Macroeconomic Dynamics, **4**, 2000, 448–466. Printed in the United States of America.

EXPECTATIONS, CREDIBILITY, AND TIME-CONSISTENT MONETARY POLICY

PETER N. IRELAND
Boston College
and
National Bureau of Economic Research

This paper addresses the problem of multiple equilibria in a model of time-consistent monetary policy. It suggests that this problem originates in the assumption that agents have rational expectations and proposes several alternative restrictions on expectations that allow the monetary authority to build credibility for a disinflationary policy by demonstrating that it will stick to that policy even if it imposes short-run costs on the economy. Starting with these restrictions, the paper derives conditions that guarantee the uniqueness of the model's steady state; monetary policy in this unique steady state involves the constant deflation advocated by Milton Friedman.

Keywords: Expectations, Credibility, Time Consistency, Monetary Policy

1. INTRODUCTION

Kydland and Prescott (1977) identify the inflationary bias that results when a monetary authority cannot precommit to a policy rule. In their model, the monetary authority desires to reduce unemployment and can do so by creating surprise inflation. Private agents with rational expectations recognize that the monetary authority has this incentive to inflate, however, and build the effects of that inflation into their decisions. In equilibrium, therefore, unemployment is no lower than it would otherwise be; the monetary authority's discretionary policy leads only to excessive inflation. This is the classic time-consistency problem for monetary policy.

Barro and Gordon (1983) extend Kydland and Prescott's (1977) work, demonstrating that reputational considerations can reduce the inflationary bias stemming from the time-consistency problem. In particular, Barro and Gordon consider cases in which inflationary expectations remain low so long as the monetary authority

I would like to thank Jon Faust and two referees, along with seminar participants at the Federal Reserve Bank of Cleveland, New York University, the University of Iowa, the University of Quebec at Montreal, the Wharton School of the University of Pennsylvania, and Yale University, for very helpful comments and suggestions. Some of this research was completed while I was visiting the Research Department at the Federal Reserve Bank of Cleveland; I would like to thank the Bank and its staff for their hospitality and support. The views expressed herein are my own and do not necessarily represent those of the Federal Reserve Bank of Cleveland or the Federal Reserve System. Address correspondence to: Peter N. Ireland, Boston College, Department of Economics, Carney Hall, 140 Commonwealth Avenue, Chestnut Hill, MA 02467-3806, USA; e-mail: irelandp@bc.edu.

 1365-1005/00 $9.50

has kept inflation low in the past but jump higher for a period of time should the government deviate from its low-inflation policy. The costs imposed by this episode of higher inflationary expectations induce the monetary authority to adhere to the low-inflation policy.

Barro and Gordon (1983) also recognize, however, that these trigger-like reputational mechanisms support a multiplicity of equilibria featuring a range of inflation rates. In fact, this large number of possible equilibria expands still further when more general approaches are taken to characterize the set of reputational equilibria. Ireland (1997), for instance, uses methods developed by Chari and Kehoe (1990) and Stokey (1991) to trace out the entire set of time-consistent monetary policies in a version of the Barro–Gordon model; policies in this set allow any rate of inflation to arise in equilibrium.

As noted by Stokey (1991), whether this multiplicity of equilibria represents a serious problem for models of time-consistent monetary policy depends partly on how these models are interpreted. If, for example, one interprets the models as describing a positive theory of monetary policy, then one might even regard the multiplicity as a virtue: from this perspective, the theory explains both why central bankers in low-inflation countries choose to maintain their reputations as inflation fighters and why central bankers in high-inflation countries find it so difficult to bring inflation down. If one interprets the models as offering a normative theory, however, then one must admit that the multiplicity is a shortcoming, for the theory provides no guidance as to how a central banker who is stuck in a high-inflation equilibrium might steer the economy toward a preferred, low-inflation equilibrium.

This paper takes the normative view and addresses the multiplicity of equilibria as a problem for models of time-consistent monetary policy. The paper suggests that this problem originates in the assumption, made throughout the literature since Kydland and Prescott (1977), that private agents have rational expectations. As emphasized by Sargent (1993), the rational expectations assumption becomes, in many settings, a convenient and powerful tool for sharpening the predictions of economic theory. In reputational models of time-consistent monetary policy, however, the rational expectations assumption may be less appropriate. In the trigger-strategy equilibria studied by Barro and Gordon (1983), for instance, the rational expectations assumption allows inflationary expectations to jump higher not just when the monetary authority surprises private agents by creating too much inflation but also when it surprises private agents by attempting to disinflate; rational expectations provide no scope for the monetary authority to work inflationary expectations down by actually adopting and building credibility for a disinflationary program.

Thus, the analysis in this paper departs from the rational expectations assumption in three ways. First, the analysis requires expected inflation to move together with actual inflation: Inflationary expectations may still rise if the monetary authority attempts to create surprise inflation, but they must begin to ease if the monetary authority attempts to disinflate. Second, and related, the analysis requires the expected rate of inflation to converge to the actual rate of inflation, provided that the monetary authority acts to keep inflation constant for a sufficient length of time; inflationary expectations have Cho and Matsui's (1995) inductive property. Together, these first two restrictions allow the monetary authority to build credibility for a disinflationary policy, in a manner suggested by Taylor (1982) and McCallum (1995), by demonstrating that it will stick to that policy even if it imposes short-run costs on the economy. Third, and finally, the analysis requires inflationary expectations to be formed as continuously differentiable functions of past inflation rates. Rogoff (1989) suggests that by restricting the extent to which expected inflation can jump in response to a change in actual inflation, the multiplicity of equilibria in models of time-consistent monetary policy might be reduced and, indeed, this conjecture proves useful here.

The paper imposes these three restrictions on private expectations in a model of time-consistent monetary policy developed by Ireland (1997), which, unlike the original Barro–Gordon model, begins with a complete description of a general-equilibrium environment featuring utility-maximizing households and profit-maximizing firms. This model draws a tight link between the government's objectives and those of the private sector: the monetary authority seeks to adopt a policy that maximizes a representative household's utility function. By identifying this welfare criterion for policy, the model facilitates the type of normative analysis performed here. The original Barro–Gordon model, in contrast, does not explicitly tie the government's objectives to the preferences of the private sector, making a normative interpretation of its implications more difficult.

