THE OPTIMAL MONETARY RESPONSE TO TECHNOLOGY SHOCKS

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This paper develops a model in which both technology and monetary shocks are important sources of variation in aggregate output and employment. The model rationalizes a policy under which money responds actively to technology shocks. The welfare cost of adopting the constant money growth rule advocated by Milton Friedman rather than the optimal activist policy is small, however. (*JEL* E52, E32)

I. INTRODUCTION

During the past decade, many economists adopted the view that shocks to the aggregate production function, or technology shocks, represent the dominant source of fluctuations in the United States economy. As noted by Shapiro [1987], a number of elements came together to shape this view. Most important, a series of severe supply shocks hit the economy during the 1970s; standard Keynesian models, with their emphasis on the demand side, had great difficulty tracking the economy's response to these shocks. The view solidified after Kydland and Prescott [1982] demonstrated that many features of the business cycle can be replicated in a model where technology shocks are the only source of fluctuations. Kydland and Prescott's work launched the now extensive literature on real business cycles, surveyed by McCallum [1989].

The observation that supply-side disturbances are important in generating business cycles, however, need not imply that monetary factors play no role in accounting for variations in economic activity. Greenwood and Huffman [1987] and Cooley and Hansen [1989] add cash-in-advance constraints to otherwise standard real business cycle models as a simple way of including the nominal sector. In these cash-in-advance models, monetary shocks have little effect on the cyclical behavior of real variables; these models preserve the emphasis on technology shocks in explaining the business cycle. Nevertheless, the models show how sustained inflation can affect the real economy by acting as a distortionary tax on productive activity.

The models of Greenwood and Huffman and Cooley and Hansen imply that optimal monetary policy involves a steady contraction of the money supply that keeps the nominal interest rate at zero, as called for by Friedman [1969]. This optimal policy makes no attempt to respond to technology shocks. Instead, it simply removes the distortionary inflation tax.

More recently, Cho and Cooley [1992], Yun [1994a], and Dow [1995] have investigated the effects of introducing various forms of price rigidity into monetary versions of the real business cycle model. Technology shocks continue to play a major role in these studies, but the presence of nominal rigidities allows monetary shocks to affect the business cycle as well. In fact, these studies show that the addition of price rigidity improves the model's ability to match the cyclical properties of the data. This paper complements these studies by deriving the implications of nominal price rigidity for optimal monetary policy.

I begin by outlining a model that shares the basic features of those cited above, including technology shocks, a cash-in-advance constraint, and nominal price rigidity. In the model, both technology and monetary shocks have important effects on aggregate output and employment. Unlike flexible-price cash-in-advance models, for instance, the model developed here gives rise to a Phillips curve relationship between money and output.

The model is used to answer the following question: Does the presence of nominal price

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rigidity rationalize a monetary policy that responds actively to technology shocks, or does the optimality of Friedman's constant money growth rule still apply? The results show that optimal policy does, in fact, exploit the monetary authority's ability to influence the real economy when prices are sticky. That is, the optimal policy is activist. On the other hand, the results also show that the Friedman rule, while no longer optimal, comes very close to the optimum in welfare terms. In this sense, the main policy implication of flexible-price cash-in-advance models appears robust to the introduction of nominal price rigidity.

Section II presents the model; section III describes the key features of its equilibrium. Section IV derives the optimal activist policy and compares its performance to that of the Friedman rule. Section V concludes.

II. THE MODEL

The Economic Environment

Infinitely lived households, firms, and a monetary authority populate an economy where time periods are indexed by $t = 0,1,2,\ldots$. There are N identical households. As in Lucas [1980], each consists of two members: a worker and a shopper. The firms are of two types; there are N identical firms of each type. Firms of different types produce different nonstorable consumption goods. By assumption, type 1 firms must set the nominal price for their good one period in advance; they produce the sticky-price good. Type 2 firms can adjust their price freely at any time; they produce the flexible-price good.

This form of nominal rigidity, in which some firms set nominal prices one period in advance, is also used by Dow [1995]. Cho and Cooley [1992] and Yun [1994a] introduce somewhat different forms of nominal rigidity; section IV, below, briefly considers these alternative specifications to establish the robustness of the results derived here.

The monetary authority controls the money supply by making lump-sum transfers to each household. The money stock evolves according to

$$M_{t+1}^s = x_t M_t^s,$$

where M_t^s is the per-household money supply at the beginning of time t and $(x_t - 1)M_t^s$ is the transfer received by the representative household during time t. Since all households are identical, equilibrium requires that $M_t = M_t^s$ for all $t = 0,1,2,\ldots$, where M_t denotes the representative household's money balances at the beginning of time t.

Households borrow and lend by trading in one-period, nominally denominated discount bonds. A bond returning one dollar at time t+1 sells for $1/(1+r_t)$ dollars at time t, where r_t is the nominal interest rate between t and t+1. The representative household owns B_t bonds at the beginning of time t. Since the monetary authority does not borrow or lend, bonds are available in zero net supply; hence, $B_t = 0$ must hold in equilibrium for all $t = 0,1,2,\ldots$.

Households also trade shares in firms of both types. Shares in the representative type i firm sell for Q_{it} dollars and pay a dividend of D_{it} dollars during time t. The representative household owns a_{it} shares in the representative type i firm at the beginning of time t. By choice of units, each firm is assumed to have one share of stock outstanding; the equilibrium conditions $a_{it} = 1$ for $i = 1,2$ and $t = 0,1,2,\ldots$ indicate that the representative household owns the representative firms.

Timing of Events

As noted above, the representative household enters time t with money M_t, bonds B_t, and shares a_{it}. The representative type 1 firm enters time t having set a sticky nominal price P_{1t} for its output.

