

ECON 337901

FINANCIAL ECONOMICS

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Consumer Optimization: The Risk Dimension

The consumer's problem is to choose c_0 , c_1^G , and c_1^B to maximize expected utility

$$u(c_0) + \beta\pi u(c_1^G) + \beta(1 - \pi)u(c_1^B),$$

subject to the budget constraint

$$Y_0 + q^G Y_1^G + q^B Y_1^B \geq c_0 + q^G c_1^G + q^B c_1^B.$$

This was Arrow and Debreu's key insight: that finance is like grocery shopping. Mathematically, making decisions over time and under uncertainty is no different from choosing apples, bananas, and pears!

Consumer Optimization: The Risk Dimension

The Lagrangian is

$$L = u(c_0) + \beta\pi u(c_1^G) + \beta(1 - \pi)u(c_1^B) \\ + \lambda (Y_0 + q^G Y_1^G + q^B Y_1^B - c_0 - q^G c_1^G - q^B c_1^B),$$

and the first-order conditions are

$$u'(c_0^*) - \lambda^* = 0 \\ \beta\pi u'(c_1^{G*}) - \lambda^* q^G = 0 \\ \beta(1 - \pi)u'(c_1^{B*}) - \lambda^* q^B = 0$$

Consumer Optimization: The Risk Dimension

The first-order conditions

$$u'(c_0^*) - \lambda^* = 0$$

$$\beta\pi u'(c_1^{G*}) - \lambda^* q^G = 0$$

$$\beta(1 - \pi)u'(c_1^{B*}) - \lambda^* q^B = 0$$

imply that marginal rates of substitution equal relative prices:

$$\frac{u'(c_0^*)}{\beta\pi u'(c_1^{G*})} = \frac{1}{q^G} \text{ and } \frac{u'(c_0^*)}{\beta(1 - \pi)u'(c_1^{B*})} = \frac{1}{q^B}$$

$$\text{and } \frac{\pi u'(c_1^{G*})}{(1 - \pi)u'(c_1^{B*})} = \frac{q^G}{q^B}.$$

Consumer Optimization: The Risk Dimension

Do we really observe consumers trading in contingent claims?

Yes, if we think of financial assets as “bundles” of contingent claims.

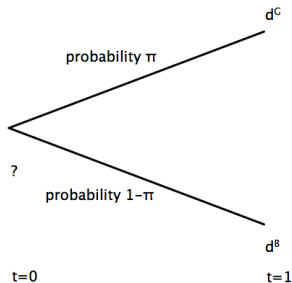
This insight is also Arrow and Debreu's.

Consumer Optimization: The Risk Dimension

A “stock” is a risky asset that pays dividend d^G next year in the good state and d^B next year in the bad state.

These payoffs can be replicated by buying d^G contingent claims for the good state and d^B contingent claims for the bad state.

Consumer Optimization: The Risk Dimension



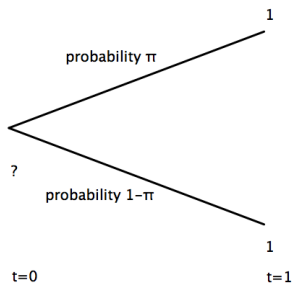
Payoffs for the stock.

Consumer Optimization: The Risk Dimension

A “bond” is a safe asset that pays off one next year in the good state and one next year in the bad state.

These payoffs can be replicated by buying one contingent claim for the good state and one contingent claim for the bad state.

Consumer Optimization: The Risk Dimension



Payoffs for the bond.

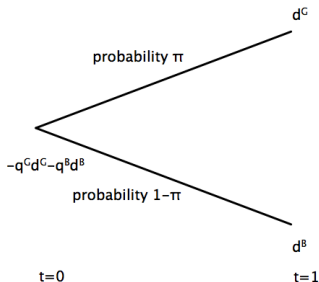
Consumer Optimization: The Risk Dimension

If we start with knowledge of the contingent claims prices q^G and q^B , then we can infer that the stock must sell today for

$$q^{stock} = q^G d^G + q^B d^B.$$

Since if the stock cost more than the equivalent bundle of contingent claims, traders could make profits for sure by short selling the stock and buying the contingent claims; and if the stock cost less than the equivalent bundle of contingent claims, traders could make profits for sure by buying the stock and selling the contingent claims.

Consumer Optimization: The Risk Dimension



“Pricing” the stock.

Consumer Optimization: The Risk Dimension

Likewise, if we start with knowledge of the contingent claims prices q^G and q^B , then we can infer that the bond must sell today for

$$q^{bond} = q^G + q^B.$$

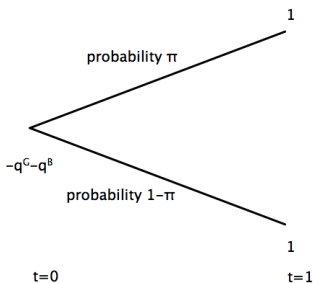
Since the bond pays off one for sure next year, the interest rate, defined as the return on the risk-free bond, is

$$1 + r = \frac{1}{q^{bond}} = \frac{1}{q^G + q^B}.$$

The bond price relates to the interest rate via

$$q^{bond} = \frac{1}{1 + r}.$$

Consumer Optimization: The Risk Dimension



Pricing the bond.