Starting with the three restrictions on expectations, the paper derives conditions that guarantee the uniqueness of the model's steady state; monetary policy in this unique steady state involves the constant deflation advocated by Milton Friedman (1969). The paper goes on to present a pair of examples, in which the model is solved numerically. These examples show that when the economy is initially in a position away from its unique steady state, with a positive rate of inflation, the monetary authority can implement a successful disinflationary program under which monetary policy is ultimately given by the Friedman rule. In both examples, however, output and employment fall in the short run as the monetary authority builds credibility for the optimal policy; in the second example, the monetary authority optimally smooths these short-run costs over time by taking a gradual approach to disinflation.

The paper is related to several lines of recent research. First, in the game theory literature, work following that of Rubinstein (1986) takes an approach similar to the one used here by showing that the introduction of boundedly rational agents reduces the number of equilibria in settings where a severe multiplicity arises under rational expectations. Similarly, in macroeconomics, Sargent (1993) discusses a number of models in which equilibria with strange or undesirable features appear under rational expectations but can be ruled out or replaced by more conventional outcomes when assumptions of bounded rationality are made instead. And in work that is most closely related to this, Cho and Matsui (1995) show how the number of equilibria in Stokey's (1991) model of time-consistent public policy can be reduced to one when agents are constrained to use inductive forecasting rules. However, although some of Cho and Matsui's assumptions are weaker than those used here, their results apply only in the case of no discounting; the analysis performed here, in contrast, allows private agents to discount future payoffs.

Work by Backus and Driffill (1985) and Barro (1986) also addresses the problem of multiple equilibria in models of time-consistent monetary policy. These authors modify the infinite-horizon Barro–Gordon model by making the horizon finite. They then assume that the monetary authority may be of two types, one that has the conventional objectives and the other that is more averse to inflation; private agents do not know the policymaker's true type. This variant of the model succeeds in identifying a unique equilibrium, in which the conventional policymaker chooses to keep inflation low in order to convince private agents that he is of the inflation-averse type. As noted by Blackburn and Christensen (1989), however, the assumption that policymakers may be of different types necessarily means that not all can share the private sector's objectives; since government and private objectives need not coincide, these models become difficult to interpret along normative lines. Furthermore, as noted by Rogoff (1989), the precise features of the unique equilibrium in these models depends crucially on the characteristics of the alternative policymaker that is introduced; by varying the preferences of the alternative type, a large number of equilibria can again be produced.

Finally, al-Nowaihi and Levine (1994) explore the implications of various equilibrium refinements in the Barro–Gordon model and succeed at identifying a unique outcome that satisfies their chisel-proof criterion. But their analysis requires the private sector to act collectively; the uniqueness result need not extend to the case in which private agents operate in a decentralized environment where the coordination of their actions becomes difficult.

Thus, although some progress has been made at confronting the problem of multiple equilibria in models of time-consistent monetary policy, a full resolution of this problem has yet to be reached, leaving room for this paper's contribution to the literature.

2. MODEL

2.1. Economic Environment

The model resembles the one developed by Ireland (1997). Households face cash-in-advance constraints on their purchases of consumption goods. These constraints give rise to an interest-elastic demand for real balances; expected inflation causes agents to inefficiently economize on their money holdings. Firms operate in monopolistically competitive markets and must set prices for their output one period in advance. Monopolistic competition implies that equilibrium output falls below its efficient level, while sticky prices allow unanticipated money to have real effects; the monetary authority can push output closer to its efficient level by creating surprise inflation. Thus, the monetary authority faces a trade-off between the costs of expected inflation and the benefits of unanticipated inflation; this trade-off gives rise to the time-consistency problem in cases in which the monetary authority cannot commit to a policy rule.

The economy consists of a representative household, a continuum of firms indexed by $i \in [0, 1]$, and a monetary authority. Each firm produces a distinct, perishable consumption good. Hence, goods also may be indexed by $i \in [0, 1]$, where firm i produces good i. Preferences and technologies display enough symmetry, however, to allow the analysis to focus on the activities of a representative firm, identified by the generic index i.

The monetary authority makes a lump-sum transfer $(x_t - 1)M_t^s$ to the representative household during each period $t = 0, 1, 2, \ldots$. Thus, the per-household money stock M_t^s at the beginning of period t obeys

$$M_{t+1}^s = x_t M_t^s$$

for all $t = 0, 1, 2, \ldots$, where a choice of nominal units provides the initial condition $M_0^s = 1$. Thus, if M_t denotes the money carried by the representative household into period t, market clearing requires that $M_t = M_t^s$ for all $t = 0, 1, 2, \ldots$.

The representative household trades bonds as well as money. Bonds costing the household B_{t+1}/R_t dollars during period t return B_{t+1} dollars during period $t + 1$, where R_t denotes the gross nominal interest rate between t and $t + 1$. Bonds are available in zero net supply, hence market clearing requires that $B_t = 0$ for all $t = 0, 1, 2, \ldots$.

2.2. Timing of Events

As suggested above, the representative household enters period t with money M_t and bonds B_t. The representative firm enters period t having set a nominal price $P_t(i)$ for its output.

At the beginning of period t, the representative household receives the nominal transfer $(x_t - 1)M_t^s$. Next, the household's bonds mature, bringing its total money holdings to $M_t + (x_t - 1)M_t^s + B_t$. The household uses some of this cash to purchase new bonds at cost B_{t+1}/R_t and carries the rest into the goods market.

The description of goods production and trade draws on Lucas' (1980) interpretation of the cash-in-advance model. The representative household consists of two members: a shopper and a worker. During period t, the shopper purchases $c_t(i)$ units of each good i from firm i at the nominal price $P_t(i)$, subject to the cash-in-advance constraint

$$M_t + (x_t - 1)M_t^s + B_t - \frac{B_{t+1}}{R_t} \geq \int_0^1 P_t(i)c_t(i)\,di.$$

The worker, meanwhile, supplies $n_t(i)$ units of labor to each firm i and receives the nominal wage W_t. The household's preferences are described by the utility function

$$\sum_{t=0}^{\infty} \beta^t[\ln(c_t) - n_t], \tag{1}$$

where $1 > \beta > 0$ and the composite goods c_t and n_t are defined by

$$c_t = \left[\int_0^1 c_t(i)^{(\theta-1)/\theta} \, di \right]^{\theta/(\theta-1)}$$

with $\theta > 1$ and

$$n_t = \int_0^1 n_t(i) \, di$$

for all $t = 0, 1, 2, \dots$.