At the beginning of time t, a technology shock z_t is realized. Next, the money supply shock x_t occurs, and the representative household receives the transfer $(x_t - 1)M_t^s$. The vector of shocks (z_t, x_t) follows a first-order Markov process over time.

After both shocks are realized, the representative household's bonds B_t mature, bringing its money holdings to $M_t + (x_t - 1)M_t^s + B_t$. The household uses some of this cash to purchase new bonds of value $B_{t+1}/(1 + r_t)$ and carries the rest into the goods market.

In the goods market, the representative household's shopper purchases c_{1t} units of the sticky-price good at the sticky price P_{1t} and c_{2t} units of the flexible-price good at the flexible price P_{2t}. He must make all of these purchases with money; he faces the cash-in-advance constraint

$$M_t + (x_t - 1)M_t^s + B_t - B_{t+1}/(1+r_t)$$
$$\geq P_{1t}c_{1t} + P_{2t}c_{2t}.$$

The household's worker, meanwhile, supplies n_{1t} units of labor to the representative type 1 firm and n_{2t} units of labor to the representative type 2 firm. He receives the nominal wage W_t.

The household's preferences are described by the expected utility function

$$E\sum_{t=0}^{\infty}\beta^t[\alpha\ln(c_{1t}) + (1-\alpha)\ln(c_{2t})$$
$$+ \gamma(1 - n_{1t} - n_{2t})],$$

where the discount factor β lies between zero and one. The parameter α measures the importance of sticky prices in the economy. With $\alpha = 1$, all prices are sticky. With $\alpha = 0$, all prices are flexible, and the model resembles those studied by Greenwood and Huffman [1987] and Cooley and Hansen [1989]. With α between zero and one, some prices are sticky while others are flexible. The positive parameter γ measures the importance of leisure relative to consumption in utility; the household's time endowment is normalized to unity.

The representative type i firm produces output y_{it} with labor n_{it} using the technology described by the production function

$$y_{it} = z_t n_{it}.$$

At the end of time t, after goods production, trade, and consumption take place, firms make their wage payments and distribute any profits in the form of dividends to their shareholders. The linear production function implies that the representative type i firm's dividend is

$$D_{it} = P_{it}y_{it} - W_t n_{it} = (P_{it} - W_t/z_t)y_{it}.$$

If $D_{it} < 0$, the representative type i firm incurs a loss at time t, and its shareholders are responsible for covering this loss.

Households trade shares at the end of time t. At this time, the representative type 1 firm sets next period's price P_{1t+1}, as described below. The representative household uses its unspent cash, its shares and dividends, and its wage receipts as sources of funds to acquire the shares and money that it will carry into time $t+1$; it faces the budget constraint

$$M_t + (x_t - 1)M_t^s + B_t + (Q_{1t} + D_{1t})a_{1t}$$
$$+ (Q_{2t} + D_{2t})a_{2t} + W_t(n_{1t} + n_{2t})$$
$$\geq P_{1t}c_{1t} + P_{2t}c_{2t} + Q_{1t}a_{1t+1} + Q_{2t}a_{2t+1}$$
$$+ B_{t+1}/(1+r_t) + M_{t+1}.$$

The analysis begins by considering household behavior, given some arbitrary nominal prices P_{1t} and P_{2t}, and then shows how those prices are determined by firm behavior.

Household Behavior

To derive a recursive formulation of the representative household's problem, define the scaled nominal variables $m_t = M_t/M_t^s$, $b_t = B_t/M_t^s$, $q_{it} = Q_{it}/M_t^s$, $d_{it} = D_{it}/M_t^s$, $w_t = W_t/M_t^s$, and $p_{it} = P_{it}/M_t^s$. Dividing the household's budget constraint through by M_t^s yields

$$(1) \qquad m + x - 1 + b + (q_1 + d_1)a_1$$
$$+ (q_2 + d_2)a_2 + w(n_1 + n_2)$$
$$\geq p_1c_1 + p_2c_2 + q_1a_1' + q_2a_2'$$
$$+ b'x/(1+r) + m'x,$$

where unprimed variables refer to time t values and primed variables refer to time $t+1$ values. Dividing the cash-in-advance constraint through by M_t^s yields

$$(2) \quad m + x - 1 + b - b'x/(1+r) \geq p_1c_1 + p_2c_2.$$

Since type 1 firms must set their prices one period in advance, a full description of the aggregate state of the economy at time t must include not only the realization of the current-period shocks, but also the realization of the shocks from time $t-1$. Accordingly, denote the aggregate state at time t by $s_t = (z_t, x_t, z_{t-1}, x_{t-1})$ and, as above, let s refer to s_t and s' to s_{t+1}.

In addition to the aggregate state, the representative household's decisions at time t depend on its beginning-of-period asset holdings. Thus, the Bellman equation

$$(3)\qquad v(m, b, a_1, a_2, s) = \max \alpha \ln(c_1) + (1-\alpha)\ln(c_2) + \gamma(1 - n_1 - n_2) + \beta \mathrm{E}[v(m', b', a_1', a_2', s') \mid s]$$

summarizes the household's problem, where the maximization is by choice of c_1, c_2, n_1, n_2, m', b', a_1', and a_2' that satisfy (1) and (2).

