Economic growth, financial evolution, and the long-run behavior of velocity

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A general equilibrium model is presented in which buyers and sellers find it increasingly difficult to make private credit arrangements as economic activity spreads into new markets; hence, the demand for money increases as the structure of production and trade becomes more complex. At the same time, however, financial innovations allow agents to economize on their cash balances. Together, these effects of technological progress account for two empirical regularities, the U-shaped pattern in velocity and the ever-increasing share of productive resources devoted to the private financial sector, that are associated with the process of real economic growth.

Key words: Economic growth; Financial innovation; Velocity of money
JEL classification: E41; O42

1. Introduction

In a very real sense, alterations in the economy, and hence economic development, consist in an alteration in the system of exchange. Economic growth (an increase in per capita income or wealth) is usually based upon institutional changes, of which the growing complexity and sophistication of the exchange system is the major index.

C.S. Belshaw, Traditional Exchange and Modern Markets

This paper presents a general equilibrium model in which, as suggested above, the process of real economic growth is intimately related to that of evolution in the structure of decentralized trade.

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Begin by considering a small, rural, predominantly agricultural economy in which, to again use Belshaw’s (1965, pp. 66–67) words, ‘people are known to one another and there is much continual interdependence, forces of reciprocal obligation create a sanction system so that traders and peasants can depend on the fulfillment of obligations, for example the repayment of debts’. By making the production and transportation of goods less costly over time, technological progress may work in this economy not only to increase activity in existing markets but to open new markets as well: trading regions may expand geographically, an ever-widening range of goods may be bought and sold, and the structure of exchange may thereby become increasingly complex.

One might expect these aspects of growth to make it progressively more difficult for agents to establish the strong personal ties that guarantee the repayment of debts and hence to increase the demand for money as a portable, easily identified, anonymous accounting device. In fact, Simon and Phoebe Ottenberg (1962) find this to be true in their detailed study of the development and monetization of the Afikpo market system in southern Nigeria, while Warburton (1949) and Friedman and Schwartz (1982) attribute the secular decline in the income velocity of money in the nineteenth century United States to similar factors.

On the other hand, technological progress may also reduce the costs of processes through which agents become known to one another mechanically or electronically, restoring their ability to trade without money as credit card networks have recently done in the US economy. These two effects of technological change on the demand for money, though potentially offsetting, need not operate symmetrically over time. The U-shaped patterns in velocity found by Bordo and Jonung (1987) to appear both within economies as they grow and across economies ranked by income per capita suggest that the first is more important in the early stages of development but is eclipsed later by the second. Government-issued money comes to dominate a payments system but is eventually reduced to being just one of many competing means of exchange.

The model is designed to capture each of the aspects of growth noted above. Buyers and sellers may use private credit agreements as a substitute for money but find it increasingly difficult to do so as technological change permits the spread of economic activity into new markets. Agents therefore use money more frequently as the structure of production and trade in the economy becomes more complex. The resource costs of overcoming the informational constraints that inhibit the use of credit may decrease over time, however; this financial innovation allows agents to economize on their cash balances. Together, these effects of technological progress determine how the role of money changes over time in a growing economy with an evolving system of payments.

Section 2 reviews the evidence cited above, while in section 3 the model is specified formally. Section 4 makes some progress in describing analytically a class of competitive equilibria for the model economy. A uniqueness result is
established and an operational procedure for finding equilibria is developed. Section 5 then applies this procedure to explore numerically the effects generated by technological change in the model economy. Section 6 concludes by pointing to directions for future research.

2. **Long-run patterns in monetary velocity and financial evolution**

Fig. 1 displays the long-run behavior of the income velocities of the United States monetary aggregates M1 and M2, using gross national product as the measure of income.¹ For either empirical definition of money, velocity declined secularly until the end of World War II and has risen since then, with the postwar increase being far more dramatic for M1 than for M2. An extensive study by Bordo and Jonung (1987) documents the existence of similar U-shaped velocity patterns in time series going back into the nineteenth century for a number of other countries as well, including Australia, Canada, Denmark, Denmark,

¹ Data sources for figs. 1–4 are given in the appendix.
Norway, and Sweden [see also Bordo and Jonung (1990)]. For the United Kingdom, Collins (1983, p. 388, table 5) reports that much of the initial decline in velocity predates the start of Bordo and Jonung's series (shown in fig. 2), reflecting the early development of the English banking system. There, too, velocity increased from the late 1940's until 1979, when the legal and institutional changes began that ultimately led the Bank of England to discontinue the publication of this series [see Bank of England (1986, 1989)].

U-shaped velocity patterns have been found in cross-sectional data as well. Ezekiel and Adekunle (1969) discover a negative correlation between velocity and income across 37 countries for the period 1950–1964 except in the highest product per capita group where the correlation is positive. Bordo and Jonung (1987) use a regression equation to uncover a U-shaped 'global velocity curve' when data are pooled from 74 countries, 1952–1982. Most recently, Ireland (1991b) finds evidence consistent with U-shaped velocity patterns using cross-sectional regression equations estimated with US regional data, 1929–1988. Thus, both the secular and the cross-sectional evidence suggest a link between the U-shaped pattern in velocity and the process of real economic growth.

In fact, that velocity tends to fall (or, equivalently, that the demand for money relative to income tends to increase) in the early stages of economic growth is
often referred to in the development literature as the process of monetization, i.e., Belshaw (1965, p. 10):

One aspect of the transformation of simple economies into complex, dynamic ones is the increase in the extent of the market, that is, an increase in the range and quantity of transactions to which market principles apply, and an intensification of those principles. To a large extent, this is linked with an increase in exchange liquidity which can be described as the monetization of the economy.

A particularly detailed account of the monetization process is provided by Ottenberg and Ottenberg (1962) in their study of the Afikpo market system in Nigeria. Before the turn of the present century, Afikpo villagers rarely ventured far from home. Intervillage trade was very limited; a small range of locally produced goods was exchanged within the village, in part through barter but primarily through ritual and ceremony. The arrival of the British in 1902, however, leading to the cessation of intervillage warfare and to the construction of bicycle paths, soon established Afikpo's link with the developing Nigerian economy. An increasingly diverse range of goods, imported from an ever-widening geographic area, began to appear in the Afikpo market. Traders from outside of Afikpo arrived in increasing numbers; a 'stranger's quarter' was founded in the village. By 1952, although ceremonial trade remained important among Afikpo natives, British West African currency completely dominated as a means of exchange in the marketplace. Change along these lines merely accelerated after 1952 with the widening and paving of roads, permitting an increase in lorry traffic and the establishment of daily bus service through Afikpo. Growth in this community involved far more than an increase in per capita income. As trading regions expanded, as an ever-increasing range of goods was brought to market, and as strangers became more numerous, the economy was transformed from one in which exchange was almost entirely governed by ceremony and ritual – a sanction system among continually interdependent agents – into one in which the use of money – an anonymous accounting device – was widespread.

