ECON 337901
FINANCIAL ECONOMICS

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Consumer Optimization: The Time Dimension

The problem is to choose c_0 and c_1 to maximize utility

$$u(c_0) + \beta u(c_1)$$

subject to the budget constraint

$$Y_0 + \frac{Y_1}{1+r} \geq c_0 + \frac{c_1}{1+r}.$$

Consumer Optimization: The Time Dimension

The budget constraint

$$Y_0 + \frac{Y_1}{1+r} \geq c_0 + \frac{c_1}{1+r}$$

says that the present value of income must be sufficient to cover the present value of consumption over the two periods. It also shows that the "price" of consumption today relative to the "price" of consumption next year is related to the interest rate via

$$\frac{p_0}{p_1} = 1 + r.$$

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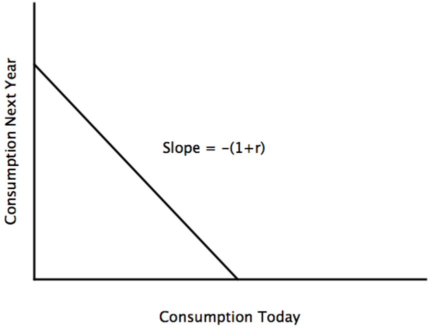

The slope of the intertemporal budget constraint is $-(1 + r)$.

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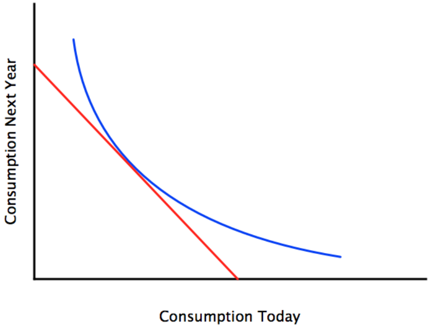

At the optimum, the intertemporal marginal rate of substitution equals the slope of the intertemporal budget constraint.

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We now know the answer ahead of time: if we take an algebraic approach to solve the consumer's problem, we will find that the IMRS equals the slope of the intertemporal budget constraint:

$$\frac{u'(c_0)}{\beta u'(c_1)} = 1 + r.$$

But let's use calculus to derive the same result.

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The Lagrangian is

$$L = u(c_0) + \beta u(c_1) + \lambda \left(Y_0 + \frac{Y_1}{1+r} - c_0 - \frac{c_1}{1+r} \right).$$

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$$L = u(c_0) + \beta u(c_1) + \lambda \left(Y_0 + \frac{Y_1}{1+r} - c_0 - \frac{c_1}{1+r} \right).$$

The first-order conditions

$$u'(c_0^*) - \lambda^* = 0$$

$$\beta u'(c_1^*) - \lambda^* \left(\frac{1}{1+r} \right) = 0.$$

lead directly to the graphical result

$$\frac{u'(c_0^*)}{\beta u'(c_1^*)} = 1 + r.$$

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At first glance, Fisher's model seems unrealistic, especially in its assumption that the consumer can borrow at the same interest rate r that he or she receives on his or her savings.

A reinterpretation of saving and borrowing in this framework, however, can make it more applicable, at least for some consumers.

Investment Strategies and Cash Flows

Investment Strategy	Cash Flow at $t = 0$	Cash Flow at $t = 1$
Saving	-1	$+(1+r)$
Buying a bond (long position in bonds)	-1	$+(1+r)$

Investment Strategies and Cash Flows

Investment Strategy	Cash Flow at $t = 0$	Cash Flow at $t = 1$
Borrowing	$+1$	$-(1 + r)$
Issuing a bond	$+1$	$-(1 + r)$
Short selling a bond (short position in bonds)	$+1$	$-(1 + r)$
Selling a bond (out of inventory)	$+1$	$-(1 + r)$

Consumer Optimization: The Time Dimension

Someone who already owns bonds can "borrow" by selling a bond out of inventory. In fact, theories like Fisher's work better when applied to consumers who already own stocks and bonds.

Greg Mankiw and Stephen Zeldes, "The Consumption of Stockholders and Nonstockholders," *Journal of Finance*, 1991.

Investment Strategies and Cash Flows

Investment Strategy	Cash Flow at $t = 0$	Cash Flow at $t = 1$
Buying a stock (long position in stocks)	$-P_0^s$	$+P_1^s$
Short selling a stock (short position in stocks)	$+P_0^s$	$-P_1^s$
Selling a stock (out of inventory)	$+P_0^s$	$-P_1^s$