The representative firm must sell output on demand at its price $P_t(i)$ during period t. It produces this output, denoted $y_t(i)$, according to a linear technology that yields one unit of good i for every unit of labor input. After goods production and trade take place, the firm makes its wage payment and distributes any profit as a dividend to the representative household. In light of the linear technology, this dividend $D_t(i)$ equals price minus wage times quantity sold:

$$D_t(i) = [P_t(i) - W_t]y_t(i).$$

At the end of period t, the representative firm sets its nominal price $P_{t+1}(i)$ for period $t+1$. The representative household uses its unspent cash and its wage and dividend receipts as sources of funds with which it accumulates the money M_{t+1} that it carries into period $t+1$; it faces the budget constraint

$$M_t + (x_t - 1)M_t^s + B_t + W_t n_t + \int_0^1 D_t(i) \, di$$
$$\geq \int_0^1 P_t(i)c_t(i) \, di + \frac{B_{t+1}}{R_t} + M_{t+1}.$$

As a first step in characterizing an equilibrium for this economy, define $m_t = M_t/M_t^s$, $b_t = B_t/M_t^s$, $w_t = W_t/M_t^s$, $d_t(i) = D_t(i)/M_t^s$, and $p_t(i) = P_t(i)/M_t^s$. In terms of these scaled nominal variables, the representative household's budget and cash-in-advance constraints become

$$m_t + x_t - 1 + b_t + w_t n_t + \int_0^1 d_t(i) \, di \geq \int_0^1 p_t(i)c_t(i) \, di + \frac{b_{t+1}x_t}{R_t} + m_{t+1}x_t$$

and

$$m_t + x_t - 1 + b_t - \frac{b_{t+1}x_t}{R_t} \geq \int_0^1 p_t(i)c_t(i) \, di,$$

while the representative firm's dividend payment becomes

$$d_t(i) = [p_t(i) - w_t]y_t(i).$$

2.3. Household Optimization

During each period $t = 0, 1, 2, \dots$, the representative household chooses sequences of current and future consumptions, labor supplies, and holdings of money and bonds to maximize its utility subject to its budget and cash-in-advance constraints. When it solves this problem, the household knows the value $(x_t - 1)M_t^s$ of the current period's monetary transfer but must form expectations of money growth in all future periods. Thus, suppose that the household believes that with probability 1, x_{t+j} will equal some constant z_{t+j}^t; then for $t = 0, 1, 2, \ldots$ and $j = 1, 2, 3, \ldots$, z_{t+j}^t denotes the household's expectation during period t of money growth during period $t + j$, while for $j = 0$, $z_{t+j}^t = z_t^t = x_t$.

During each period $t = 0, 1, 2, \ldots$, therefore, the household chooses sequences $\{c_{t+j}^t\}_{j=0}^{\infty}$, $\{c_{t+j}^t(i)\}_{j=0}^{\infty}$, $\{n_{t+j}^t\}_{j=0}^{\infty}$, $\{m_{t+j+1}^t\}_{j=0}^{\infty}$, and $\{b_{t+j+1}^t\}_{j=0}^{\infty}$ to maximize

$$\sum_{j=0}^{\infty} \beta^j \left[\ln\left(c_{t+j}^t\right) - n_{t+j}^t \right] \tag{2}$$

subject to the constraints

$$\left[\int_0^1 c_{t+j}^t(i)^{(\theta-1)/\theta}\, di \right]^{\theta/(\theta-1)} \geq c_{t+j}^t, \tag{3}$$

$$\begin{aligned} & m_{t+j}^t + z_{t+j}^t - 1 + b_{t+j}^t + w_{t+j}^t n_{t+j}^t + \int_0^1 d_{t+j}^t(i)\, di \\ & \geq \int_0^1 p_{t+j}^t(i) c_{t+j}^t(i)\, di + \frac{b_{t+j+1}^t z_{t+j}^t}{R_{t+j}^t} + m_{t+j+1}^t z_{t+j}^t, \end{aligned} \tag{4}$$

and

$$m_{t+j}^t + z_{t+j}^t - 1 + b_{t+j}^t - \frac{b_{t+j+1}^t z_{t+j}^t}{R_{t+j}^t} \geq \int_0^1 p_{t+j}^t(i) c_{t+j}^t(i)\, di \tag{5}$$

for all $j = 0, 1, 2, \ldots$, where w_{t+j}^t, $d_{t+j}^t(i)$, $p_{t+j}^t(i)$, and R_{t+j}^t denote the household's expectations of w_{t+j}, $d_{t+j}(i)$, $p_{t+j}(i)$, and R_{t+j}, respectively, during period t, and where $d_t^t(i) = d_t(i)$, $w_t^t = w_t$, $p_t^t(i) = p_t(i)$, and $R_t^t = R_t$.

In equilibrium, the values of c_t^t, $c_t^t(i)$, n_t^t, m_{t+1}^t, and b_{t+1}^t that solve this problem become the actual values of c_t, $c_t(i)$, n_t, m_{t+1}, and b_{t+1} chosen by the household during period t. Thus, Section A.1 of the Appendix demonstrates that

$$c_t(i) = \left(z_t^t / p_t\right)[p_t(i)/p_t]^{-\theta}, \tag{6}$$

$$c_t = z_t^t / p_t, \tag{7}$$

$$w_t = z_t^t z_{t+1}^t / \beta, \tag{8}$$

and

$$R_t = z_{t+1}^t / \beta, \tag{9}$$

where the scaled nominal price index p_t is defined by

$$p_t = \left[\int_0^1 p_t(i)^{1-\theta} \, di \right]^{1/(1-\theta)} \tag{10}$$

for all $t = 0, 1, 2, \ldots$.

2.4. Firm Optimization

At the end of period $t - 1$, the representative firm must set its nominal price $P_t(i)$ for period t. When it chooses this price, it knows the value of the money stock M_t^s. Hence, the firm also can be depicted as choosing a scaled nominal price $p_t(i)$ for period t.

Looking ahead to period t, the firm knows that it will be required to satisfy the representative household's demand $c_t(i)$ for good i, described by (6). The firm also knows that to produce this output, it will have to hire labor at the scaled nominal wage w_t, given by (8). Thus, at the end of period $t-1$, the firm must form expectations of the household's expectations z_t^t and z_{t+1}^t. Here, it is assumed that the firm's expectations are consistent with those of the household, so that the firm's expectation of z_t^t during $t-1$ is given by z_t^{t-1}, while its expectation of z_{t+1}^t is given by z_{t+1}^{t-1}.