Although accounts of the monetization process are most frequently found in studies of contemporary developing economies, the factors underlying velocity's initial decline in the United States have been described in similar terms. Warburton (1949), for instance, discovers that the downward trend seen in fig. 1 extends back to 1799 and attributes the trend to such changes as the increase in the share of national output sold in organized markets, the increase in the fraction of the population working for wages instead of producing for their own consumption, and the increase due to specialization in the number of intermediate payments required in production. Friedman and Schwartz (1982, p. 144) contrast the late nineteenth century behavior of money demand in the United States, where velocity fell rapidly (see fig. 1), with that in the United Kingdom, where it actually increased slightly (fig. 2), and note that while by 1870 the 'money economy' had spread widely throughout
Britain, in the United States industrialization, urbanization, and the development of the banking system had only just begun. Quantitative evidence of ongoing structural change in the US economy during this period is given in table 1. In the United States as in Afikpo village, economic growth in its early stages involved changes far beyond those in product per capita. With an increase in the volume of trade conducted in environments where agents were more likely to meet as strangers than as lifelong neighbors came an increase in the demand for money, the value of which, Ben-Porath (1980, p. 13) emphasizes, 'is independent of the identity of the seller'.

Considering now the subsequent reversal in velocity’s initial downward trend, there is considerable evidence to suggest that the postwar increase in velocity in the United States is at least partially the result of the development and proliferation of assets that substitute for money as means of exchange. Some of these new financial instruments, such as money market mutual funds and bank credit cards, allow households to carry lower cash balances. Others, such as repurchase agreements, Eurodollar deposits, cash concentration accounts, and zero balance accounts, provide ways for firms to economize on their money holdings. Two surveys of the recent empirical money demand literature, the first by Judd and Scadding (1982) and the second by Goldfeld and Sichel (1990), find the consensus to be that these innovations have had quantitatively important effects on velocity. Some of the new assets (money market mutual funds, Eurodollar deposits, and repurchase agreements) are now accounted for in M2, which may explain why the rise in the velocity of M2 has been slight compared to that of M1.

The hypothesis that technological innovations in manufacturing and finance give rise to a U-shaped pattern in velocity as an economy grows also appears in

### Table 1
Institutional changes, United States, 1840–1910.

<table>
<thead>
<tr>
<th>Year</th>
<th>Population in rural areas (% of total)</th>
<th>Labor force in agriculture (% of total)</th>
<th>Assets of all banks ($ per capita)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1840</td>
<td>89.2</td>
<td>63.1</td>
<td>38.55</td>
</tr>
<tr>
<td>1850</td>
<td>84.7</td>
<td>54.8</td>
<td>22.94</td>
</tr>
<tr>
<td>1860</td>
<td>80.2</td>
<td>52.9</td>
<td>31.80</td>
</tr>
<tr>
<td>1870</td>
<td>72.0</td>
<td>52.5</td>
<td>44.73</td>
</tr>
<tr>
<td>1880</td>
<td>71.8</td>
<td>51.3</td>
<td>67.77</td>
</tr>
<tr>
<td>1890</td>
<td>64.9</td>
<td>42.7</td>
<td>101.00</td>
</tr>
<tr>
<td>1900</td>
<td>60.3</td>
<td>40.2</td>
<td>149.85</td>
</tr>
<tr>
<td>1910</td>
<td>54.3</td>
<td>31.4</td>
<td>249.23</td>
</tr>
</tbody>
</table>

the work of Goldsmith (1969), McKinnon (1973), and Shaw (1973). All argue that in the early stages of growth the expansion of market activity is facilitated by the spread of the monetary and banking systems, while in later stages nonbank intermediaries emerge to provide alternative financial instruments in progressively greater quantities and of progressively higher quality. According to this hypothesis, government-issued money first comes to dominate the national system of payments, then is reduced to being just one of many competing assets. Also according to this hypothesis, growth in the private financial sector ought to accompany the U-shaped pattern in velocity.

Kuznets (1971), in fact, does find a positive correlation between the share of the finance industry in GDP and the level of real per capita GDP. Kuznets's secular picture (p. 274, table 41.B) shows that the percentage of all workers employed in banking and insurance increased from 1.1 to 2.1 in France, 1906–1954; from 0.5 to 1.8 in the Netherlands, 1899–1947; and from 0.3 to 1.7 in Norway, 1920–1960. In addition, the share of banking, insurance, and real estate is found to have an elasticity of 0.68 – higher than any other subdivision of any major sector – with respect to product per capita in a cross-section of 57 countries sampled in 1958 (p. 108, table 13). Also, in the 1957 cross-section, the share of the commerce subdivision in total employment is found to be over 350% higher for the wealthiest countries than for the lowest income group (p. 200, table 28).

Fig. 3 shows that a secular increase in the share of banks and credit agencies in US labor markets has been interrupted during the past 120 years only by the Great Depression and World War II. Fig. 4, meanwhile, reveals a positive correlation between the share of GDP originating in finance, insurance, and real estate (FIRE) and the level of per capita GNP in US dollars across 103 countries in 1986. Once again, both secular and cross-sectional evidence link the changing role of the financial sector to the process of economic growth.

The model is designed to capture the two empirical regularities identified here with the growth process: the U-shaped pattern in velocity and the ever-increasing share of aggregate resources devoted to financial activity. In the model agents may use private credit agreements as a substitute for government-issued money only through the allocation of productive resources to a financial, or intermediation, technology. Real economic growth involves not only an increase in aggregate output but also a continual spread of economic activity into new markets, a geographic expansion of trading regions, and an increased diversity in the range of goods produced and sold. As suggested by the Ottenbergs (1962), Warburton (1949), and Friedman and Schwartz (1982), these additional aspects of growth make it more difficult for agents to establish and maintain the relationships necessary to trade without money and thus contribute to a downward trend in velocity. Technological progress also reduces the costs of overcoming the informational constraints that inhibit the use of credit, however; innovations in the financial sector increase velocity in the model as they have in
recent US monetary history. If these two effects of technological change proceed at the appropriate rates, not only does velocity trace out a U-shaped pattern, but the fraction of the model economy's productive resources allocated to financial activities increases steadily over time as well. The artificial series generated by the model share both of the stylized features found in the data.

Another branch of literature to which the present work is related begins with Levhari and Patinkin (1968), who include real balances as an argument in an aggregate production function based upon the observation that 'an economy without money would have to devote effort (read: labor and physical capital) in order to achieve the multitude of 'double coincidences' ... on which successful barter is based' (pp. 737–738). Analogously, Saving (1971) and Niehans (1971) develop models in which consumers hold money in order to reduce the time and effort of the exchange process; McCallum and Goodfriend (1987) also use a model of this type. As shown by Nadiri (1969) for the case of firms and by Dutton and Gramm (1973) for consumers, these models have the testable implication that the real wage belongs in a correctly specified money demand equation; in fact, both studies find the coefficient on the wage rate to be positive and statistically significant. Karni (1974) defines the brokerage fee parameter in the Baumol–Tobin inventory model of money demand to include the real value
of time spent by agents on trips to the bank; the average level of real money holdings therefore depends positively on both the wage rate and the length of time required for each visit to the bank. Most recently, Gillman (1987) presents a version of the cash-in-advance model in which credit, as an alternative means of exchange, is produced through the allocation of productive time to a household technology, so that the real wage is again predicted to enter into the money demand function.