As demonstrated in the appendix, the solution to (3) implies that in equilibrium the household's consumptions, the wage rate, and the interest rate can all be expressed as functions of the current state and the scaled nominal prices. In particular,

$$(4)\qquad c_1 = c_1(s, p_1) = (\alpha x / p_1)\min[1, 1/\beta A(s)],$$

$$(5)\qquad c_2 = c_2(s, p_2) = [(1-\alpha)x/p_2]\min[1, 1/\beta A(s)],$$

$$(6)\qquad w = w(s) = \gamma x / \beta A(s),$$

and

$$(7)\qquad r = r(s) = \max[1/\beta A(s) - 1, 0],$$

where the function $A(s)$ solves

$$(8)\qquad A(s) = \mathrm{E}\{(1/x')\max[1, \beta A(s')] \mid s\}.$$

Equations (4) and (5) show that the household's demand for each good is decreasing as a function of its price and, holding price constant, increasing in size of the monetary shock. Equation (7) implies that the nominal interest rate is positive if and only if $1 > \beta A(s)$; in this case, the representative household has an incentive to economize on its real balances and the cash-in-advance constraint binds. Hence, (4) and (5) also show that the household's demand for each good depends on whether or not $r(s) > 0$.

Firm Behavior

By assumption, each type 1 firm must set its nominal price one period in advance and sell output on demand at its sticky price. The type 1 firm that sets the lowest price attracts the demand of all N households, while all firms that set higher prices attract no demand. If multiple firms set the same price, those with the lowest price split market demand evenly.

Type 1 firms know the value of the per household money supply M_t^s when they set their prices for time t at the end of time $t-1$. Thus, these firms can be depicted as setting scaled nominal prices $p_{1t} = P_{1t}/M_t^s$ for their output. Neither z_t nor x_t, the current technology and monetary shocks, is known at the end of the previous period $t-1$ however; current price p_{1t} can depend only on the previous period's shocks.

Given this informational restriction, three conditions define a symmetric outcome of this price-setting game. The first requires each firm to behave optimally: taking all other prices as given, the representative type 1 firm chooses its own price for time t to maximize its stock price q_{1t-1} at time $t-1$. The second is analogous to a zero-profit condition: in equilibrium, the representative type 1 firm's stock price must equal zero, for if $q_{1t-1} > 0$, new firms would have an incentive to enter the market. The third is a symmetry condition: since all type 1 firms are identical, each must choose the same price in equilibrium.

The appendix shows that in the symmetric outcome of this price-setting game, all type 1 firms set the scaled nominal price

$$(9)\qquad p_1 = p_1(z_{-1}, x_{-1}) = \gamma \mathrm{E}\{(x/z)\min[1, 1/\beta A(s)] \mid z_{-1}, x_{-1}\} / \mathrm{E}\{\min[1, \beta A(s)] \mid z_{-1}, x_{-1}\}$$

for time t, where z_{-1} and x_{-1} refer to time $t-1$ values.

Type 2 firms, meanwhile, can change their prices freely at any time. Hence, they behave as standard, profit-maximizing firms; in light of the linear production function, the representative type 2 firm can be viewed as choosing labor input n_{2t} to maximize its current-period profits $(P_{2t}z_t - W_t)n_{2t}$. In equilibrium, these profits must equal zero; otherwise, new firms would have an incentive to enter the market. Hence, $P_{2t} = W_t / z_t$ or, using the recursive solution (6),

$$p_2 = p_2(s) = \gamma x / \beta A(s)z. \tag{10}$$

Equilibrium

Equations (4) through (7), (9), and (10) reveal that all equilibrium prices and quantities can be easily constructed once a solution $A(s)$ is found to the functional equation (8). Thus, the next section uses (4) through (10) to derive some key properties of the economy's equilibrium.

III. PROPERTIES OF THE EQUILIBRIUM

The presence of a Phillips curve relationship between innovations in money or prices on the one hand and output or employment on the other distinguishes the equilibrium in this economy with nominal price rigidity from equilibria in flexible-price, cash-in-advance versions of the real business cycle model. The nature of this Phillips curve is best illustrated when there are no technology shocks, so that $z_t = 1$ for all $t = 0,1,2,...$, and when the monetary shock is identically and independently distributed (i.i.d.) with $1 \geq \beta E(1/x_t)$ for all $t = 0,1,2,...$. In this case, (8) implies that $A(s) = A = E(1/x_t)$.

Equations (4), (5), (9), and (10) then imply that nominal expenditures at time t are

$$P_{1t}c_{1t} + P_{2t}c_{2t} = M_t^s(p_{1t}c_{1t} + p_{2t}c_{2t}) = M_t^s x_t,$$

while real output at time t is

$$y_t = c_{1t} + c_{2t} = (\beta A / \gamma x^e)[(1-\alpha)x^e + \alpha x_t],$$

where $x^e = E(x_t)$. With $z_t = 1$, y_t also measures employment at time t. Dividing nominal expenditures by real output yields an expression for the price level:

$$P_t = M_t^s x_t(\gamma x^e / \beta A) / [(1-\alpha)x^e + \alpha x_t].$$

Hence,

$$\partial \ln(y_t) / \partial \ln(x_t) = \alpha x_t / [(1-\alpha)x^e + \alpha x_t]$$

and

$$\partial \ln(P_t) / \partial \ln(x_t) = 1 - \alpha x_t / [(1-\alpha)x^e + \alpha x_t],$$

so that an unanticipated increase in the money supply results in less than proportional increases in both real output and the price level. The slope of this short-run Phillips curve depends in a natural way on the importance of sticky prices in the economy. With $\alpha = 0$, the model reduces to a flexible-price cash-in-advance model; unanticipated money has no effect on output and yields an immediate proportional increase in the price level. With $\alpha = 1$, all prices are sticky. In this case, the price level itself is sticky, so that all unanticipated money shows up in the form of higher output. The expressions for y_t and P_t also indicate that the long-run Phillips curve is vertical; after one period, an increase in M_t^s results only in higher prices.

