00
$\Delta\pi_{t-2}^c$	-0.48	-4.34	0.00	-0.33	-3.23	0.00	-0.30	-2.98	0.00
$\Delta\pi_{t-3}^c$	-0.22	-1.95	0.05						
$\Delta\pi_{t-4}^c$	-0.31	-2.99	0.00						
$v_{t-1}^{c*} - v_{t-1}^c$	0.22	3.92	0.00	0.16	3.12	0.00	0.14	2.71	0.01
$c_{t-1} - c_{t-1}^*$	0.38	2.39	0.02	0.20	1.25	0.21	0.16	1.01	0.31

Note: Maximal lag length for changes in inflation selected based on the Schwarz Information Criterion.

easing” or large-scale asset purchases, slow rates of money growth put little if any upward pressure on inflation during the sluggish recoveries that, in all three economies, followed the 2008-9 global financial crisis and recession.

Second, price gaps rivaling or exceeding their historical peaks in 2020-1 show that, by sharp contrast, low interest rates and large-scale asset purchases *did* generate rapid money growth and strong upward pressure on inflation during and after the 2020 economic closures. Taken together, these observations point to the usefulness of the quantity-theoretic perspective provided by the P-star model, as an alternative to or at least a cross-check against more conventional analyses that focus on the behavior of short and long-term interest rates alone.

Table 14. Estimated P-star models: longest sample periods, consumption, and velocity gaps

Dependent variable: Change in inflation $\Delta\pi_t^c$ based on the consumption deflator									
Euro Area, 2001:1–2023:2									
	Divisia M1			Divisia M2			Divisia M3		
	Estimate	t stat	p value	Estimate	t stat	p value	Estimate	t stat	p value
Constant	0.17	1.24	0.22	0.12	0.83	0.41	0.09	0.67	0.51
$\Delta\pi_{t-1}^c$	-0.28	-2.85	0.01	-0.21	-2.09	0.04	-0.20	-1.95	0.05
$\Delta\pi_{t-2}^c$	-0.23	-2.26	0.03						
$v_{t-1}^{c*} - v_{t-1}^c$	0.20	4.11	0.00	0.29	3.95	0.00	0.23	3.52	0.00
$c_{t-1} - c_{t-1}^*$	0.39	4.39	0.00	0.48	4.30	0.00	0.41	3.89	0.00
UK, 1977:1–2023:4									
	Divisia M4								
	Estimate	t stat	p value	Estimate	t stat	p value	Estimate	t stat	p value
Constant				-0.01	-0.06	0.95			
$\Delta\pi_{t-1}^c$				-0.61	-8.87	0.00			
$\Delta\pi_{t-2}^c$				-0.31	-4.58	0.00			
$v_{t-1}^{c*} - v_{t-1}^c$				0.16	2.97	0.00			
$c_{t-1} - c_{t-1}^*$				0.25	3.01	0.00			
USA, 1967:1–2023:4									
	Divisia M2			Divisia MZM			Divisia M4		
	Estimate	t stat	p value	Estimate	t stat	p value	Estimate	t stat	p value
Constant	0.00	0.01	0.99	-0.00	-0.02	0.98	-0.00	-0.04	0.97
$\Delta\pi_{t-1}^c$	-0.36	-5.61	0.00	-0.36	-5.53	0.00	-0.35	-5.41	0.00
$\Delta\pi_{t-2}^c$	-0.26	-4.20	0.00	-0.26	-4.14	0.00	-0.26	-4.04	0.00
$v_{t-1}^{c*} - v_{t-1}^c$	0.13	4.19	0.00	0.09	3.67	0.00	0.11	3.81	0.00
$c_{t-1} - c_{t-1}^*$	0.12	1.80	0.07	0.11	1.59	0.11	0.13	1.87	0.06

Note: Maximal lag length for changes in inflation selected based on the Schwarz Information Criterion.

Third, even as unusually large and positive price gap measures signal the episode of vigorous monetary expansion that presaged the rise in inflation in 2021 and 2022, large (in magnitude) but negative (in sign) values for the same price gap measures suggest that monetary policies in all three countries are putting strong downward pressure on inflation today. These results illustrate, once again, how a quantity-theoretic approach, combined with more careful attempts at measurement, can help guide central bankers in their quest to restore and maintain a more favorable environment of low and stable inflation.

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Notes

- 1 Borio et al. (2023) emphasize this point and verify that it applies to data from Canada, Brazil, and Thailand as well as the three major economies studied here. Along the same lines, it is worth noting that the strongest evidence supporting Friedman (1956)'s "restatement" of the quantity theory of money, presented by Cagan (1956), comes from episodes of hyperinflation.
- 2 Barnett (2012) provides a more recent and comprehensive overview of the logic and methods behind Divisia monetary aggregation and a discussion of the usefulness of Divisia monetary aggregates in monetary policy analysis.
- 3 Fleissig and Jones (2024) also provide a detailed description and analysis of the Bank of England's Divisia monetary aggregates.
- 4 Duca (2000) and Bachmeier and Swanson (2005) discuss this "case of the missing M2" in more detail, attributing the rise in velocity to financial innovations that increased the liquidity of bond mutual funds in the early 1990s.
- 5 Taking a much longer historical perspective, Nelson (2012) discusses a range of measurement problems that obscure the statistical connections between UK money growth and output in data extending back into the 19th century.
- 6 This paper, like all of those listed in Table 1, applies the P-star model to data from individual economies, one at a time. Recent work by DeSantis (2015) shows, however, that shifts in money demand and hence velocity across the EA and USA are linked by cross-border stock and bond portfolio shifts. This work suggests that there might be gains in predictive accuracy from extensions of the P-star model that consider data from the EA, USA, and UK jointly, to account for common trends in long-run velocities.
- 7 As noted previously, Hallman et al. (1991) used real GNP as their measure of Y_t . In 1991, GDP replaced GNP as the principal measure of aggregate output in the US National Income and Product Accounts; for details, see United States Department of Commerce (1991) and Ramey (2021).
- 8 These measures of EA Divisia money are updated regularly and made freely available through Bruegel at <http://www.bruegel.org/dataset/divisia-monetary-aggregates-euro-area>.
- 9 Details can be found in European Central Bank (1999).
- 10 These measures of US Divisia money are updated monthly and made freely available through the Center for Financial Stability at http://centerforfinancialstability.org/amfm_data.php.
- 11 Hodrick and Prescott (1997) present the HP filter in its original, two-sided form and also provide the suggested setting for the smoothing parameter, $\lambda = 1600$, used with quarterly data in the one-sided variant employed here.
- 12 In each case, the sample period is limited by the availability of data on Divisia money: as noted above, starting in 2001 for the EA, 1977 for the UK, and 1967 for the USA. Since the graphs show year-over-year growth rates of money and prices, however, the series plotted begins one year later.
- 13 For the USA, a series for simple-sum MZM was compiled by the Federal Reserve Bank of St. Louis, but discontinued in 2021. Similarly, a simple-sum aggregate resembling Divisia M4, labeled L for "liquid assets," was reported by the Federal Reserve Board but discontinued in 1998. Hence, of the three measures of money for the USA considered here, only Divisia M2 has a simple-sum analog with data available through the present.
- 14 Note that by switching the order of c_t^* and c_t in (7), the price gap can be written as the sum of instead of the difference between the two terms involving velocity and consumption. In addition, the consumption gap $c_t - c_t^*$ is thereby defined analogously to standard definitions of the output gap, with positive values indicating that real spending has risen above its long-term level.

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