In all of these models, agents make decisions based upon a trade-off between the opportunity costs of holding money and the resource costs of economizing on cash balances. Because in each case the resource costs are specified in terms of labor, the wage rate is relevant in determining money demand. This trade-off also plays a central role in the model presented here. By explicitly considering a general equilibrium environment with a large number of agents, however, it is possible to imagine that a subset of these agents, comprising a distinct financial sector, specialize in the business of producing substitutes for money. The predicted relationship between the demand for money relative to income and the fraction of the labor force employed in the financial sector is then an additional implication of this model that may, in principle at least, be tested.
Finally, the present work is related to a number of recent studies that consider various activities performed by the private financial sector during the course of long-run economic growth. Levine (1990), Greenwood and Jovanovic (1990), and Bencivenga and Smith (1991) focus on the role of financial intermediaries in allocating savings toward their most efficient use. As these studies examine how the financial sector facilitates sustained economic growth, the present study examines how economic growth, in turn, affects the financial sector's role in the operation of the payments system. In this respect, this paper's immediate predecessor is work by Townsend (1983). In Townsend's model, agents begin by trading in decentralized markets where cash-in-advance constraints apply. As part of the growth process, an ever-increasing fraction of these agents gain access to a centralized Arrow–Debreu market where a means of exchange is not needed. Thus, Townsend captures the upward trend that forms the second half of velocity's U-shaped pattern. The model presented here goes beyond Townsend's to account for both parts of the U-shape in velocity as well as for the growth of the private financial sector that accompanies velocity's long-run pattern.

3. A model of economic growth and financial evolution

3.1. The economic environment

The model economy consists of a continuum of identical infinitely-lived households that inhabit an environment in which production and trade take place in a continuum of spatially distinct markets. Markets are indexed by $i \in [0, \infty)$, where market 0 is the economy's financial center at which all agents begin and end every period, and where the index $i$ measures the distance between the center and the $i$th market. In each market other than the financial center, a distinct perishable consumption good may potentially be produced and sold. Thus, the consumption goods are indexed by $i \in (0, \infty)$ corresponding to the markets in which they are produced. There is perfect certainty; time is discrete and indexed by $t = 0, 1, \ldots$

Preferences, endowments, and technologies are specified below so that economic growth in this model involves not only an increase in activity within any given market, but also a continual spread of activity into new, more distant markets. Consequently, increases in aggregate output are accompanied by the geographic expansion of trading regions as well as by increases in the range of distinct goods bought and sold. As the structure of production and trade becomes more complex in these ways, agents find it increasingly difficult to establish the relationships necessary to trade on credit, and hence find government-issued money increasingly useful as a means of exchange. At the same time, however, innovation in the financial sector may help to restore their ability to trade without money. Thus, both of the effects of technological progress on exchange possibilities discussed in the previous section are present here.
3.2. Preferences and endowments

Since households are identical, the analysis to follow considers the behavior of a single representative household, endowed with one unit of productive time in each period \( t \geq 1 \) and having preferences over leisure and all of the consumption goods.

Letting \( c_t : (0, \infty) \mapsto \mathbb{R} \) denote consumption of each good and \( x_t \in \mathbb{R} \) denote leisure taken by the representative household at time \( t \), the commodity space is defined as \( L = \prod_{t=1}^{\infty} L_t \), where

\[
L_t = \{(c_t, x_t) : c_t : (0, \infty) \mapsto \mathbb{R} \text{ piecewise continuous}; x_t \in \mathbb{R}\}.
\]

The household's consumption set is \( X = \prod_{t=1}^{\infty} X_t \), where

\[
X_t = \{(c_t, x_t) : c_t : (0, \infty) \mapsto [0, \infty) \text{ piecewise continuous}
\]

and \( \exists y_t \in (0, \infty) \) such that \( c_t(i) = 0 \ \forall \ i \geq y_t; \ x_t \in [0, 1] \).

For technical purposes, \( c_t(0) \) is defined for all \( t \geq 1 \) as \( \lim_{i \to 0} c_t(i) \).

The household's preferences are represented by the additively time-separable utility function

\[
U(\{c_t, x_t\}_{t=1}^{\infty}) = \sum_{t=1}^{\infty} \beta^t \left\{ v(x_t) + \int_0^{\infty} u[c_t(i)] \, di \right\},
\]

where \( \beta \in (0, 1) \). The function \( u(\cdot) \) is strictly increasing, strictly concave, and twice continuously differentiable with \( u(0) = 0 \) and \( u'(0) < \infty \). As in Stokey (1988), agents enjoy variety in their consumption bundles but will not purchase a particular good if its price is too high. The function \( v(\cdot) \) is strictly increasing, strictly concave, and twice continuously differentiable.

3.3. Technologies

As in Lucas and Stokey (1983), each household consists of two members: a worker and a shopper. In each period \( t \geq 1 \) the worker must specialize in the production of one of the consumption goods. Hence, in each period \( t \geq 1 \) the representative worker proceeds to some market \( i > 0 \), where good \( i \) may be produced according to a constant returns to scale technology yielding one unit of output for every \( q(i, \theta_t) \) units of labor input. The argument \( \theta_t \) summarizes the aggregate state of technology at time \( t \) and takes on values in \( \Theta \), a subset of some finite-dimensional Euclidean space. Hence, \( q \) maps \((0, \infty) \times \Theta\) into \((0, \infty)\). The function \( q(i, \cdot) \) is nonincreasing for all \( i \geq 0 \), so that technological progress, represented by increases over time in \( \theta_t \), permits more output to be obtained per
dose of labor. For all \( \theta \in \Theta \), \( q(\cdot, \theta) \) is strictly increasing and twice continuously differentiable with \( \lim_{i \to \infty} q(i, \theta) = \infty \). The technology resembles that used by Stokey (1988) to model the production of goods indexed by quality. Here, however, the increasing cost reflects, perhaps because of transportation costs, the increasing remoteness of markets.

While the worker is engaged in production, the representative shopper travels from market to market, acquiring output for the household's consumption. Workers are unable to individually identify shoppers outside of the financial center so that, in Lucas and Stokey's (1983) terminology, the shopper is 'unknown' to the workers in every market that he visits. One means of overcoming this informational constraint is through the use of government-issued money. By carrying positive quantities of money, the shopper has evidence that his household's production in the past has been large enough to entitle it to consumption in the present; receiving cash that can be used to make future purchases, the worker is willing to hand over some of his output. Thus, the government, which otherwise plays no role in this economy, is assumed to supply noninterest-bearing fiat money in quantity \( M_0 \) per household at time \( t = 0 \) and to augment (or contract) this supply with lump-sum transfers (or taxes) \( H_t \) per household in each period \( t \geq 1 \).

It is also assumed that a second means of overcoming the informational constraints to trade in this environment exists in a set of intermediation technologies through which, in any market \( i > 0 \), a third party (the intermediary) may work to establish a shopper's identity and ability to pay and to provide a bill of exchange serving as an acceptable substitute for cash to a participating worker, thereby facilitating a trade on credit. Like a credit card transaction, this process might involve reading a magnetic card carried by the shopper on which his name is encoded, sending electronic messages to and from a computer at the financial center to guard against fraud and overspending, and leaving the worker with a signed receipt. This process is feasible only at the expense of real resources.