In addition to these Phillips-curve dynamics, the inflation tax effects that appear in Greenwood and Huffman's [1987] and Cooley and Hansen's [1989] flexible-price cash-in-advance models are also present here. To focus on these, continue to abstract from technology shocks, but now let money growth be constant as well, with $x_t = x^e \geq \beta$ for all $t = 0,1,2,...$. Equation (8) then implies that $A(s) = 1/x^e$, while (4), (5), (9), and (10) imply

$$y_t = c_{1t} + c_{2t} = \beta / \gamma x^e.$$

This expression for y_t, which again measures both output and employment in the absence of technology shocks, shows that anticipated money growth x^e acts as a distortionary tax on real balances and hence on productive activity. It encourages agents to take more leisure, reducing output and employment.

Finally, the effects of technology shocks in this economy differ from those in flexible-price cash-in-advance models. To isolate these effects, let the money supply be constant, with $x_t = 1$ and $M_t^s = 1$ for all $t = 0,1,2,...$. Equation (8) then implies $A(s) = 1$. When z_t is i.i.d. with $E(1/z_t) = 1$ for all $t = 0,1,2,...$, (4), (5), (9), and (10) imply

$$P_{1t} = \gamma / \beta,$$

$$P_{2t} = \gamma / \beta z_t,$$

$$y_{1t} = c_{1t} = \alpha \beta / \gamma,$$

and

$$y_{2t} = c_{2t} = (1 - \alpha)\beta z_t / \gamma.$$

A positive technology shock lowers marginal costs for firms of both types. This lower marginal cost is passed along to households in the form of lower prices only for the flexible-price good produced by type 2 firms, however. While P_{2t} falls and y_{2t} rises after an unexpectedly high realization of z_t, neither the price P_{1t} nor the demand-determined output y_{1t} responds. This result suggests a potential role for activist monetary policy in this economy: to induce output of the sticky-price good to vary optimally with the technology shock in a way that, because of nominal rigidity, the price system cannot. The next section shows how optimal policy performs this role.

IV. MONETARY POLICY

Optimal Activist Policy

A Pareto optimal allocation maximizes the representative household's utility subject to the resource constraints $z_t n_{it} \geq c_{it}$ for $i = 1,2$ and $t = 0,1,2,...$. Under this optimal allocation, consumptions depend only on the current technology shock z:

(11) $$c_1 = c_1^*(z) = \alpha z / \gamma$$

and

(12) $$c_2 = c_2^*(z) = (1 - \alpha) z / \gamma.$$

When the monetary authority sets

(13) $$x = \beta E(1/z \mid z_{-1}) z,$$

(8) is solved with $A(s) = 1/\beta$, and (4), (5), (9), and (10) imply that equilibrium consumptions coincide with (11) and (12). Equation (13) therefore describes the utility-maximizing, or optimal, monetary policy in this economy.

With money growth governed by (13), (7) implies that $r(s) = 0$; the optimal policy keeps the nominal interest rate at zero. Thus, although (13) is more complicated than the steady withdrawal of money advocated by Friedman [1969], Friedman's conclusion that the nominal interest rate should be zero still applies. On the other hand, the Friedman rule $x_t = \beta$ for all $t = 0,1,2,...$ also yields $r(s) = 0$. Zero nominal interest rates are therefore a necessary, but not sufficient, feature of the optimal monetary policy. Here, the presence of nominal price rigidity rationalizes an activist policy that responds to technology shocks in a special way.

The optimal activist policy (13) calls for higher rates of money growth when the technology shock turns out to be higher than expected. Under the optimal allocation, consumption of both goods rises after a positive technology shock. In equilibrium, however, nominal rigidity prevents P_{1t} from falling so as to induce higher consumption of the sticky-price good. Instead, faster money growth generates the appropriate response in purchases of the sticky-price good, c_{1t}.

Relative to flexible-price cash-in-advance specifications, therefore, this model identifies a second role for monetary policy. In addition to removing the inflation tax, optimal policy allows output to fluctuate as it would in an economy without nominal rigidities.

The Friedman Rule

The optimal policy (13) requires money to respond contemporaneously to technology shocks. Friedman [1968] argues that the monetary authority is unlikely to possess the detailed information about the economy that would allow it to implement such a policy and advocates one of constant money growth instead. The utility-maximizing policy in this class sets $x_t = \beta$ for all $t = 0,1,2,...$, as called for by Friedman [1969]. Under this rule, (8) has the solution $A(s) = 1/\beta$ and (4), (5), (9), and (10) imply that consumptions are given by

(14) $c_1 = c_1^F(z_{-1}) = \alpha / \gamma \mathrm{E}(1/z \mid z_{-1})$

and

(15) $c_2 = c_2^F(z) = (1-\alpha)z/\gamma.$

The Friedman rule keeps the nominal interest rate at zero and thereby accomplishes one goal of monetary policy. It fails, however, to accomplish the second goal of allowing the economy to respond optimally to technology shocks.

Welfare Comparison

Equations (12) and (15) show that $c_2^*(z) = c_2^F(z)$. Equations (11) and (14), however, imply $c_1^*(z) \neq c_1^F(z_{-1})$ except in two special cases. The first special case occurs when $\alpha = 0$. In this case, the model reduces to a flexible-price cash-in-advance economy; the Friedman rule is optimal, just as in Greenwood and Huffman [1987] and Cooley and Hansen [1989]. The second special case arises when $z_t = z^e = \mathrm{E}(z_t)$ for all $t = 0,1,2,\ldots$. In this case, there are no technology shocks for policy to respond to; the Friedman rule is again optimal.