Thus, the firm chooses $p_t(i)$ at the end of period $t-1$ to maximize its expected scaled nominal dividend

$$d_t^{t-1}(i) = \left(z_t^{t-1}/p_t\right)\left[p_t(i) - z_t^{t-1} z_{t+1}^{t-1}/\beta\right][p_t(i)/p_t]^{-\theta}.$$

The solution to this problem implies that

$$p_t(i) = [\theta/(\theta - 1)]\left(z_t^{t-1} z_{t+1}^{t-1}/\beta\right). \tag{11}$$

Because the right-hand side of (11) does not depend on i, all firms $i \in [0, 1]$ set the same price in equilibrium; hence, (10) implies that

$$p_t = [\theta/(\theta - 1)]\left(z_t^{t-1} z_{t+1}^{t-1}/\beta\right) \tag{12}$$

for all $t = 0, 1, 2, \ldots$.

2.5. Equilibrium

Substituting the solutions for $p_t(i)$ and p_t given by (11) and (12) into the solutions for $c_t(i)$ and c_t given by (6) and (7) and using the definition $z_t^t = x_t$ reveals that $c_t(i) = c_t$ for all $t = 0, 1, 2, \ldots$, where

$$c_t = \beta[(\theta - 1)/\theta]\left(x_t/z_t^{t-1}\right)\left(1/z_{t+1}^{t-1}\right) \tag{13}$$

for all $t = 0, 1, 2, \ldots$. The equilibrium conditions $c_t(i) = y_t(i) = n_t(i)$ for all $i \in [0, 1]$ then imply that $n_t = c_t$, so that (13) also describes the household's total labor supply during each period $t = 0, 1, 2, \ldots$. Hence, (13) provides solutions for aggregate consumption, output, and employment.

Equation (13) highlights the source of the time-consistency problem for monetary policy in this model. Because firms set prices one period in advance, the monetary authority can increase output and employment by setting actual money growth x_t above its expected value z_t^{t-1}. However, because households face cash-in-advance constraints, they inefficiently economize on their real balances by substituting out of market activity and into leisure in the face of higher expected inflation. Thus, output and employment fall when z_{t+1}^{t-1} rises.

3. EXPECTATIONS, CREDIBILITY, AND TIME-CONSISTENT MONETARY POLICY

According to (13), consumption, output, and employment during each period $t = 0, 1, 2, \ldots$ depend not only on actual money growth during period t, but also on private agents' expectations during period $t-1$ of money growth during periods t and $t+1$. Hence, to complete the description of an equilibrium for this economy, it is necessary to specify how agents form these expectations.

The typical approach taken in the literature on time-consistent monetary policy assumes that agents have rational expectations or, in cases like this where there are no sources of uncertainty, perfect foresight. With perfect foresight, agents' expectations of money growth during period t and $t+1$ coincide with the actual values of money growth during these periods, so that $z_t^{t-1} = x_t$ and $z_{t+1}^{t-1} = x_{t+1}$ for all $t = 0, 1, 2, \ldots$.

As emphasized by Chari and Kehoe (1990) and Stokey (1991), one must consider two distinct environments in which optimal policy may be formulated under rational expectations. In the first environment, the government has access to a technology that allows it to announce, at the beginning of period $t = 0$, a sequence $\{x_t\}_{t=0}^{\infty}$ of planned money growth rates and to commit to actually following that plan in all future periods. Ireland (1997) shows that, in this case with commitment, the optimal policy in the model considered here sets $x_t = \beta$ for all $t = 0, 1, 2, \ldots$, as called for by Friedman (1969), to make the net nominal interest rate $R_t - 1$ constant and equal to zero.

In the second environment, the government lacks a commitment technology. The government can still announce a sequence of planned money growth rates $\{x_t\}_{t=0}^{\infty}$ at the beginning of period $t = 0$ but is free to rechoose the sequence $\{x_{t+j}\}_{j=0}^{\infty}$ at the beginning of each period $t = 1, 2, 3, \ldots$. Thus, the government has no mechanism for committing itself to a future plan for monetary policy and can instead be viewed as choosing a value for x_t at the beginning of each period $t = 0, 1, 2, \ldots$.

In the case without commitment, the time-consistency problem arises. The benefits from creating surprise inflation provide the monetary authority with an incentive to choose a rate of money growth that is higher than expected in each period. However, with rational expectations, private agents recognize that the monetary authority has this incentive and adjust their behavior accordingly. In equilibrium, therefore, the time-consistency problem may lead to the outcome first described by Kydland and Prescott (1977), in which the monetary authority attempts to increase output and employment by creating surprise inflation but finds that its efforts lead only to a higher rate of expected inflation.

By applying methods developed by Chari and Kehoe (1990) and Stokey (1991) to the model used here, Ireland (1997) shows that there are, in fact, many possible outcomes in the case in which the monetary authority lacks a commitment technology and agents have rational expectations. All of these outcomes can be supported in equilibria where private expectations display an extreme form of trigger-like behavior: A single deviation by the monetary authority away from its proposed policy causes expected inflation to jump permanently to a very high level. In one such equilibrium, policy follows the Friedman (1969) rule, even without commitment: The single-period gain from setting x_t above β is more than offset by the costs of higher expected inflation forever after. However, these trigger-strategy equilibria also support many other outcomes with higher rates of inflation.

Thus, to reduce the number of equilibria in this model of time-consistent monetary policy, suppose that instead of having perfect foresight, agents must form their expectations in period $t-1$ of money growth during periods t and $t+1$ as stationary functions of actual money growth during periods $t-1$ through $t-N$, so that

$$z_t^{t-1} = \psi^1(x_{t-1}, x_{t-2}, \ldots, x_{t-N}) \tag{14}$$

and

$$z_{t+1}^{t-1} = \psi^2(x_{t-1}, x_{t-2}, \ldots, x_{t-N}) \tag{15}$$

for all $t = 0, 1, 2, \ldots$, where $N < \infty$ is a positive integer and where the expectations functions $\psi^1\colon R_{++}^N \to R_{++}$ and $\psi^2\colon R_{++}^N \to R_{++}$ satisfy the following three restrictions:

(R1) ψ^1 and ψ^2 are nondecreasing in each of their arguments.
(R2) For all $x \in R_{++}$, $\psi^1(x, x, \ldots, x) = \psi^2(x, x, \ldots, x) = x$.
(R3) ψ^1 and ψ^2 are continuously differentiable on R_{++}^N.