In particular, let these financial technologies be constant returns to scale, so that the intermediation activity may be performed for a transaction of real value (in terms of labor) \( V \) in market \( i \) at time \( t \) using \( V \tau(i, \theta_t) \) units of labor. The function \( \tau: (0, \infty) \times \Theta \mapsto (0, \infty) \) depends nontrivially on \( i \) since intermediation involves communication with the financial center, the cost of which increases with distance. Thus, for all \( \theta \in \Theta \), \( \tau(i, \cdot) \) is strictly increasing and twice continuously differentiable with \( \lim_{i \to \infty} \tau(i, \theta) = \infty \). Technological progress may lower

\(^2\)Ireland (1991a) presents a model that, while analytically equivalent to the one presented here, derives this informational constraint to trade endogenously. This more elaborate model combines the idea contained in Kiyotaki and Wright (1991), that agents who meet anonymously in spatially distinct markets will require a means of exchange, with the idea contained in Williamson (1986), that one agent can establish the identities of other agents and enter into private credit arrangements with them only at a cost in terms of real resources.
the resource costs of financial activity, so that for all $i \in (0, \infty)$, $\tau(i, \cdot)$ is non-increasing. This specification of financial technologies is similar to Gillman’s (1987).

Just as some workers specialize in the production of a single consumption good, others specialize in the business of providing intermediation services in a single market $i > 0$. Workers who choose to produce and sell a consumption good must neither gain nor lose from the substitution of private trade credit for money in exchange. Consequently, all intermediation costs are passed along to the shopper. Because there is a continuum of households in the economy, there will be many workers in each market, some producing goods and others acting as intermediaries; thus, perfect competition will prevail in all markets. Letting $w_t$ denote the nominal wage (the price of labor in terms of money) at time $t$, amount $c_t(i)$ of good $i$ may be obtained by a shopper at time $t$ either in exchange for $w_t q_t(i, \theta_t) c_t(i)$ units of money or on credit of nominal value $w_t [1 + \tau_t(i, \theta_t)] q_t(i, \theta_t) c_t(i)$, amount $w_t q_t(i, \theta_t) c_t(i)$ to pay for the goods themselves, and $w_t \tau_t(i, \theta_t) q_t(i, \theta_t) c_t(i)$ to compensate the intermediary for his services.

3.4. Asset markets and the cash-credit trade-off

At the end of each period $t \geq 1$, after output has been produced and sold, everyone returns to the financial center. Households consume their purchases, then convene in a centralized asset market to settle outstanding debts made earlier through intermediaries and to accumulate the money balances needed to make cash purchases in the following period. The government also participates in this market by making the lump-sum transfers $H_t$ of money to each household. The representative household leaves the asset market at the end of time $t$ with cash holdings denoted $M_{t+1}$.

Households are able to borrow and lend among themselves in the end-of-period asset market without the help of intermediaries by trading in one-period nominally denominated pure discount bonds. The representative household purchases bonds paying $B_{t+1}$ units of money in the time $t+1$ asset market for $B_{t+1}/R_t$ units of money in the time $t$ asset market, where $R_t$ is the gross nominal interest rate between $t$ and $t+1$. The asset market is also open in period 0, when households receive their initial transfer of money $M_0$. Bonds may be traded at this time as well; the representative household’s initial bond

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3In contrast to Williamson (1986), who specifies a fixed cost, the cost of the financial activity here is proportional to the size of the transaction. In Williamson’s model, the fixed cost is crucial in giving rise to specialized financial institutions. Here, however, replacing the proportional cost with a fixed cost is unlikely to change any of the key results and would complicate the analysis considerably, since a competitive equilibrium could no longer be found by solving a concave program. Hence, the assumption of proportional costs is retained for technical reasons.
holdings are denoted $B_0$, and the prevailing interest rate is $R_0$. Since bonds are available in zero net supply, $B_t = 0$ must hold in equilibrium for all $t \geq 0$.

Holding money balances involves a cost to the representative household in terms of interest foregone, since the funds could otherwise be lent out at rate $R$. On the other hand, to economize on money the household must make more of its purchases on credit and hence must use more of its resources to pay for intermediation services. It is the trade-off between these costs that determines the final mixture of cash and credit used in exchange.

The nature of the cash-credit trade-off is illustrated in fig. 5. Whereas the opportunity cost of using money, $R$, is constant across goods, the resource cost of using credit, $1 + \tau(i, \theta)$, is strictly increasing as a function of $i$. Thus, in each period $t \geq 1$, shoppers will find it optimal to use credit to buy goods $i \in (0, s_i)$ for some $\infty > s_i \geq 0$ and to use money for purchases of the remaining goods; as in Schreft (1992), agents use credit close to home and cash far from home.

The government influences the cash-credit trade-off through the effects of its choice of monetary policy on the nominal rate of interest. An increase in $R$, all else equal, will increase the range of goods purchased on credit. Changes over time in the parameter $\theta$ also influence the cash-credit trade-off. Since the function $\tau(\cdot, \theta)$ is increasing for all $\theta \in \Theta$, agents find it more difficult to trade without money as technological progress facilitates the introduction of new goods from more distant markets. Thus, the demand for money tends to increase relative to income as the structure of production and trade becomes more complex. The function $\tau(i, \cdot)$ is nonincreasing for all $i \in (0, \infty)$, however, so financial innovation increases the borderline index $s_i$ (again, see fig. 5) and thereby leads to a decreased role for money. These are precisely the effects of technological change on money demand identified in section 2.
3.5. Competitive equilibrium

It is now possible to state formally the problem facing the representative household and to define a competitive equilibrium for this economy. In the time 0 asset market, the representative household faces the budget constraint

$$B_0 + M_0 \geq \frac{B_1}{R_0} + M_1. \quad (2)$$

As sources of funds at time $t \geq 1$ the representative household has its labor income, its money and bond holdings, and the end-of-period government transfer. As uses of funds it has purchases of goods $i < s$, made on credit, purchases of goods $i \geq s$, made with cash, payments to intermediaries, and the money and bond holdings to be carried into the next period. Its time $t$ budget constraint is therefore

$$\left(1 - x_t\right) + \frac{B_t + M_t + H_t}{w_t} \geq \int_{0}^{\infty} \left[1 + \tau(i, \theta_t)\right] q(i, \theta_t) c_t(i) \, di$$

$$+ \int_{t}^{\infty} q(i, \theta_t) c_t(i) \, di + \frac{M_{t+1}}{w_t} + \frac{B_{t+1}}{R_t w_t}, \quad (3)$$

where $q(0, \theta_t)$ and $\tau(0, \theta_t)$ are defined for all $t \geq 1$ by

$$q(0, \theta_t) = \lim_{i \to 0} q(i, \theta_t), \quad \tau(0, \theta_t) = \lim_{i \to 0} \tau(i, \theta_t).$$

The household’s money balances at time $t$ must be sufficient to cover any purchases of goods $i \geq s$, that it decides to make with cash. This requirement gives rise to the cash-in-advance constraint

$$\frac{M_t}{w_t} \geq \int_{s_t}^{\infty} q(i, \theta_t) c_t(i) \, di, \quad t = 1, 2, \ldots. \quad (4)$$