Thus, in the presence of both nominal price rigidity and technology shocks, the Friedman rule fails to implement the optimal resource allocation. How far from optimal the Friedman rule is in welfare terms remains to be determined, however. Here, the welfare cost of adopting the Friedman rule is measured by the permanent percentage increase in consumption that makes the representative household as well off under the Friedman rule as it is under the optimal policy. That is, the welfare cost ω is defined implicitly by

(16) $$U^* = \mathrm{E}\sum_{t=0}^{\infty} \beta^t[\alpha \ln(\omega c_{1t}^F) + (1-\alpha)\ln(\omega c_{2t}^F) + \gamma(1 - n_{1t}^F - n_{2t}^F)],$$

where U^* denotes expected utility under the optimal policy and c_1^F, c_2^F, n_1^F, and n_2^F are consumptions and labor supplies under the Friedman rule. The equilibrium conditions $y_{it} = c_{it}$, $i = 1,2$, imply that this percentage increase in consumption translates directly into a percentage increase in output.

The production functions $y_{it} = z_t n_{it}$ and the equilibrium conditions $y_{it} = c_{it}$ determine labor supplies as $n_{it} = c_{it}/z_t$ for $i = 1,2$ and $t = 0,1,2,\ldots$. Hence, (11), (12), (14), and (15) can be used to solve (16) for ω:

(17) $$\omega = \exp(\alpha \mathrm{E}\{\ln(z) + \ln[\mathrm{E}(1/z \mid z_{-1})]\}).$$

Equation (17) confirms that $\omega = 1$, so that the welfare cost is zero, when $\alpha = 0$ or when z is nonstochastic. In general, the welfare cost is increasing in α; the more important sticky prices become, the more costly it is to follow the Friedman rule.

Suppose that the logarithm of z_t follows the first-order autoregression

(18) $$\ln(z_t) = \rho \ln(z_{t-1}) + \varepsilon_t,$$

where $1 > \rho > -1$ and ε_t is i.i.d. and normally distributed with mean zero and standard deviation σ. In this case, (17) specializes to

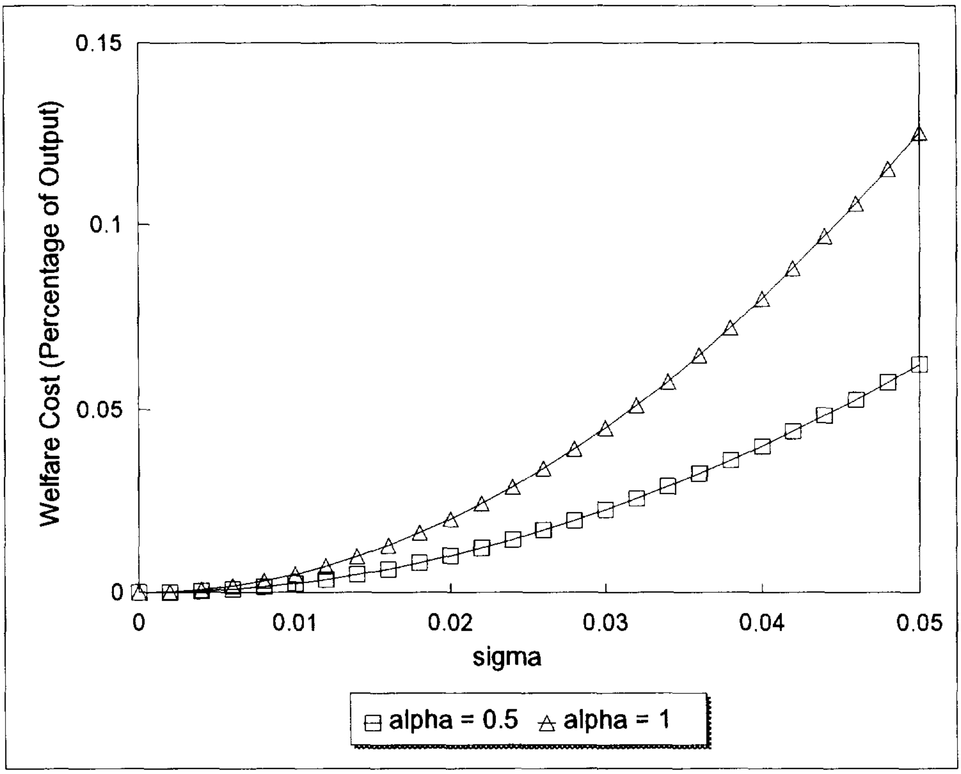
(19) $$\omega = \exp(\alpha\, \sigma^2/2).$$

The persistence parameter ρ does not enter (19); the welfare cost of the Friedman rule depends only on the variance of unanticipated changes in technology.

In the real business cycle literature, Prescott [1986] sets the standard deviation of the technology shock equal to 0.0076, matching the standard deviation of measured innovations to the Solow residual. Using this value of σ, (19) implies $100(\omega = 1) = 0.0014$ when $\alpha = 0.5$; the welfare cost of adopting the Friedman rule instead of the optimal policy is just 0.0014% of output. Even in the extreme case where all prices are sticky, so that $\alpha = 1$, the Friedman rule costs only 0.0029% of output.

Figure 1 plots the percentage welfare cost $100(\omega - 1)$ for various values of σ when $\alpha = 0.5$ and $\alpha = 1$. With technology shocks twice the size of those used by Prescott, $\sigma = 0.0152$, the welfare cost remains trivial: 0.0058% of output when $\alpha = 0.5$% and 0.012% of output when $\alpha = 1$. With $\sigma = 0.05$, more than 6½ times Prescott's value, the cost when $\alpha = 1$ is still only 0.125% of output.

