Problem Set 3

Illustrates that Hotelling's lemma, Shephard's lemma, Roy's identity, and the Slutsky equation can all be derived with reference to the envelope thorem.

Next:

Proof of the Kuhn-Tucker theorem

The Ramsey Model

Berge's Maximum Theorem

The Maximum Principle

The Envelope Theorem

Suppose the price of a good that you like to consumer falls slightly. How much do you benefit?

This is a question that can be answered by taking the derivative of the indirect utility function, using the envelope theorem.

The Envelope Theorem

$$v(p_1, p_2, I) = \max_{c_1, c_2} U(c_1, c_2) \text{ subject to } I \geq p_1 c_1 + p_2 c_2$$

$$v(p_1, p_2, I) = U[c_1^*(p_1, p_2, I), c_2^*(p_1, p_2, I)]$$

$$\begin{aligned} v(p_1, p_2, I) = \; & U[c_1^*(p_1, p_2, I), c_2^*(p_1, p_2, I)] \\ & + \lambda^*(p_1, p_2, I)[I - p_1 c_1^*(p_1, p_2, I) - p_2 c_2^*(p_1, p_2, I)] \end{aligned}$$

The Envelope Theorem

$$v(p_1, p_2, I) = U[c_1^*(p_1, p_2, I), c_2^*(p_1, p_2, I)] \\ + \lambda^*(p_1, p_2, I)[I - p_1 c_1^*(p_1, p_2, I) - p_2 c_2^*(p_1, p_2, I)]$$

$$\frac{\partial v(p_1, p_2, I)}{\partial p_1} = -\lambda^*(p_1, p_2, I) c_1^*(p_1, p_2, I)$$

$$\frac{\partial v(p_1, p_2, I)}{\partial I} = \lambda^*(p_1, p_2, I)$$

$$\frac{\partial v(p_1, p_2, I)}{\partial p_1} = -\left[\frac{\partial v(p_1, p_2, I)}{\partial I}\right] c_1^*(p_1, p_2, I)$$

The Envelope Theorem

Roy's identity

$$\frac{\partial v(p_1, p_2, I)}{\partial p_1} = -\left[\frac{\partial v(p_1, p_2, I)}{\partial I}\right] c_1^*(p_1, p_2, I)$$

says that the utility gain from a one dollar reduction in the price of good 1 is the same as the utility gain from a $c_1^*(p_1, p_2, I)$ increase in your income.

So long as the price reduction is small, this gain depends only on the amount of the good you purchased before the price change and not on your willingness or ability to substitute towards the good and out of other goods in response to the price change.