Finally, no household is permitted to engage in Ponzi schemes through which it can borrow more than it will ever be able to repay. The no-Ponzi-game requirement enters into the representative household's problem as a non-negativity constraint on the sum of its current nominal asset position and the
nominal value of its future wage receipts and government transfers, all discounted back to data 0 using the nominal interest rate:

\[
W_t = \left[ \prod_{s=0}^{t-1} R_s \right]^{-1} \left[ M_{t+1} + \frac{B_{t+1}}{R_t} \right] + \sum_{k=t+1}^{\infty} \left\{ \prod_{s=0}^{k-1} R_s \right\}^{-1} \left[ w_k (1 - x_k) + H_k \right] \geq 0, \quad t = 0, 1, \ldots .
\]  

(5)

The representative household solves:

**Problem.** Maximize by choice of \( \{c_t, x_t\}_{t=1}^{\infty} \in X \), nonnegative scalars \( \{M_t\}_{t=1}^{\infty} \) and \( \{s_t\}_{t=1}^{\infty} \), and scalars \( \{B_t\}_{t=1}^{\infty} \) the objective function (1) subject to the constraints (2)–(5), taking \( B_0, M_0, \{H_t\}_{t=1}^{\infty}, \{w_t\}_{t=1}^{\infty}, \{R_t\}_{t=0}^{\infty}, \) and \( \{\theta_t\}_{t=1}^{\infty} \) as given.

Denoting the nominal money supply per household at each date \( t \geq 1 \) by \( M_t^* = M_0 \) and

\[
M_t^* = M_0 + \sum_{k=1}^{t-1} H_k, \quad t = 2, 3, \ldots ,
\]

a perfect foresight competitive equilibrium is defined as:

**Definition.** A perfect foresight competitive equilibrium consists of the initial condition \( B_t = 0 \) and sequences of quantities \( \{c_t, x_t, s_t, M_t, M_t^*, B_t\}_{t=1}^{\infty} \), wages \( \{w_t\}_{t=1}^{\infty} \), and interest rates \( \{R_t\}_{t=0}^{\infty} \) such that given the sequence of technological parameters \( \{\theta_t\}_{t=1}^{\infty} \):

(i) The sequences \( \{c_t, x_t, s_t, M_t, B_t\}_{t=1}^{\infty} \) solve the representative household's problem given \( B_0, \{M_t^*\}_{t=1}^{\infty}, \{w_t\}_{t=1}^{\infty}, \) and \( \{R_t\}_{t=0}^{\infty} \).

(ii) Markets clear in every period:

(a) \( 1 - x_t = \int_0^{s_t} [1 + \tau(i, \theta_t)] q(i, \theta_t) c_t(i) \, di + \int_{s_t}^{\infty} q(i, \theta_t) c_t(i) \, di, \quad t = 1, 2, \ldots , \)

(b) \( M_t^* = M_t, \quad t = 1, 2, \ldots , \)

(c) \( B_t = 0, \quad t = 1, 2, \ldots . \)

Competitive equilibria for this model economy are characterized analytically and numerically in the next two sections.
4. Analytic results

Since the model is designed specifically to study the dynamic effects of technological change on the structure of production and trade, it is not appropriate here to confine the analysis, as is often done, to equilibria in which real variables are stationary over time. Recent work by Woodford (1988), which demonstrates the potential existence of a large number of equilibria outside of the stationary class for the more conventional cash-in-advance model used by Lucas and Stokey (1983), suggests that uniqueness will not be a general property of equilibria in the model just described. However, a uniqueness result can be established when an additional assumption is made on the nature of monetary policy. Specifically, it will henceforth be assumed that monetary policy is conducted so as to determine a sequence \( \{R_t\}_{t=0}^{\infty} \) of nominal interest rates; the money supply is permitted to expand or contract as necessary to clear markets given these interest rates.

Under this additional assumption, equilibrium quantities must satisfy:

\[
u'[c_t(i)] - \lambda_t[1 + \tau(i, \theta_t)] q(i, \theta_t) \leq 0, \quad (6)
\]

\(i \in (0, s_t), \) with equality if \( c_i(i) > 0, \ \forall \ t \geq 1;\)

\[
u'[c_t(i)] - \lambda_t R_{t-1} q(i, \theta_t) \leq 0, \quad (7)
\]

\(i \in (0, \infty) \cap [s_t, \infty), \) with equality if \( c_i(i) > 0, \ \forall \ t \geq 1;\)

\[
u'(x_t) - \lambda_t + \alpha \phi_t - \phi_t = 0, \quad (8)
\]

where \( \alpha \phi_t \geq 0, \beta \phi_t \geq 0, \alpha \phi_t = 0 \) if \( x_t > 0, \beta \phi_t = 0 \) if \( 1 > x_t, \ \forall \ t \geq 1;\)

\[
- [1 + \tau(s_t, \theta_t) + R_{t-1}] \leq 0, \quad (9)
\]

with equality if \( s_t > 0, \ \forall \ t \geq 1;\)

\[
1 - x_t = \int_0^{s_t} [1 + \tau(i, \theta_t)] q(i, \theta_t) c_i(i) \, di + \int_{s_t}^{\infty} q(i, \theta_t) c_i(i) \, di, \quad (10)
\]

\( \forall \ t \geq 1;\)

\[
\int_{s_t}^{\infty} q(i, \theta_t) c_i(i) \, di - m_t \leq 0, \quad (11)
\]

with equality if \( R_{t-1} > 1, \ \forall \ t \geq 1;\)
for some nonnegative sequence \( \{A_t\}_{t=1}^\infty \), where \( m_t = M_t^*/w_t \) denotes the level of real balances held by the representative household at time \( t \). Eqs. (6)–(9) follow from the household's first-order conditions, (10) is the aggregate resource constraint, and (11) is the cash-in-advance constraint. There is, in addition, a restriction on the representative household's asset holdings emerging from the no-Ponzi-game constraint that implies that the transversality condition,

\[
\lim_{T \to \infty} \beta^T A_T m_T = 0, \tag{12}
\]

must also be satisfied.

The well-known indeterminacy of nominal quantities under interest rate targeting policies [see Olivera (1970)] is apparent in eq. (11): if \( \{M_t^*\}_{t=1}^\infty \) and \( \{w_t^*\}_{t=1}^\infty \) satisfy (11) given a particular sequence \( \{R_t^*\}_{t=0}^\infty \), then so do \( \{M_t^{**}\}_{t=1}^\infty \) and \( \{w_t^{**}\}_{t=1}^\infty \) where \( M_t^{**} = \omega M_t^* \) and \( w_t^{**} = \omega w_t^* \) for all \( t \geq 1 \) and any positive constant \( \omega \). This indeterminacy may be eliminated, of course, by allowing the government to choose a value for \( M_1^* = M_0 \) in addition to the sequence \( \{R_t^*\}_{t=0}^\infty \).