FIGURE 1
Welfare Cost of the Friedman Rule for Various Values of Sigma



Robustness

The results derived above show that while the Friedman rule is no longer optimal in the presence of nominal price rigidity and technology shocks, the welfare cost of adopting this simple policy is very small. Thus, flexible-price cash-in-advance models that identify the Friedman rule as optimal continue to provide useful guidance for monetary policy, even when some prices are sticky. This final section establishes the robustness of these results by considering a number of alternative assumptions regarding the form of the nominal rigidity and the information available to the monetary authority.

Cho and Cooley [1992] generalize the form of nominal rigidity used here and in Dow [1995] by requiring some firms to set their prices $J \geq 1$ periods in advance. When this more general form of price rigidity is imposed on the model developed here, the implications of the representative household's problem remain as before, except that the description of the aggregate state must include the realizations of shocks from the previous J periods; equations (4) through (8) continue to apply, but with $s_t = (z_t, x_t, z_{t-1}, x_{t-1}, \ldots, z_{t-J}, x_{t-J})$. Similarly, the prices charged by type 2 firms continue to be given by (10).

The price-setting game played by type 1 firms also remains as before, except that these firms choose nominal prices for time t at the end of time $t - J$. In the symmetric outcome of this price-setting game, (9) generalizes to

(20) $$p_1 = p_1(z_{-J}, x_{-J})$$
$$= \gamma \mathrm{E}\{M^s(x/z)\min[1,1/\beta A(s)] \mid z_{-J}, x_{-J}\}$$
$$/ M^s \mathrm{E}\{\min[1,\beta A(s)] z_{-J}, x_{-J}\},$$

where z_{-J} and x_{-J} refer to time $t - J$ values and M^s refers to the money supply at the beginning of time t.

When $J > 1$, Cho and Cooley's form of nominal rigidity becomes more severe than Dow's so that in general, monetary policy can no longer implement the optimal allocation described by (11) and (12). In fact, the welfare-maximizing policy is difficult to characterize when $J > 1$. Since the level of welfare achieved by the best monetary policy can be no higher than that achieved under the optimal allocation, however, it is still possible to place an upper bound on the welfare cost of following the Friedman rule instead of the optimal activist policy by comparing the allocation achieved by the Friedman rule to (11) and (12).

Under the Friedman rule, (8) has the solution $A(s) = 1/\beta$. Thus, when prices are set J periods in advance, (4), (5), (10), and (20) imply that consumptions are given by

$$c_1 = c_1^{FJ}(z_{-J}) = \alpha/\gamma E(1/z|z_{-J})$$

and

$$c_2 = c_2^{FJ}(z) = (1-\alpha)z/\gamma.$$

Substituting these expressions, along with (11) and (12), into (16) yields

$$\omega = \exp\{\alpha[1 + \rho^2 + \ldots + \rho^{2(J-1)}]\sigma^2/2\},$$

when z_t follows the autoregressive process (18). Since $1 > \rho > -1$, this value of ω is bounded above by

$$\omega^b = \exp(\alpha J \sigma^2/2). \tag{21}$$

Hence, (21) gives an upper bound on the welfare cost of following the Friedman rule instead of the optimal policy when firms set prices J periods in advance.

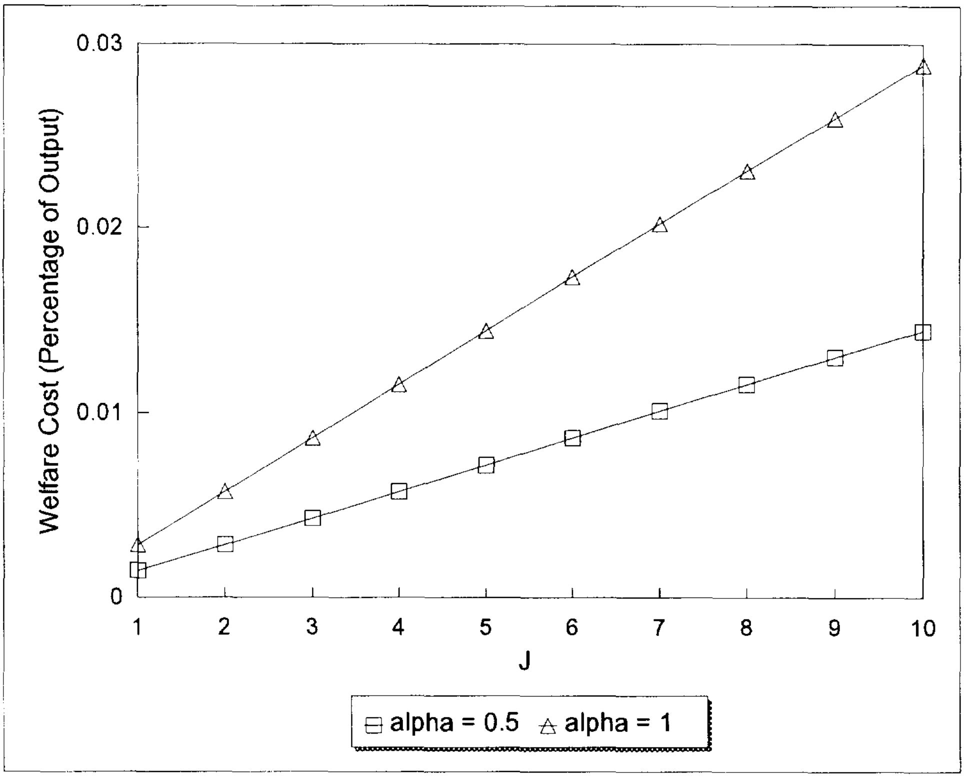
Figure 2 plots the upper bound (21), in percentage terms, for various values of J when σ equals Prescott's value of 0.0076 for the cases $\alpha = 0.5$ and $\alpha = 1$. The bound is increasing in J, suggesting that with prices set farther in advance, it becomes more costly to follow the Friedman rule. However, even with $J = 8$, the highest value considered by Cho and Cooley, the welfare cost of adopting the Friedman rule instead of the optimal policy can be no larger than 0.012% of output when $\alpha = 0.5$% and 0.023% of output when $\alpha = 1$. Thus, the results derived above appear quite robust to generalizing the form of nominal rigidity as suggested by Cho and Cooley.

