Problem Set 7

ECON 772001 - Math for Economists — Peter Ireland
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For Extra Practice – Not Collected or Graded

1. The Permanent Income Hypothesis

The permanent income hypothesis describes how a forward-looking consumer optimally saves or borrows to smooth out his or her consumption in the face of a fluctuating income stream. This problem formalizes the permanent income hypothesis using a two-period model. So consider a consumer who lives for two periods, earning income w_0 during period $t = 0$ and w_1 during period $t = 1$. Let c_0 and c_1 denote his or her consumption during periods $t = 0$ and $t = 1$ and let s denote his or her amount saved (or borrowed, if negative) during period $t = 0$. Suppose that savings earn interest between $t = 0$ and $t = 1$ at the constant rate r. Then the consumer faces the budget constraints

$$w_0 \geq c_0 + s \tag{1}$$

at $t = 0$ and

$$w_1 + (1 + r)s \geq c_1 \tag{2}$$

at $t = 1$. Finally, suppose that the consumer's preferences are described by the utility function

$$\ln(c_0) + \beta \ln(c_1), \tag{3}$$

where the discount factor lies between zero and one: $0 < \beta < 1$.

a. Find the values for c_0^*, c_1^*, and s^* that solve the consumer's problem – choose c_0, c_1, and s to maximize the utility function in (3) subject to the constraints in (1) and (2) – in terms of the parameters w_0, w_1, r, and β.

b. Suppose now that the market interest rate and the consumer's discount factor satisfy $\beta(1 + r) = 1$. Although it might seem that this condition will hold only through a rare coincidence, in fact as problem set 6 showed, it is often satisfied in more elaborate general equilibrium models that feature a large numbers of consumers who borrow and lend in a competitive market. With this extra condition imposed, what do your solutions from part (a) above imply for the optimal behavior of consumption: does it rise, fall, or stay the same moving from period $t = 0$ to period $t = 1$?

c. Continuing to assume that $\beta(1 + r) = 1$, what determines whether the consumer will borrow (choosing $s < 0$) or save (choosing $s > 0$) during period $t = 0$?

2. Habit Formation

Consider the behavior of a consumer that faces the same budget constraints (1) and (2) shown above, but instead of (3) has the utility function

$$\ln(c_0) + \beta \ln(c_1 - \gamma c_0), \tag{4}$$

where $0 < \gamma < 1$. The preferences represented by this utility function display "habit formation" or capture an "addictive" element of consumption by implying that the more the individual consumers during period $t = 0$ the more he or she will want to consume during period $t = 1$.

a. Assuming once more that β and r are separate parameters, find the values for c_0^*, c_1^*, and s^* that solve the consumer's problem – choose c_0, c_1, and s to maximize the utility function in (4) subject to the constraints in (1) and (2) – in terms of the parameters w_0, w_1, r, β, and γ.

b. Now assume that $\beta(1 + r) = 1$ holds again. What happens to the optimal behavior of consumption now: is c_1^* larger than, smaller than, or the same as c_0^*?

3. Durable Consumption

Now replace the utility function in (4) with

$$\ln(c_0) + \beta \ln(c_1 + \theta c_0), \tag{5}$$

where $0 < \theta < 1$. This specification might capture an aspect of "durability" in the consumption good, whereby a purchase made at $t = 0$ continues to yield utility at $t = 1$, in which case the new parameter θ can be interpreted as measuring the depreciation rate for the durable item.

a. Assuming again that β and r are separate parameters, find the values for c_0^*, c_1^*, and s^* that solve the consumer's problem – choose c_0, c_1, and s to maximize the utility function in (5) subject to the constraints in (1) and (2) – in terms of the parameters w_0, w_1, r, β, and θ.

b. Now assume for the last time that $\beta(1 + r) = 1$ holds. What happens to the optimal behavior of consumption in this case: is c_1^* larger than, smaller than, or the same as c_0^*?