The following uniqueness result for real quantities follows from eqs. (6)–(11):

Proposition. Let monetary policy determine a sequence of nominal interest rates \( \{R_t\}_{t=0}^\infty \), where \( R_t > 1 \), \( \forall t \geq 0 \). Then the perfect foresight equilibrium sequences \( \{c_t, x_t, s_t, m_t\}_{t=1}^\infty \) are uniquely determined. If \( u'(0) \leq \min \{R_{t-1}, \frac{1}{1 - \tau(0, \theta_t)} \} v'(1)q(0, \theta_i), \) then \( s_t \geq 0, m_t = 0, x_t = 1, \) and \( c_t(i) = 0, \forall i \in (0, \infty) \). If \( u'(0) > R_{t-1} v'(1)q(0, \theta_i) \) and \( \left[ 1 - \tau(0, \theta_i) \right] \geq R_{t-1}, \) then \( s_t = 0, m_t > 0, x_t \in [0, 1], \) and \( c_t(i) \) is continuous, positive, and strictly decreasing on \( (0, y) \) for some \( \infty > y > 0 \) and zero on \( [y, \infty) \). If \( u'(0) > \left[ 1 + \tau(0, \theta_i) \right] v'(1)q(0, \theta_i) \) and \( \left[ 1 + \tau(0, \theta_i) \right] < R_{t-1}, \) then \( s_t > 0, m_t \geq 0, x_t \in [0, 1], \) and \( c_t(i) \) is continuous, positive, and strictly decreasing on \( (0, y) \) for some \( \infty > y > 0 \) and zero on \( [y, \infty) \).

Proof. For each \( t \geq 1 \), define

\[
\chi_t = \max \{u'(0)/R_{t-1} q(0, \theta_t), u'(0)/[1 + \tau(0, \theta_i)] q(0, \theta_i)\}.
\]

If \( v'(1) \geq \chi_t \), then (6)–(11) are solved for period \( t \) with \( c_t(i) = 0, \forall i \in (0, \infty), x_t = 1, m_t = 0, \) and \( s_t \geq 0 \). That is, no goods production takes place. It might seem strange to take seriously a solution in which consumption is zero. However, if leisure is interpreted as time allocated to the household production of nonmarket goods that, by choice of units, may be produced from labor with a one-to-one constant returns to scale technology and that enter into preferences through the utility function \( v(\cdot) \), then meaning may be attached to such
solutions. Stokey (1988) identifies goods of this type with a traditional agricultural sector; her interpretation will be used in considering how equilibrium quantities vary with the parameters $\theta$ and $R$.

If $\chi_t > v'(1)$, then market production takes place. In this case, (6)–(11) are solved by finding functions $c_t(i) = c(i, \lambda_t, R_{t-1})$, $x_t = x_t(\lambda_t)$, and $s_t = s_t(R_{t-1})$ that satisfy (6)–(9) for given values of $\lambda_t$ and $R_{t-1}$. Eq. (10) is used to determine $\lambda_t$, while $R_{t-1}$ is given exogenously. Assuming that it holds with equality, eq. (11) then determines $m_t$.

Eq. (9) implicitly defines the functions $s_t: [1, \infty) \mapsto [0, \infty)$ as

$$s_t(R_{t-1}) = \begin{cases} s \text{ satisfying } 1 + \tau(s, \theta_t) = R_{t-1} & \text{if } R_{t-1} > 1 + \tau(0, \theta_t), \\ 0 & \text{if } 1 + \tau(0, \theta_t) \geq R_{t-1}. \end{cases}$$

For each $t \geq 1$, $s_t(\cdot)$ is continuous and increasing and is strictly increasing on $[1 + \tau(0, \theta_t), \infty)$.

Since $v(\cdot)$ is strictly concave and since $x_t > 1$ if goods production takes place, eq. (8) implies that $\lambda_t > v'(1)$. Moreover, since $c_t(0)$ is strictly positive (if any goods are produced, then the lowest-indexed goods will be among them) and $u(\cdot)$ is strictly concave, (6) and (7) imply that $\chi_t > \lambda_t$. Hence functions of $\lambda_t$ need only be defined on $(v'(1), \chi_t)$.

The functions $x_t: (v'(1), \chi_t) \mapsto [0, 1)$ are defined by eq. (8) as

$$x_t(\lambda) = \begin{cases} x \text{ satisfying } v'(x) = \lambda & \text{if } \min\{v'(0), \chi_t\} > \lambda > v'(1), \\ 0 & \text{if } \chi_t > \lambda \geq v'(0). \end{cases}$$

For all $t \geq 1$, $x_t(\cdot)$ is continuous and decreasing and is strictly decreasing on $(v'(1), \min\{v'(0), \chi_t\})$.

The functions $c_t: [0, \infty) \times (v'(1), \chi_t) \times [1, \infty) \mapsto [0, \infty)$ are defined as

$$c_t(i, \lambda_t, R_{t-1}) = \begin{cases} c \text{ such that } u'(c) = \lambda_t[1 + \tau(i, \theta_t)] q(i, \theta_t) & \text{for } \min\{s_t, y_t\} > i > 0, \\ c \text{ such that } u'(c) = \lambda_t R_{t-1} q(i, \theta_t) & \text{for } y_t > i \geq s_t, \\ 0 & \text{for } i \geq y_t, \end{cases}$$

$$c_t(0, \lambda_t, R_{t-1}) = \lim_{i \to 0} c_t(i, \lambda_t, R_{t-1}),$$

where $y_t: (v'(1), \chi_t) \times [1, \infty) \mapsto (0, \infty)$ are defined by

$$y_t(\lambda_t, R_{t-1}) = \{ y: u'(0) = \min\{R_{t-1}, 1 + \tau(y, \theta_t)\} \lambda_t q(y, \theta_t) \}.$$
For all \( t \geq 1 \), \( c_t(\cdot, \lambda_t, R_{t-1}) \) is continuous and decreasing in all three arguments; \( c_t(\cdot, \lambda_t, R_{t-1}) \) is strictly decreasing on \([0, y_t(\lambda_t, R_{t-1})]\); \( c_t(i, \lambda_t, R_{t-1}) \) is strictly decreasing for \( i \in [0, y_t(\lambda_t, R_{t-1})] \); \( y_t(\lambda_t, R_{t-1}) \) is strictly decreasing.\footnote{\( y_t(\cdot, \cdot) \) is continuous and decreasing in both arguments; \( y_t(\cdot, R_{t-1}) \) is strictly decreasing.}

By construction, \( s_t = s_t(R_{t-1}) \), \( x_t = x_t(\lambda_t) \), and \( c_t(i) = c_t(i, \lambda_t, R_{t-1}) \) satisfy eqs. (6)-(9) given \( \lambda_t \in (v'(1), \varphi_t) \). Substituting these functions into eq. (10) yields

\[
1 = \int_0^{s_t(R_{t-1})} \tau(i, \theta) q(i, \theta) c_t(i, \lambda_t, R_{t-1}) \, di \\
+ \int_0^{y_t(\lambda_t, R_{t-1})} q(i, \theta) c_t(i, \lambda_t, R_{t-1}) \, di + x_t(\lambda_t)
\]

For each \( t \geq 1 \) and any \( R_{t-1} \geq 1 \), \( \psi_t(\cdot, R_{t-1}) \) is continuous and strictly decreasing on \((v'(1), \varphi_t)\) with \( \lim_{t \to v'(1)} \psi_t(\lambda_t, R_{t-1}) > 1 \) and \( \lim_{t \to \varphi_t} \psi_t(\lambda_t, R_{t-1}) < 1 \). Hence, there exists a unique \( \lambda_t \in (v'(1), \varphi_t) \) such that \( \psi_t(\lambda_t, R_{t-1}) = 1 \); this solution uniquely determines \( x_t = x_t(\lambda_t) \) and \( c_t(i) = c_t(i, \lambda_t, R_{t-1}) \). \( s_t = s_t(R_{t-1}) \) is uniquely determined by \( R_{t-1} \) and, if \( R_{t-1} > 1 \), eq. (11) uniquely determines \( m_t \). This completes the proof.