Yun [1994a] uses a form of nominal rigidity that is somewhat different from those of Dow and Cho and Cooley; he assumes that individual firms can change their prices only at randomly determined dates. Yun [1994b] demonstrates that under this alternative form of nominal rigidity, optimal policy is activist, just as it is here. In fact, Yun's optimal activist policy calls for an increase in money growth following a positive technology shock, just as (13) does here. Thus, these conclusions appear robust to changing the form of nominal rigidity to the one suggested by Yun. Yun does not compute the welfare cost of following the Friedman rule instead of his optimal activist policy, however; determining whether this cost is small, as it is here, remains a task for future research.

Finally, it is worth emphasizing that the optimal activist policy described by (13) requires the money supply to respond immediately to the technology shock and therefore assumes that the monetary authority receives perfect information about the current state of the economy. In terms of informational requirements, the Friedman rule lies at the opposite extreme: it does not require the monetary authority to have any knowledge of the economy's past or present states. Intermediate to these two extremes are cases where the monetary authority receives only partial information, either because it observes the technology shock with a lag or because it observes a noisy signal of the current technology shock.

In these cases with partial information, the monetary authority may no longer be able to implement the optimal allocation described by (11) and (12). Once again, however, the level of welfare achieved by the best policy under partial information can be no higher than that achieved by (11) and (12). Thus, under partial information, (19) becomes an upper bound on the welfare cost of adopting the Friedman rule instead of the optimal activist policy. And since the values of ω implied by (19) are small, the cost of following the Friedman rule must continue to be small.

FIGURE 2
Upper Bound on the Welfare Cost of the Friedman Rule When Prices are Set J Periods in Advance



V. CONCLUSION

Following in the real business cycle tradition of Kydland and Prescott [1982], the model developed here begins by identifying technology shocks as important sources of fluctuations in output and employment. The model departs from the real business cycle framework, however, not only by providing a role for money as a means of exchange, but also by introducing an element of nominal price rigidity. In fact, the addition of sticky prices yields an equilibrium with features—including the presence of a Phillips curve relationship between the money supply and aggregate output—far different from those found in the standard real business cycle model.

The results show that the joint presence of sticky prices and technology shocks rationalizes an activist monetary policy. They also show, however, that the constant money growth rule advocated by Friedman [1969] comes very close to the optimum in welfare terms. Flexible-price, monetary versions of the real business cycle model, like those of Greenwood and Huffman [1987] and Cooley and Hansen [1989], identify the Friedman rule as optimal; the results derived here confirm that this policy implication is still useful in the presence of nominal price rigidity.

Hansen and Prescott [1993] interpret technology shocks very broadly, incorporating in their definition anything that impacts on private production possibilities, including various political and institutional changes. To the extent that this broad interpretation is warranted, the results derived here describe the appropriate monetary policy under a wide range of circumstances.

Rasche and Tatom [1981] present evidence that energy price increases have lowered productivity in the United States economy, just as technology shocks do in the real business cycle model; their work suggests that the results derived here also describe the optimal monetary response to an oil price shock. In fact, Rotemberg [1983] develops a sticky-price model in which energy price shocks are considered explicitly. He concludes that an increase in oil prices calls for a decrease in money growth. The results here confirm Rotemberg's, but add one important qualification: while the optimal monetary response to an oil price shock is activist, there is little welfare loss in holding money growth constant instead. Indeed, by showing that a constant money growth rule comes close to the optimum, the results support Friedman's [1968] view: the simplest monetary policies rank among the best.

APPENDIX

Household Optimization

Combining the first-order and envelope conditions for (3) with the equilibrium conditions $m = m' = 1$, $b = b' = 0$, $a_1 = a_1' = 1$, and $a_2 = a_2' = 1$ yields

(A1) $$\alpha = (\lambda + \mu)p_1 c_1,$$

(A2) $$1 - \alpha = (\lambda + \mu)p_2 c_2,$$

(A3) $$\gamma = \lambda w,$$

(A4) $$\lambda x = \beta \mathrm{E}(\lambda' + \mu' \mid s),$$

(A5) $$(\lambda + \mu)x = \beta(1 + r)\mathrm{E}(\lambda' + \mu' \mid s),$$

(A6) $$\lambda q_1 = \beta \mathrm{E}[\lambda'(q_1' + d_1') \mid s],$$

(A7) $$\lambda q_2 = \beta \mathrm{E}[\lambda'(q_2' + d_2') \mid s],$$

and

(A8) $$x \geq p_1 c_1 + p_2 c_2,\ \mu \geq 0,$$

$$\mu(x - p_1 c_1 - p_2 c_2) = 0,$$

where λ and μ are nonnegative multipliers on the constraints (1) and (2).

Let

(A9) $$A(s) = \mathrm{E}(\lambda' + \mu' \mid s).$$

Then (A4) implies

(A10) $$\lambda = \lambda(s) = \beta A(s)/x.$$

Substituting (A10) into (A3) yields (6) in the text.

