

ECON 337901

FINANCIAL ECONOMICS

Peter Ireland

Boston College

February 12, 2026

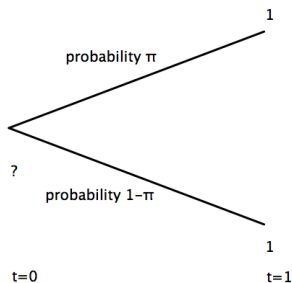
These lecture notes by Peter Ireland are licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 4.0 International (CC BY-NC-SA 4.0) License. <http://creativecommons.org/licenses/by-nc-sa/4.0/>.

Consumer Optimization: The Risk Dimension

A “bond” is a safe asset that pays off one next year in the good state and one next year in the bad state.

These payoffs can be replicated by buying one contingent claim for the good state and one contingent claim for the bad state.

Consumer Optimization: The Risk Dimension



Payoffs for the bond.

Consumer Optimization: The Risk Dimension

If we start with knowledge of the contingent claims prices q^G and q^B , then we can infer that the bond must sell today for

$$q^{bond} = q^G + q^B.$$

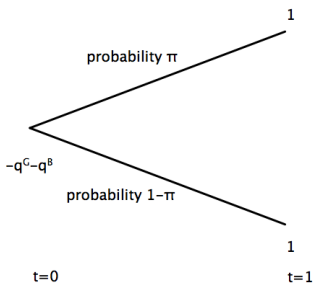
Since the bond pays off one for sure next year, the interest rate, defined as the return on the risk-free bond, is

$$1 + r = \frac{1}{q^{bond}} = \frac{1}{q^G + q^B}.$$

The bond price relates to the interest rate via

$$q^{bond} = \frac{1}{1 + r}.$$

Consumer Optimization: The Risk Dimension



Pricing the bond.

Consumer Optimization: The Risk Dimension

We've already seen how contingent claims can be used to replicate the stock and the bond.

Now let's see how the stock and the bond can be used to replicate the contingent claims.

Consumer Optimization: The Risk Dimension

Consider buying s shares of stock and b bonds, in order to replicate the contingent claim for the good state.

In the good state, the payoffs should be

$$sd^G + b = 1$$

and in the bad state, the payoffs should be

$$sd^B + b = 0$$

since the contingent claim pays off one in the good state and zero in the bad state.

Consumer Optimization: The Risk Dimension

To replicate the contingent claim for the good state:

$$sd^G + b = 1$$

$$sd^B + b = 0 \Rightarrow b = -sd^B$$

Substitute the second equation into the first to solve for

$$s = \frac{1}{d^G - d^B} \text{ and } b = \frac{-d^B}{d^G - d^B}$$

Since s and b are of opposite sign, this requires going “long” one asset and “short” the other.

Consumer Optimization: The Risk Dimension

To replicate the contingent claim for the good state:

$$s = \frac{1}{d^G - d^B} \text{ and } b = \frac{-d^B}{d^G - d^B}$$

If we know the prices q^{stock} and q^{bond} of the stock and bond, we can infer that in the absence of arbitrage, the claim for the good state would have price

$$q^G = q^{stock} s + q^{bond} b = \frac{q^{stock} - d^B q^{bond}}{d^G - d^B}.$$