The consequences of the policy choice \( \{R_t\}_{t=0}^\infty \) are immediately evident. If intermediation costs are high, then \( u'(0) \leq \min\{R_{t-1}, 1 + \tau(0, \theta_t)\} v'(1) q(0, \theta_t) \lvert u'(0) \rvert \) will hold when interest rates are sufficiently high. In this case, monetary policy drives out all market activity. Under more modest intermediation costs, high interest rates may lead to a situation in which production takes place but \( s_t(R_{t-1}) > y_t(s_t, R_{t-1}) > 0 \), so that the household makes all of its purchases on credit at time \( t \). In this case, policy drives the economy to its nonmonetary equilibrium.

Since \( s_t = 0 \) if \( [1 + \tau(0, \theta_t)] \geq R_{t-1} \), then especially low interest rates will eliminate the demand for intermediation. This result implies that the upward trend that forms the latter half of velocity's U-shaped pattern – the trend toward a cashless society – can be avoided simply by controlling inflation. In particular, since \( \tau(0, \theta) \geq 0 \) for all \( \theta \in \Theta \), cash is used in all transactions and hence velocity is fixed at unity under Friedman's (1969) rule for the optimal quantity of money, which sets \( R_t = 1 \) for all \( t \geq 0 \).

In fact, when \( R_t = 1 \) for all \( t \geq 0 \), the resulting equilibrium allocation described by \( c_t(i, \lambda_t, 1), x_t(\lambda_t), \) and \( \psi_t(\lambda_t, 1) \) for all \( t \geq 1 \) corresponds to the full-information Pareto-optimal allocation under which all households are treated alike. That is, under the Friedman rule agents are able to completely circumvent
the informational constraints to trade in this environment at absolutely no cost. The costs of higher rates of inflation include both the welfare cost resulting from the wedge implied by (6)–(8) between marginal rates of transformation and the corresponding marginal rates of substitution and the real resource costs resulting from the allocation of scarce labor to the intermediation activity.

Little can be said about the effects of technological change at this level of generality. As well as yielding the uniqueness proposition stated above, however, the procedure developed in this section to find equilibria may be used to generate numerical solutions to the representative household’s problem when specific functional forms are chosen to describe preferences and technologies. Once the equations $\psi_t(\lambda_t, R_{t-1}) = 1$ are solved for $\{\lambda_t\}_{t=1}^{\infty}$, the equilibrium quantities $c_t(i), x_t$, and $s_t$ are easily found by evaluating the functions $c_t(i, \lambda_t, R_{t-1}), x_t(\lambda_t)$, and $s_t(R_{t-1})$. When $R_{t-1} > 1$ and $y_t(\lambda_t, R_{t-1}) \geq s_t(R_{t-1})$, real money demand may be calculated as

$$m_t(\lambda_t, R_{t-1}) = \int_{s_t(R_{t-1})}^{\psi_t(\lambda_t, R_{t-1})} q(i, \theta_t) c_t(i, \lambda_t, R_{t-1}) \, di.$$ 

Since, using the resource constraint (10), the real value of total market production is equal to $1 - x_t(\lambda_t)$, the income velocity of money is given by

$$V_t(\lambda_t, R_{t-1}) = \frac{1 - x_t(\lambda_t)}{m_t(\lambda_t, R_{t-1})},$$

and since the representative household’s time endowment is fixed at unity, the share of economy-wide productive resources allocated to financial intermediation is

$$F_t(\lambda_t, R_{t-1}) = \int_{0}^{\psi_t(R_{t-1})} \tau(i, \theta_t) q(i, \theta_t) c_t(i, \lambda_t, R_{t-1}) \, di.$$ 

Section 5 applies this numerical procedure to examine the effects of technological change on these variables in a parameterized example of the model economy.

5. Numerical results

5.1. A parametric example

To perform the numerical work, preferences and technologies are specialized to

$$u(c) = \ln(c + 1),$$

$$v(x) = \ln(x),$$
\[ q(i, \theta) = A_i [\exp(2i)], \quad A_i > 0, \]
\[ \tau(i, \theta) = 0.02 [\exp(D_i i)], \quad D_i > 0, \]
so that
\[ \theta_t = [1/A_t, 1/D_t], \]
\[ \Theta = (0, \infty) \times (0, \infty) \subset \mathbb{R}^2. \]

The choice of \( u(c) = \ln(c + 1) \) is such that \( u(0) = 0 \) and \( u'(0) = 1 < \infty \) as required. For all \( \theta \in \Theta, q(\cdot, \theta) \) and \( \tau(\cdot, \theta) \) are strictly increasing; for all \( i \in (0, \infty) \), \( q(i, \cdot) \), and \( \tau(i, \cdot) \) are nonincreasing. Technological progress in goods production processes, captured by decreases in the parameter \( A \), is modeled as affecting costs proportionately in all markets, reflecting general improvements in labor productivity. Technological progress in finance, captured by decreases in \( D \), affects costs disproportionately in higher-indexed markets, reflecting the development of technologies that greatly reduce the costs of communication over long distances.

5.2. Comparative dynamics

Ideally, the approach advocated by Prescott (1986), according to which technology parameters are assigned values based upon the results of econometric studies using cross-sectional or panel data, could be used to calibrate this model. The functional forms employed here, however, are sufficiently stylized to limit the applicability of this approach. Thus, the starting values \( A_1 = 0.5 \) and \( D_1 = 5 \) are chosen simply so that velocity and the size of the financial sector at \( t = 1 \) in the model are of roughly the same orders of magnitude as M1 velocity and the share of finance in total employment were in the nineteenth century United States (see figs. 1 and 3). The preference parameter \( \beta \) is fixed at 0.95, so that one period in the model represents one year in real time.