Restriction (R1) requires the expected rate of future money growth to move together with the actual rate of money growth; it still allows inflationary expectations to rise if the monetary authority creates too much actual inflation, but also implies that inflationary expectations will begin to ease if the monetary authority acts to bring actual inflation down. Restriction (R2) requires that expectations have Cho and Matsui's (1995) inductive property: If the monetary authority holds money growth constant at any rate x for at least N consecutive periods, then private agents will come to expect that it will continue to hold money growth constant at x. Thus, (R1) and (R2) allow the monetary authority to build credibility for a disinflationary policy by simply adopting and following that policy for a sufficient length of time, as suggested by Taylor (1982) and McCallum (1995). The rational expectations assumption, in contrast, may make it impossible for the monetary authority to build credibility; the trigger-like mechanisms used to support the multitude of equilibria in Ireland (1997), for instance, require expected inflation to rise even when the monetary authority surprises private agents by attempting to lower the current inflation rate. Restriction (R3) limits the extent to which expectations of future money growth can jump following any unexpected change in policy; Rogoff (1989) suggests that the number of equilibria in models of time-consistent monetary policy might be reduced under such a restriction. Expectations functions satisfying restrictions (R1)–(R3) also appear throughout the literature on temporary general equilibrium theory; see, for example, Fuchs and Laroque (1976), Fuchs (1979), Tillmann (1983), and Grandmont and Laroque (1986).

Equation (9) links the gross nominal interest rate to the expected future money growth rate via $R_t = z^t_{t+1}/\beta$. If $R_t < 1$, then the net nominal interest rate becomes negative, and the representative household can make infinite profits by selling bonds and using the proceeds to accumulate hoards of cash balances. In this case, the household's problem fails to have a well-defined solution. Thus, a fourth and final restriction on the expectations functions ψ^1 and ψ^2 is required:

(R4) For all $(x_1, x_2, \ldots, x_N) \in R^N_{++}$ satisfying $x_i \geq \beta$ for all $i = 1, 2, \ldots, N$, $\psi^1(x_1, x_2, \ldots, x_N) \geq \beta$ and $\psi^2(x_1, x_2, \ldots, x_N) \geq \beta$.

Restriction (R4) states that if the monetary authority always chooses a rate of money growth that is greater than or equal to the household's discount factor β, as it must to guarantee that the net nominal interest rate is nonnegative under perfect foresight, then private agents who form their expectations using the functions ψ^1 and ψ^2 will always expect future rates of money growth to be greater than or equal to β, so that nominal interest rates will be nonnegative here as well. Thus, when coupled with the constraints

$$x_t \geq \beta \tag{16}$$

for all $t = 0, 1, 2, \ldots$ imposed on the monetary authority's choice of policy, (R4) performs the role of Marcet and Sargent's (1989) projection facility by ensuring that private expectations remain consistent with the conditions required for the existence of an equilibrium in this model.

Combining (13)–(15), the representative household's consumption and employment are determined as

$$c_t = n_t = \beta\left(\frac{\theta - 1}{\theta}\right)\left[\frac{x_t}{\psi^1(x_{t-1}, x_{t-2}, \ldots, x_{t-N})}\right]\left[\frac{1}{\psi^2(x_{t-1}, x_{t-2}, \ldots, x_{t-N})}\right] \tag{17}$$

for all $t = 0, 1, 2, \ldots$. An equilibrium for this model can now be defined as follows:

DEFINITION 1. *An equilibrium consists of a pair of expectations functions* ψ^1 *and* ψ^2 *and a sequence of money growth rates* $\{x_t\}^\infty_{t=0}$ *such that*: (*i*) *the functions* ψ^1 *and* ψ^2 *satisfy* (R1)–(R4) *and* (*ii*) *the sequence* $\{x_t\}^\infty_{t=0}$ *maximizes the representative household's utility function* (1) *subject to the constraints* (16) *and* (17) *for all* $t = 0, 1, 2, \ldots,$ *taking the initial conditions* $x_{-N}, x_{-N+1}, \ldots, x_{-1}$ *as given.*

The definition indicates that the government in this model is benevolent: it chooses a policy $\{x_t\}_{t=0}^{\infty}$ to maximize the household's utility, given that consumption, output, and employment are determined by (17). Since private expectations of future money growth depend only on past rates of actual money growth, the solution to this problem is time consistent; optimal policy is the same, regardless of whether the monetary authority chooses the entire sequence $\{x_t\}_{t=0}^{\infty}$ at the beginning of period $t = 0$ or whether it chooses each individual x_t at the beginning of each period $t = 0, 1, 2, \ldots$. However, although the alternative restrictions on expectations eliminate the time-consistency problem, they still allow for a multiplicity of equilibria, with different outcomes associated with different choices of ψ^1 and ψ^2 that satisfy (R1)–(R4). If agents in the model are sufficiently patient, however, all of these equilibria share the same steady state, as the following proposition shows:

PROPOSITION 1. *Suppose that $\beta^N > 1/2$. Then any equilibrium that converges to a steady state, with*

$$\lim_{t\to\infty} x_t = x$$

for some constant x, must have $x = \beta$.

A complete proof of this proposition is contained in Section A.2 of the Appendix; here, a less formal argument indicates why the result must hold. Imagine that the economy begins in an initial steady state, in which both actual and expected money growth are constant at some rate $x > \beta$, so that

$$x_t = z_t^{t-1} = z_{t+1}^{t-1} = x$$

for all $t \leq 0$. If the monetary authority decides to keep the rate of money growth constant at x for all $t = 0, 1, 2, \ldots$, then (17) implies that output remains constant, with

$$c_t = c^0 = \beta[(\theta - 1)/\theta](1/x)$$

for all $t = 0, 1, 2, \ldots$. However, suppose that, instead, the monetary authority immediately and permanently reduces the rate of money growth, so that

$$x_t = x - \varepsilon < x$$

for all $t = 0, 1, 2, \ldots$. Restriction (R1) implies that expected inflation will not jump higher in response to this change in policy, whereas (R2) guarantees that, by period $t = N$, expected money growth will have converged to the lower rate $x - \varepsilon$. Thus, using (17), output must satisfy

$$c_t \geq c^L = \beta[(\theta - 1)/\theta][(x - \varepsilon)/x^2]$$

for all $t = 0, 1, \ldots, N - 1$ and

$$c_t = c^H = \beta[(\theta - 1)/\theta][1/(x - \varepsilon)]$$

for all $t = N, N + 1, N + 2, \ldots$.

Because $c^L < c^0$, the disinflation may be accompanied, at first, by a recession. Consider measuring the potential cost of this recession as

$$C = \sum_{t=0}^{N-1} \beta^t [\ln(c^0) - \ln(c^L)] = [(1-\beta^N)/(1-\beta)][\ln(x) - \ln(x-\varepsilon)].$$

Likewise, because $c^H > c^0$, consider measuring the benefit of the subsequent decline in inflationary expectations by

$$B = \sum_{t=N}^{\infty} \beta^t [\ln(c^H) - \ln(c^0)] = [\beta^N/(1-\beta)][\ln(x) - \ln(x-\varepsilon)].$$

Comparing these last two expressions reveals that the benefits exceed the costs whenever $\beta^N > 1 - \beta^N$ or, more simply, whenever $\beta^N > 1/2$.

