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FINANCIAL ECONOMICS

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Consumer Optimization: The Risk Dimension

We've already seen how contingent claims can be used to replicate the stock and the bond.

Now let's see how the stock and the bond can be used to replicate the contingent claims.

Consumer Optimization: The Risk Dimension

Consider buying s shares of stock and b bonds, in order to replicate the contingent claim for the good state.

In the good state, the payoffs should be

$$sd^G + b = 1$$

and in the bad state, the payoffs should be

$$sd^B + b = 0$$

since the contingent claim pays off one in the good state and zero in the bad state.

Consumer Optimization: The Risk Dimension

To replicate the contingent claim for the good state:

$$sd^G + b = 1$$

$$sd^B + b = 0 \Rightarrow b = -sd^B$$

Substitute the second equation into the first to solve for

$$s = \frac{1}{d^G - d^B} \text{ and } b = \frac{-d^B}{d^G - d^B}$$

Since s and b are of opposite sign, this requires going "long" one asset and "short" the other.

Consumer Optimization: The Risk Dimension

To replicate the contingent claim for the good state:

$$s = \frac{1}{d^G - d^B} \text{ and } b = \frac{-d^B}{d^G - d^B}$$

If we know the prices q^{stock} and q^{bond} of the stock and bond, we can infer that in the absence of arbitrage, the claim for the good state would have price

$$q^G = q^{stock} s + q^{bond} b = \frac{q^{stock} - d^B q^{bond}}{d^G - d^B}.$$

Consumer Optimization: The Risk Dimension

Consider buying s shares of stock and b bonds, in order to replicate the contingent claim for the bad state.

In the good state, the payoffs should be

$$sd^G + b = 0$$

and in the bad state, the payoffs should be

$$sd^B + b = 1$$

since the contingent claim pays off one in the bad state and zero in the good state.

Consumer Optimization: The Risk Dimension

To replicate the contingent claim for the bad state:

$$sd^G + b = 0 \Rightarrow b = -sd^G$$

$$sd^B + b = 1$$

Substitute the first equation into the second to solve for

$$s = \frac{-1}{d^G - d^B} \text{ and } b = \frac{d^G}{d^G - d^B}$$

Once again, this requires going long one asset and short the other.

Consumer Optimization: The Risk Dimension

To replicate the contingent claim for the bad state:

$$s = \frac{-1}{d^G - d^B} \text{ and } b = \frac{d^G}{d^G - d^B}$$

Once again, if we know the prices q^{stock} and q^{bond} of the stock and bond, we can infer that in the absence of arbitrage, the claim for the bad state would have price

$$q^B = q^{stock}s + q^{bond}b = \frac{d^G q^{bond} - q^{stock}}{d^G - d^B}.$$