Problem Set 2

Ties up some loose ends associated with the Kuhn-Tucker theorem.

Question 1: Second-Order Conditions

Question 2: Complementary Slackness

Question 3: The Constraint Qualification

Problem Set 2

Question 4: Perfect Substitutes

An interesting example where the consumer flips between corner solutions depending on prices.

You can use your intuition to figure out what the solution looks like, then use the KT conditions to see how different configurations of binding and nonbinding constraints arise, depending on prices.

We'll come back to this example in our discussion of Berge's maximum theorem, because it produces a case in which optimal consumer depends take the form of multi-valued correspondences instead of single-valued functions. A generalized notion of continuity applies to them.

Problem Set 2

Question 5: Restrictions on elasticities of demand implied by weak assumptions about preferences and rationality.

If more is preferred to less, the budget constraint will bind:

$$I = p_1 c_1^*(p_1, p_2, p_3, I) + p_2 c_2^*(p_1, p_2, p_3, I) + p_3 c_3^*(p_1, p_2, p_3, I)$$

Differentiate with respect to one of the prices and manipulate the result algebraically to obtain the "Cournot aggregation" condition

$$s_1 \varepsilon_{1,j} + s_2 \varepsilon_{2,j} + s_3 \varepsilon_{3,j} = -s_j$$

If there is a Giffen good, with $\varepsilon_{j,j} > 0$, then one of the other goods must have $\varepsilon_{i,j} < -0$, so that it's consumption goes down when the price of the Giffen good rises.

Problem Set 2

Question 5: Restrictions on elasticities of demand implied by weak assumptions about preferences and rationality.

If more is preferred to less, the budget constraint will bind:

$$I = p_1 c_1^*(p_1, p_2, p_3, I) + p_2 c_2^*(p_1, p_2, p_3, I) + p_3 c_3^*(p_1, p_2, p_3, I)$$

Differentiate with respect to income I to obtain the "Engel aggregation" or "adding up" condition

$$s_1\eta_1 + s_2\eta_2 + s_3\eta_3 = 1$$

Goods can't all be inferior, with $\eta_j < 0$ for all j. Goods can't all be luxuries, with $\eta_j > 1$ for all j.

Problem Set 2

Question 5: Restrictions on elasticities of demand implied by weak assumptions about preferences and rationality.

A weak assumption about rationality is that Marshallian demand curves are homogeneous of degree zero in income and prices:

$$c_i^*(rp_1, rp_2, rp_3, rI) = c_i^*(p_1, p_2, p_3, I).$$

Differentiate with respect to r to obtain

$$\varepsilon_{i,1} + \varepsilon_{i,2} + \varepsilon_{i,3} + \eta_i = 0$$

If you are estimating elasticities of demand, you can impose or test this restriction.

Consumer Optimization

Associated with the consumer's problem

$$\max_{c_1, c_2} \; U(c_1, c_2) \text{ subject to } I \geq p_1 c_1 + c_2 c_2$$

define the Lagrangian

$$L(c_1, c_2, \lambda) = U(c_1, c_2) + \lambda(I - p_1 c_1 - p_2 c_2)$$

Consumer Optimization

$$L(c_1, c_2, \lambda) = U(c_1, c_2) + \lambda(I - p_1 c_1 - p_2 c_2)$$

The first-order conditions and binding constraint

$$U_1(c_1^*, c_2^*) - \lambda^* p_1 = 0$$

$$U_2(c_1^*, c_2^*) - \lambda^* p_2 = 0$$

$$I = p_1 c_1^* + p_2 c_2^*$$

form a system of 3 equations in c_1^*, c_2^*, and λ^*.

Consumer Optimization

$$U_1(c_1^*, c_2^*) - \lambda^* p_1 = 0$$
$$U_2(c_1^*, c_2^*) - \lambda^* p_2 = 0$$

The envelope theorem provides an economic interpretation of λ^* as the marginal utility of income.

The prices p_1 and p_2 measure the dollar cost of buying one more unit of good 1 or 2.

The products $\lambda^* p_1$ and $\lambda^* p_2$ measure the utility cost of buying one more unit of good 1 or 2.

The FOCs show that it is optimal to equate, at the margin, the utility benefits and costs of buying one more unit of each good.

Consumer Optimization

The first-order conditions and binding constraint

$$U_1(c_1^*, c_2^*) - \lambda^* p_1 = 0$$

$$U_2(c_1^*, c_2^*) - \lambda^* p_2 = 0$$

$$I = p_1 c_1^* + p_2 c_2^*$$

Alternatively, if a solution for λ^* is not needed, divide the FOC for c_1 by the FOC for c_2 to obtain

$$\frac{U_1(c_1^*, c_2^*)}{U_2(c_1^*, c_2^*)} = \frac{p_1}{p_2}$$

which restates the tangency condition equating the slopes of the indifference curve (MRS) and budget constraint.

Consumer Optimization

$$\frac{U_1(c_1^*, c_2^*)}{U_2(c_1^*, c_2^*)} = \frac{p_1}{p_2}$$

$$\frac{U_1(c_1^*, c_2^*)}{U_2(c_1^*, c_2^*)} = \frac{\text{utility/good 1}}{\text{utility/good 2}} = \frac{\text{utility}}{\text{good 1}} \times \frac{\text{good 2}}{\text{utility}} = \frac{\text{good 2}}{\text{good 1}}$$

measures the trade-off in utility

$$\frac{p_1}{p_2} = \frac{\text{dollars/good 1}}{\text{dollars/good 2}} = \frac{\text{dollars}}{\text{good 1}} \times \frac{\text{good 2}}{\text{dollars}} = \frac{\text{good 2}}{\text{good 1}}$$

measures the trade-off in the market

The tangency condition shows that it is optimal to equate the trade-off in utility with the trade-off in the market.

Consumer Optimization

The tangency condition

$$\frac{U_1(c_1^*, c_2^*)}{U_2(c_1^*, c_2^*)} = \frac{p_1}{p_2}$$

and the binding constraint

$$I = p_1 c_1^* + p_2 c_2^*$$

form a system of 2 equations in c_1^* and c_2^*.

It's often easiest to just eliminate the multiplier and work with the optimality conditions in this form.