Thus, (R1) and (R2) give the monetary authority enough leverage over private expectations to build credibility for a disinflationary policy. The costs of disinflation are immediate but transitory; the benefits of the disinflation are permanent but delayed. However, as long as agents are sufficiently patient, the overall impact of the disinflation is positive. In the limit, therefore, monetary policy follows the Friedman (1969) rule, contracting the money supply so that the net nominal interest rate $R_t - 1$ is constant and equal to zero.

Note that, in practice, the condition $\beta^N > 1/2$ is likely to hold. Consider, for example, an annual version of the model in which $N = 10$, so that it can take up to 10 years for a disinflationary policy to gain full credibility. Then, $\beta^N > 1/2$ requires only that the annual discount factor β exceed $(1/2)^{1/10} = 0.933$.

Note, also, that neither Proposition 1 nor Definition 1 ties private agents in the model to any specific pair of expectations functions ψ^1 and ψ^2; the results hold for any ψ^1 and ψ^2 that satisfy (R1)–(R4). This means, of course, that the plausibility of the results ultimately depends on the plausibility of (R1)–(R4). As argued earlier, however, (R1) and (R2) formalize the idea from Taylor (1982) and McCallum (1995) that a monetary authority should be able to build credibility for a disinflationary policy by actually adopting and sticking to that policy, while (R3) formalizes the idea from Rogoff (1989) that private inflationary expectations ought to move smoothly in response to changes in monetary policy.

Two final examples illustrate how (R1)–(R4) allow the monetary authority to build credibility for a disinflationary policy when the economy is initially in a position away from its unique steady state, with a positive rate of inflation. Both examples use an annual version of the model, with $\beta = 0.95$, $N = 10$, and

$$\psi^1(x_{t-1}, x_{t-2}, \ldots, x_{t-10}) = \psi^2(x_{t-1}, x_{t-2}, \ldots, x_{t-10}) = \prod_{j=1}^{10} x_{t-j}^{\alpha_j},$$

so that α_j represents the elasticity of z_t^{t-1} and z_{t+1}^{t-1} with respect to x_{t-j}. Restriction (R1) requires that $\alpha_j \geq 0$ for all $j = 1, 2, \ldots, 10$, while (R2) requires that

$$\sum_{j=1}^{10} \alpha_j = 1.$$

Equations (8) and (12) imply that $\theta/(\theta - 1)$ measures the steady-state markup of price over marginal cost; both examples set $\theta = 6$, corresponding to a markup of 20%. Finally, both examples set $x_{-10} = x_{-9} = \cdots = x_{-1} = 1.03$, so that the

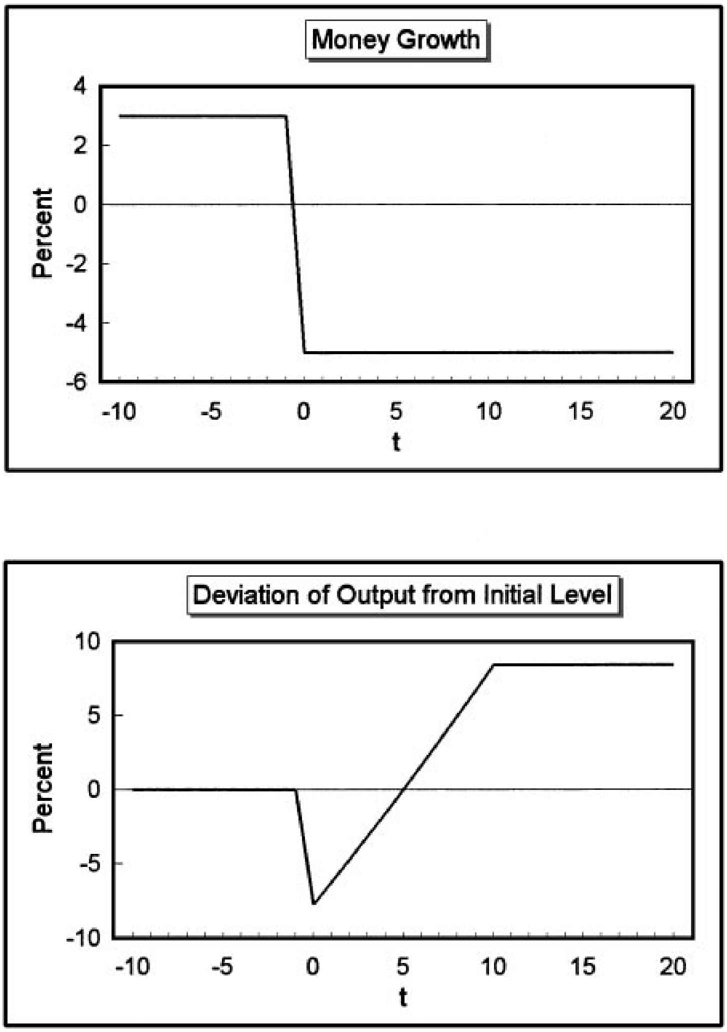

FIGURE 1. Money growth and output in Example 1.