Two comparative dynamics exercises are performed in which one of the technology parameters \( A \) or \( D \) is decreased according to
\[ A_{t+1} = (0.99) A_t \quad \text{or} \quad D_{t+1} = (0.999) D_t \]
over 200 periods, while the other is held constant. Since the effects of interest rate changes are discussed above, these exercises focus exclusively on the effects of technological change; the nominal interest rate is held constant, with \( R_t = 1.05 \) for all \( t \geq 0 \). All parameters are assumed to be constant after period 200; in both examples, this implies that \( \lambda_t \) and \( m_t \) are constant for all \( t \geq 200 \) and hence that the transversality condition (12) is satisfied. Figs. 6 and 7 show the effects of these changes on velocity \( V_t (\lambda, R_{t-1}) \), the percentage of the labor force
employed in finance $F_t(\lambda_t, R_{t-1})$, consumption across markets $c_t(i)$, and a measure of the constant-cost value of production at time 200 prices given by $P_t(\lambda_t, R_{t-1})/P_t(\lambda_{t-1}, R_0)$, where

$$P_t(\lambda_t, R_{t-1}) = \int_0^{\gamma_{y_t}(R_{t-1})} \tau(i, \theta_{200}) q(i, \theta_{200}) c_t(i, \lambda_t, R_{t-1}) \, di + \int_0^{\gamma_{y_{t-1}}(R_{t-1})} q(i, \theta_{200}) c_t(i, \lambda_t, R_{t-1}) \, di.$$ 

Decreasing costs of goods production alone drives growth in real product, which may be seen in fig. 6 to involve increases not only in the quantity of each good produced [the height of the curve $c_t(i)$], but also in the number of distinct goods produced [the range over which $c_t(i)$ is positive]. The monetization process occurs as new goods from more distant markets are introduced; hence, velocity declines as the economy grows. Wealth effects associated with real growth result in an increase in the fraction of the model economy's productive resources allocated to finance over most of the 200 periods, although substitution effects from productivity gains in manufacturing relative to finance eventually dominate, drawing resources out of the financial sector in later periods.

Meanwhile, the financial innovation represented by decreases in the parameter $D$ has little effect on output (fig. 7) but generates a strong upward trend in velocity. The financial sector expands to take advantage of the technological progress. The effects on the composition of consumption are small. However, innovation in finance encourages the consumption of goods that switch from being cash goods to being credit goods; these have indices in the middle of the entire range of goods produced, since those with low indices are always purchased on credit and those with high indices are always bought with cash.

5.3. An example with U-shaped velocity and a growing financial sector

The comparative dynamics exercises demonstrate how the two, potentially offsetting, effects of technological change on money demand identified in section 2 are captured by the model. As the innovations in production technologies that drive real economic growth also permit new markets to open, trading regions to expand, and new goods to be introduced, agents find it increasingly difficult to trade without money, and velocity falls. On the other hand, as innovations in financial technologies restore to agents the ability to purchase goods on credit, velocity rises.

Thus, in a third numerical exercise, $A$ and $D$ change simultaneously according to

$$A_{t+1} = (0.98) A_t \quad \text{and} \quad D_{t+1} = (0.99) D_t.$$
Fig. 6. The effects of technological progress in goods production.
Real Product Valued at Time 200 Prices

Consumption Across Markets

Fig. 6 (continued)
Fig. 7. The effects of technological progress in finance.
Real Product Valued at Time 200 Prices

Consumption Across Markets

Fig. 7 (continued)
Fig. 8. An example with U-shaped velocity and a growing financial sector.
Real Product Valued at Time 200 Prices

Consumption Across Markets

Fig 8 (continued)
over 200 periods, after which both are held constant. The interest rate is again held constant at 1.05 for all \( t \geq 0 \).

Fig. 8 shows that in this example velocity traces out a U-shaped pattern and the financial sector grows steadily over time. Technological progress in finance occurs at a constant rate, but the representative household does not take full advantage of this progress until production has spread into very distant markets; hence velocity declines at first and rises later. Throughout the entire process of economic growth, however, an increasing fraction of the representative household's resources is allocated to financial activity. Thus, the artificial series share both of the stylized features of the data described in section 2.

6. Conclusions and directions for future research

Conventional versions of the cash-in-advance model have been criticized [e.g., Christiano (1991)] for failing to account for all but the simplest kinds of monetary dynamics. In particular, Lucas (1988) demonstrates that standard cash-in-advance models predict that money demand will be a stable function of income and interest rates; these models cannot possibly explain systematic changes in velocity due to any other factors. This paper, however, demonstrates that the cash-in-advance model can be modified to account for a number of unconventional kinds monetary of dynamics associated with the processes of technological change and real economic growth. Ideas about monetization, innovation in the payments system, changes in the secular trend of the velocity of money, and changes in the role of the private financial sector, taken from a diverse body of literature on economic development and the demand for money, are unified into a single theory of how the structure of production and trade evolves as an economy grows.

The work presented here can be extended in two directions. One approach is to search for data with which to pin down the behavior of the technological parameters \( A \) and \( D \) introduced in section 5. Unfortunately, the industry-level indices of technological change extending back into the nineteenth century that this approach requires are difficult to construct, particularly for the financial sector where both inputs and outputs are difficult to measure. A second approach is to generalize the model so that technological change at the industry level becomes an endogenous process. Some progress in this direction is made by Marquis and Reffett (1992).

Appendix: Data sources for figs. 1–4

Fig. 1. Velocity of money, United States.

M1 velocity is computed by dividing gross national product by M1. M2 velocity is computed similarly.
P.N. Ireland, Economic growth, financial evolution, and velocity 845

Sources:  


Fig. 2. M2 velocity, United Kingdom.

Computed as gross domestic product divided by M2.

Sources:  


Fig. 3. Employment in finance, United States.

Sources:  

Persons engaged in production for banks and nonbank credit agencies as a percentage of all persons engaged in production, 1869–1918: Estimates based on figures at ten-year intervals, 1870–1910, for employment in FIRE (finance, insurance, and real estate) as a percentage of all workers ages 10 and over taken from Daniel Carson, ‘Changes in the industrial composition of manpower since the Civil War’, in: Studies in income and wealth, Vol. 11, NBER, New York, NY, 1949, table 1. It is assumed that the share of banks and credit agencies in employment in FIRE as a whole may be approximated for these years by 0.332, its level in 1929 (the first year for which US Commerce Department data on

Fig. 4. FIRE's share in GDP and GNP per capita, 1986 international cross-section.

The 103 countries are as follows: Antigua and Barbuda, Argentina, Australia, Austria, Bangladesh, Barbados, Belgium, Belize, Benin, Bhutan, Bolivia, Burundi, Canada, Columbia, Congo, Costa Rica, Cyprus, Democratic Yemen, Denmark, Dominica, Dominican Republic, Ecuador, Egypt, El Salvador, Ethiopia, Fiji, Finland, France, Gambia, Federal Republic of Germany, Ghana, Greece, Grenada, Guatemala, Guinea-Bissau, Guyana, Haiti, Honduras, Hong Kong, Hungary, Iceland, India, Indonesia, Ireland, Israel, Italy, Jamaica, Japan, Jordan, Kenya, Republic of Korea, Kuwait, Lesotho, Liberia, Luxembourg, Malawi, Malaysia, Mali, Malta, Mauritius, Mexico, Morocco, Nepal, Netherlands, New Zealand, Niger, Norway, Oman, Pakistan, Panama, Paraguay, Peru, Philippines, Portugal, Rwanda, Saint Kitts and Nevis, Saint Lucia, Saint Vincent and the Grenadines, Seychelles, Sierra Leone, Singapore, Solomon Islands, Somalia, South Africa, Spain, Sri Lanka, Suriname, Swaziland, Sweden, Syrian Arab Republic, Thailand, Trinidad and Tobago, Tunisia, Turkey, United Arab Emirates, United Kingdom, United Republic of Tanzania, United States, Venezuela, Yemen, Yugoslavia, Zambia, Zimbabwe.


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