Suppose first that $\mu > 0$. Then (A1), (A2), and (A8) imply

$$\lambda + \mu = 1/x,$$

$$c_1 = \alpha x / p_1,$$

and

$$c_2 = (1 - \alpha)x / p_2.$$

Moreover, by (A5), (A9), and (A10),

$$r = 1/\beta A(s) - 1$$

and

$$\mu = [1 - \beta A(s)]/x,$$

so that this case requires

$$1 > \beta A(s).$$

Next, suppose that $\mu = 0$. Then (A1), (A2), (A5), and (A10) imply

$$c_1 = (\alpha x / p_1)[1/\beta A(s)],$$

$$c_2 = [(1 - \alpha)x / p_2][1/\beta A(s)],$$

and

$$r = 0,$$

so that, by (A8), this case requires

$$\beta A(s) \geq 1.$$

Combining the results for $\mu > 0$ with those for $\mu = 0$ yields (4), (5), and (7) in the text. Finally, combining

$$\mu(s) = (1/x)\max[1 - \beta A(s), 0]$$

with (A10) yields

$$\lambda(s) + \mu(s) = (1/x)\max[1, \beta A(s)].$$

Substituting this result into (A9) yields (8) in the text.

Firm Optimization

In a symmetric outcome of their price-setting game, all type 1 firms set the same scaled nominal price and split market demand evenly; with N type 1 firms and N households, the representative firm satisfies the representative household's demand. Hence, the representative type 1 firm's scaled nominal dividend at time t is

$$\text{(A11)} \qquad d_{1t} = (p_{1t} - w_t / z_t) y_{1t},$$

with $y_{1t} = c_{1t}$, where (4) and (6) give the solutions for c_{1t} and w_t.

In a symmetric outcome, $q_{1t} = 0$ for all $t = 0, 1, 2, \ldots$. Hence, (A6) requires

$$\beta\, \mathrm{E}(\lambda_t d_{1t} \mid z_{t-1}, x_{t-1}) = 0,$$

where (A10) gives the solution for λ_t. Combining this expression with (4), (6), (A10), and (A11) with $y_{1t} = c_{1t}$ reveals that in a symmetric outcome, p_{1t} must be given by (9).

It remains to establish that when all other type 1 firms set the time t price given by (9), the representative firm maximizes its stock price by also setting its time t price according to (9). Thus, consider the situation faced by the representative type 1 firm when it sets the time t price $p*$ and all other type 1 firms set the time t price p_{1t} given by (9).

If $p* > p_{1t}$, then the representative firm's price is higher than those of all other firms. In this case, the representative firm attracts no demand; (A11) with $y_{1t} = 0$ implies that its scaled nominal dividend at time t is

$$\text{(A12)} \qquad d_{1t} = d_1(s_t, p*) = 0.$$

If $p* = p_{1t}$, then the representative firm's price is the same as those of all other firms. In this case, the firms split market demand evenly; the representative firm satisfies the representative household's demand. Using (4), (6), and (A11) with $y_{1t} = c_{1t}$, the firm's scaled dividend at time t is

$$\text{(A13)} \qquad d_{1t} = d_1(s_t, p*)$$

$$= \alpha\, x_t [1 - \gamma x_t / \beta A(s_t) z_t p*] \min[1, 1/\beta A(s_t)].$$

Finally, if $p* < p_{1t}$, then the representative firm's price is below those of all other firms. In this case, the representative firm attracts the demand of all N households; (4), (6), and (A11) with $y_{1t} = N c_{1t}$ imply that its scaled dividend at time t is

$$\text{(A14)} \qquad d_{1t} = d_1(s_t, p*)$$

$$= N\alpha x_t [1 - \gamma x_t / \beta A(s_t) z_t p*] \min[1, 1/\beta A(s_t)].$$

By (A10), the equilibrium value of λ_{t-1} does not depend on the representative firm's choice of $p*$. Thus, (A6) implies that the firm's problem of choosing $p*$ to maximize its stock price q_{1t-1} is equivalent to the problem of choosing $p*$ to maximize

$$\text{(A15)} \qquad \pi(z_{t-1}, x_{t-1}, p*)$$

$$= \mathrm{E}[\lambda(s_t) d_1(s_t, p*) \mid z_{t-1}, x_{t-1}].$$

Equations (9), (A10), (A13), and (A15) imply that $\pi(z_{t-1}, x_{t-1}, p*) = 0$ when the representative firm chooses $p* = p_{1t}$. Equations (A12) and (A15) imply that $\pi(z_{t-1}, x_{t-1}, p*) = 0$ when the representative firm chooses $p* > p_{1t}$. Thus, to verify that the representative firm maximizes $\pi(z_{t-1}, x_{t-1}, p*)$ by choosing $p* = p_{1t}$, it suffices to show that for any $p* < p_{1t}$, $\pi(z_{t-1}, x_{t-1}, p*) < 0$.

Suppose, therefore, that for some $p* < p_{1t}$, $\pi(z_{t-1}, x_{t-1}, p*) \geq 0$. Equations (A10), (A14), and (A15) then imply

$$0 \leq \pi(z_{t-1}, x_{t-1}, p*)$$

$$= N\alpha \mathrm{E}\{[\beta A(s_t) - \gamma x_t / z_t p*]$$

$$\min[1, 1/\beta A(s_t)] \mid z_{t-1}, x_{t-1}\}$$

and hence, using (9),

$$p_{1t} = \gamma\, \mathrm{E}\{(x_t / z_t) \min[1, 1/\beta A(s_t)] \mid z_{t-1}, x_{t-1}\}$$

$$/\mathrm{E}\{\min[1, \beta A(s_t)] z_{t-1}, x_{t-1}\} \leq p*.$$

This last inequality, however, contradicts the assumption that $p* < p_{1t}$ and thereby shows that the representative firm maximizes its stock price by choosing $p*$ according to (9) in the text.

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