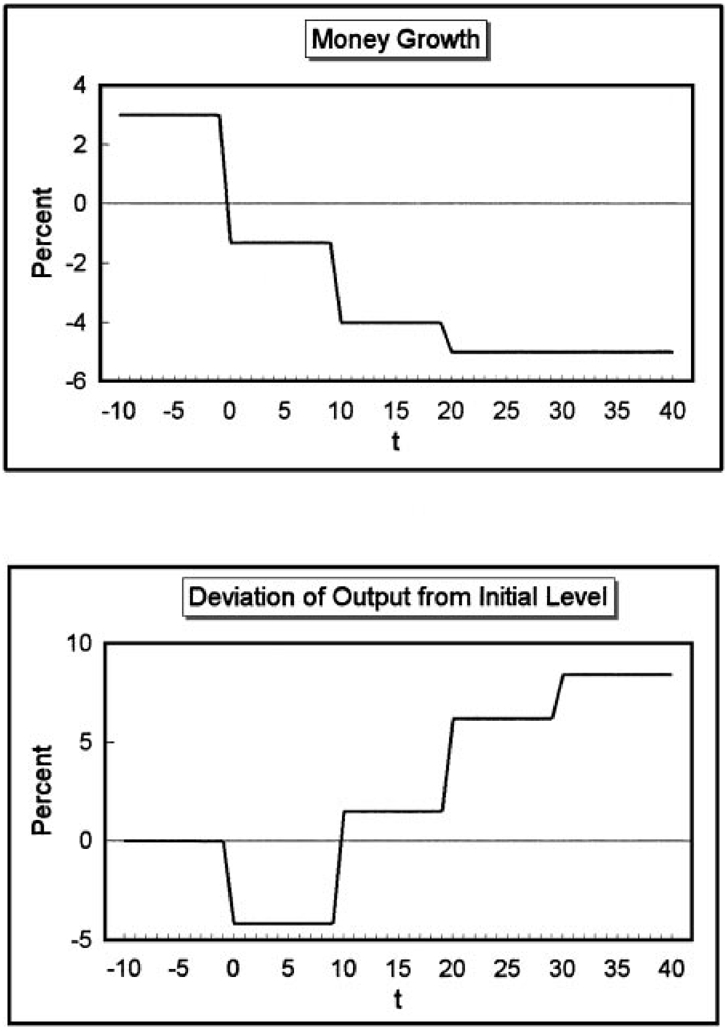

FIGURE 2. Money growth and output in Example 2.

economy begins in an initial steady state with actual and expected rates of inflation equal to 3%, the average rate of consumer price inflation in the United States since 1990.

The first example sets $\alpha_j = 0.1$ for all $j = 1, 2, \ldots, 10$. Figure 1 shows that, in this case, optimal policy immediately reduces the rate of money growth to its unique steady-state level, so that the equilibrium has $x_t = \beta$ for all $t = 0, 1, 2, \ldots$. Initially, output and employment fall, as expectations adjust only gradually to the change in policy. Eventually, however, the declining rate of expected inflation allows output to rise. Thus, in this example, the monetary authority builds credibility for its disinflationary policy, as suggested by Taylor (1982) and McCallum (1995), by demonstrating that it will stick to this policy despite the short-run costs.

The second example sets $\alpha_j = 0$ for $j = 1, 2, \ldots, 9$ and $\alpha_{10} = 1$, so that expectations adjust much more slowly to an observed change in policy. Figure 2 shows that, in this case, optimal policy smooths the short-run costs over time by taking a gradual approach to disinflation. The money growth rate reaches its unique steady-state level, but only after 20 years have passed. Output remains below its initial level for 10 years and takes 30 years to completely adjust.

4. CONCLUSION

Typically, models of time-consistent monetary policy have many equilibria. This multiplicity presents a problem if one chooses to interpret the models along normative lines, for the theory fails to indicate how a central banker who is stuck in a high-inflation equilibrium might steer the economy toward a preferred, low-inflation equilibrium.

Results derived here suggest that the assumption of rational expectations lies at the source of the multiplicity problem. Under rational expectations, the expected rate of inflation often jumps higher, not only when the monetary authority surprises private agents by creating too much inflation, but also when the monetary authority surprises private agents by attempting to disinflate. Thus, this paper replaces the rational expectations assumption with a set of alternative restrictions on expectations that allow the monetary authority to build credibility for a disinflationary policy by actually adopting and following that policy for a sufficient length of time. Under these alternative restrictions, the model used here has a unique steady state, in which monetary policy follows the Friedman (1969) rule by contracting the money supply to keep the nominal interest rate constant at zero.

Two examples show that when the economy begins away from this unique steady state, with positive inflation, the monetary authority can successfully disinflate. In both cases, however, the disinflation is accompanied by short-run losses in output and employment; in the second case, these costs are sufficient to make a gradual approach to disinflation optimal. For central bankers, therefore, the news is both good and bad: Credibility can be acquired, but only at a price.

REFERENCES

al-Nowaihi, A. & P. Levine (1994) Can reputation resolve the monetary policy credibility problem? *Journal of Monetary Economics* 33, 355–380.

Apostol, T.M. (1974) *Mathematical Analysis*. Reading, MA: Addison-Wesley.

Backus, D. & J. Driffill (1985) Inflation and reputation. *American Economic Review* 75, 530–538.

Barro, R.J. (1986) Reputation in a model of monetary policy with incomplete information. *Journal of Monetary Economics* 17, 3–20.

Barro, R.J. & D.B. Gordon (1983) Rules, discretion and reputation in a model of monetary policy. *Journal of Monetary Economics* 12, 101–121.

Blackburn, K. & M. Christensen (1989) Monetary policy and policy credibility: Theories and evidence. *Journal of Economic Literature* 27, 1–45.

Chari, V.V. & P.J. Kehoe (1990) Sustainable plans. *Journal of Political Economy* 98, 783–802.
Cho, I.-K. & A. Matsui (1995) Induction and the Ramsey policy. *Journal of Economic Dynamics and Control* 19, 1113–1140.
Friedman, M. (1969) The optimum quantity of money. In *The Optimum Quantity of Money and Other Essays*, pp. 1–50. Chicago, IL: Aldine.
Fuchs, G. (1979) Is error learning behaviour stabilizing? *Journal of Economic Theory* 20, 300–317.
Fuchs, G. & G. Laroque (1976) Dynamics of temporary equilibria and expectations. *Econometrica* 44, 1157–1178.
Grandmont, J.-M. & G. Laroque (1986) Stability of cycles and expectations. *Journal of Economic Theory* 40, 138–151.
Ireland, P.N. (1997) Sustainable monetary policies. *Journal of Economic Dynamics and Control* 22, 87–108.
Kydland, F.E. & E.C. Prescott (1977) Rules rather than discretion: The inconsistency of optimal plans. *Journal of Political Economy* 85, 473–491.
Lucas, R.E., Jr. (1980) Equilibrium in a pure currency economy. *Economic Inquiry* 18, 203–220.
Marcet, A. and T.J. Sargent (1989) Convergence of least squares learning mechanisms in self-referential linear stochastic models. *Journal of Economic Theory* 48, 337–368.
McCallum, B.T. (1995) Two fallacies concerning central-bank independence. *American Economic Review* 85, 207–211.
Rogoff, K. (1989) Reputation, coordination, and monetary policy. In R.J. Barro (ed.), *Modern Business Cycle Theory*, pp. 236–264. Cambridge, MA: Harvard University Press.
Rubinstein, A. (1986) Finite automata play the repeated prisoner's dilemma. *Journal of Economic Theory* 39, 83–96.
Sargent, T.J. (1993) *Bounded Rationality in Macroeconomics*. Oxford: Clarendon Press.
Stokey, N.L. (1991) Credible public policy. *Journal of Economic Dynamics and Control* 15, 627–656.
Taylor, J.B. (1982) Establishing credibility: A rational expectations viewpoint. *American Economic Review* 72, 81–85.
Tillmann, G. (1983) Stability in a simple pure consumption loan model. *Journal of Economic Theory* 30, 315–329.

APPENDIX

A.1. IMPLICATIONS OF HOUSEHOLD OPTIMIZATION

During each period $t = 0, 1, 2, \ldots,$ the representative household chooses sequences $\{c^t_{t+j}\}^\infty_{j=0}$, $\{c^t_{t+j}(i)\}^\infty_{j=0}$, $\{n^t_{t+j}\}^\infty_{j=0}$, $\{m^t_{t+j+1}\}^\infty_{j=0}$, and $\{b^t_{t+j+1}\}^\infty_{j=0}$ to maximize its utility function (2) subject to the constraints (3)–(5) for all $j = 0, 1, 2, \ldots.$ When the market-clearing conditions $m^t_{t+j} = 1$ and $b^t_{t+j} = 0$, $j = 0, 1, 2, \ldots,$ are imposed, the first-order conditions for this problem can be written as

$$\left(c^t_{t+j}\right)^{(1-\theta)/\theta} c^t_{t+j}(i)^{-1/\theta} = \left(\lambda^t_{t+j} + \mu^t_{t+j}\right) p^t_{t+j}(i), \qquad \textbf{(A.1)}$$

$$1 = \lambda^t_{t+j} w^t_{t+j}, \qquad \textbf{(A.2)}$$

$$\lambda^t_{t+j} z^t_{t+j} = \beta\left(\lambda^t_{t+j+1} + \mu^t_{t+j+1}\right), \qquad \textbf{(A.3)}$$

$$\left(\lambda^t_{t+j} + \mu^t_{t+j}\right) z^t_{t+j} = \beta R^t_{t+j}\left(\lambda^t_{t+j+1} + \mu^t_{t+j+1}\right), \qquad \textbf{(A.4)}$$

$$\left[\int_0^1 c_{t+j}^t(i)^{(\theta-1)/\theta}\,di\right]^{\theta/(\theta-1)} = c_{t+j}^t, \tag{A.5}$$

and

$$z_{t+j}^t = \int_0^1 p_{t+j}^t(i)c_{t+j}^t(i)\,di \tag{A.6}$$

for all $j = 0, 1, 2, \ldots$, where $\lambda_{t+j}^t > 0$ and $\mu_{t+j}^t \geq 0$ are multipliers on the budget constraint (4) and the cash-in-advance constraint (5) and where the cash-in-advance constraint is assumed to hold with equality even when it does not bind.

Multiplying both sides of (A.1) by $c_{t+j}^t(i)$, integrating over $i \in [0, 1]$, and using (A.5) and (A.6) yields

$$\lambda_{t+j}^t + \mu_{t+j}^t = 1\big/z_{t+j}^t. \tag{A.7}$$

Substituting this result back into (A.1), raising both sides to the power $1-\theta$, integrating over $i \in [0, 1]$, and using (A.5) and the definition

$$p_{t+j}^t = \left[\int_0^1 p_{t+j}^t(i)^{1-\theta}\,di\right]^{1/(1-\theta)} \tag{A.8}$$

yields

$$c_{t+j}^t = z_{t+j}^t\big/p_{t+j}^t. \tag{A.9}$$

Equations (A.8) and (A.9), with $j = 0$, coincide with (10) and (7) in the text.

Substituting (A.7) and (A.9) into (A.1), solving for $c_{t+j}^t(i)$, and setting $j = 0$ yields (6) in the text. Substituting (A.3) and (A.7) into (A.2), solving for w_{t+j}^t, and setting $j = 0$ yields (8) in the text. Finally, substituting (A.7) into (A.4), solving for R_{t+j}^t, and setting $j = 0$ yields (9) in the text.

A.2. PROOF OF PROPOSITION

The following lemma proves useful in establishing the main result:

LEMMA A.1. *Let $f\colon R_{++}^N \to R_{++}$ be a differentiable function satisfying*

$$f(x, x, \ldots, x) = x$$

for all $x \in R_{++}$. Then

$$\sum_{j=1}^N f_j(x, x, \ldots, x) = 1$$

for all $x \in R_{++}$, where f_j denotes the partial derivative of f with respect to its jth argument.

Proof. Follows from Apostol's (1974, pp. 346–348, Definition 12.2 and Theorem 12.5). ■

In any equilibrium, the monetary authority chooses $\{x_t\}_{t=0}^\infty$ to maximize the representative household's utility function (1) subject to the constraints (16) and (17) for all $t = 0, 1, 2, \ldots$, taking the initial conditions $x_{-N}, x_{-N+1}, \ldots, x_{-1}$ as given. Since, by (R2) and (R3), the expectations functions ψ^1 and ψ^2 are continuously differentiable and satisfy

$\psi^1(x, x, \ldots, x) = \psi^2(x, x, \ldots, x) = x$ for all $x \in R_{++}$, the first-order condition for this problem implies that if the solution has

$$\lim_{t\to\infty} x_t = x,$$

then the constant x must satisfy

$$x\varphi = \left[1 - \left(\frac{\theta - 1}{\theta}\right)\left(\frac{\beta}{x}\right)\right]\left\{\sum_{j=1}^{N} \beta^j \left[\psi_j^1(x, x, \ldots, x) + \psi_j^2(x, x, \ldots, x)\right] - 1\right\}, \quad \textbf{(A.10)}$$

where the φ denotes the limit of the sequence $\{\varphi_t\}_{t=0}^{\infty}$ of multipliers on the constraints $x_t \geq \beta$, $t = 0, 1, 2, \ldots$. In addition, x and φ must satisfy the complementary slackness conditions $\varphi \geq 0$, $x \geq \beta$, and $\varphi(x - \beta) = 0$.

Since $x \geq \beta$ and $\theta > 1$, it must be that

$$1 - \left(\frac{\theta - 1}{\theta}\right)\left(\frac{\beta}{x}\right) > 0.$$

Moreover,

$$\sum_{j=1}^{N} \beta^j \left[\psi_j^1(x, x, \ldots, x) + \psi_j^2(x, x, \ldots, x)\right]$$
$$\geq \beta^N \sum_{j=1}^{N} \left[\psi_j^1(x, x, \ldots, x) + \psi_j^2(x, x, \ldots, x)\right] = 2\beta^N,$$

where the first inequality follows from (R1) and the second equality follows from (R2) and the lemma. Thus, when $\beta^N > 1/2$, (A.10) requires that $\varphi > 0$ and $x = \beta$. ■