

ECON 337901

FINANCIAL ECONOMICS

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Pricing Risky Cash Flows

Now consider a risky asset, with cash flows $\tilde{C}_1, \tilde{C}_2, \dots, \tilde{C}_T$ over the next T years that are **random variables** with values that are unknown today.

Again, we might simplify the task of pricing this asset, by viewing it as a portfolio of more basic assets, each of which makes a random payment \tilde{C}_t after t years, then summing up the prices of all of these more basic assets.

Pricing Risky Cash Flows

But we still have to deal with the fact that the payoff \tilde{C}_t is risky.

And that is what the modern theory of asset pricing, on which this course is based, is really all about.

Pricing Risky Cash Flows

One possibility is to break down the random payoff \tilde{C}_t into separate components $C_{t,1}, C_{t,2}, \dots, C_{t,n}$ delivered in n different “states of the world” that can prevail t years from now.

The risky asset that delivers the random payoff \tilde{C}_t t years from now can itself be viewed as a portfolio of contingent claims: $C_{t,1}$ contingent claims for state 1, $C_{t,2}$ contingent claims for state 2, \dots , and $C_{t,n}$ contingent claims for state n .

Pricing Risky Cash Flows

This **Arrow-Debreu** approach to asset pricing then computes

$$P_t^A = q_{t,1}C_{t,1} + q_{t,2}C_{t,2} + \dots + q_{t,n}C_{t,n}$$

where $q_{t,i}$ is the price today of a contingent claim that delivers one dollar if state i occurs t years from now and zero otherwise.

This approach uses contingent claims as the “basic building blocks” for risky assets, in the same way that discount bonds can be viewed as the building blocks for coupon bonds.

Pricing Risky Cash Flows

In probability theory, if a **random variable** \tilde{X} can take on n possible values, X_1, X_2, \dots, X_n , with probabilities $\pi_1, \pi_2, \dots, \pi_n$, then the **expected value** of \tilde{X} is

$$E(\tilde{X}) = \pi_1 X_1 + \pi_2 X_2 + \dots + \pi_n X_n.$$

Pricing Risky Cash Flows

More traditional approaches to asset pricing replace the random payoff \tilde{C}_t with its expected value $E(\tilde{C}_t)$ and then “penalize” the fact that the payoff is random by discounting it at a higher rate

$$P_t^A = \frac{E(\tilde{C}_t)}{(1 + r_t + \psi_t)^t}$$

The **capital asset pricing model** (CAPM) will give us a way of determining values for the **risk premium** ψ_t .

Two Perspectives on Asset Pricing

Although all are designed to accomplish the same basic goal – to value risky cash flows – different theories of asset pricing can be grouped under two broad headings.

No-arbitrage theories take the prices of some assets as given and use those to determine the prices of other assets.

Equilibrium theories price all assets based on the principles of microeconomic theory.

Two Perspectives on Asset Pricing

No-arbitrage theories require fewer assumptions and are sometimes easier to use.

We've already used no-arbitrage arguments, for example, to price stocks and bonds as portfolios of contingent claims and to price coupon bonds as portfolios of discount bonds.

Two Perspectives on Asset Pricing

But no-arbitrage theories raise questions that only equilibrium theories can answer.

Where do the prices of the basic securities come from? And how do asset prices relate to economic fundamentals?

As an equilibrium theory of asset pricing, the CAPM will also help us answer these questions.

Two Perspectives on Asset Pricing

Whereas no-arbitrage theories require only that investors prefer more to less . . .

. . . the CAPM requires us to make assumptions about how investors trade off risk versus return.

Preferences and Utility Functions

Consumers have preferences.

Economists describe those preferences with a utility function.

What exactly does this mean? And what exactly do economists assume when they describe or “represent” preferences with a utility function?

Preferences and Utility Functions

In the standard, static setting without uncertainty, let $c^1 = (c_a^1, c_b^1)$ and $c^2 = (c_a^2, c_b^2)$ denote two bundles of apples and bananas.

“More preferred to less” is enough to predict which bundle the consumer will choose if one of the bundles has more of both goods than the other.

Even if there is a trade-off, however, we should expect the consumer to be able to say which bundle is preferred or to express indifference.

Preferences and Utility Functions

Economists say that the consumer's preferences are represented by a utility function U when:

The consumer says "I prefer c^1 to c^2 "

if and only if

$$U(c_a^1, c_b^1) > U(c_a^2, c_b^2)$$

Preferences and Utility Functions

Under what assumptions can preferences be represented by a utility function? Gerard Debreu answered this question in 1954:

Cowles Foundation Paper 97
Reprinted from Thrall, Davis and Coombs, eds.,
Decision Processes, John Wiley, 1954

CHAPTER XI

REPRESENTATION OF A PREFERENCE
ORDERING BY A NUMERICAL FUNCTION*

by

Gerard Debreu

COWLES COMMISSION FOR RESEARCH IN ECONOMICS

Theorem If preferences are complete, transitive, and continuous, then they can be represented by a continuous, real-valued utility function.

Preferences and Expected Utility Functions

Under certainty, the “goods” are described by consumption baskets with known characteristics.

Under uncertainty, the “goods” are random (state-contingent) payoffs.

Accordingly, let $p^1 = (p_G^1, p_B^1)$ and $p^2 = (p_G^2, p_B^2)$ denote the payoffs from two assets in a good state that occurs with probability π and a bad state with probability $1 - \pi$.

Preferences and Expected Utility Functions

In 1947, John von Neumann and Oskar Morgenstern worked out the conditions under which investors' preferences over risky payoffs could be described by an **expected utility function** such as

$$U(p) = E[u(p)] = \pi u(p_G) + (1 - \pi)u(p_B),$$

where the **Bernoulli utility function** (named after Daniel Bernoulli, from the 1700s) over payoffs u is increasing and concave and the **von Neumann-Morgenstern expected utility function** U is linear in the probabilities.

Preferences and Expected Utility Functions

Economists say that an investor's preferences are represented by an expected utility function U when:

The investor says "I prefer p^1 to p^2 "

if and only if

$$\begin{aligned}U(p_G^1, p_B^1) &= \pi u(p_G^1) + (1 - \pi)u(p_B^1) \\ &> \pi u(p_G^2) + (1 - \pi)u(p_B^2) \\ &= U(p_G^2, p_B^2)\end{aligned}$$

Preferences and Expected Utility Functions

Debreu's 1954 theorem says that if preferences over state-contingent payoffs (assets or "lotteries") are complete, transitive, and continuous, then they can be represented by **some** utility function.

von Neumann and Morgenstern identified the extra assumptions needed for that utility function to take the special form of an **expected** utility function.

Maurice Allais (Nobel Prize 1988) showed that these extra assumptions are quite "strong," in the sense that it is not difficult to construct examples where perfectly reasonable and rational investors make choices that violate those assumptions.

Preferences and Expected Utility Functions

Expected utility remains the dominant framework for analyzing economic decision-making under uncertainty.

But a very active line of ongoing research continues to explore alternatives and generalizations.

Preferences and Expected Utility Functions

For more detail on these issues, see:

Danthine and Donaldson, Ch 3 (“Making Choices in Risky Situations”).

Mark Machina, “Choice Under Uncertainty: Problems Solved and Unsolved,” *Journal of Economic Perspectives* (Summer 1987).

Expected Utility Functions

We have already seen that concavity of the Bernoulli utility function u in the expected utility function

$$U(p_G, p_B) = \pi u(p_G) + (1 - \pi)u(p_B)$$

captures the investor's risk aversion.

In fact, this insight was first made by Gabriel Cramer and Daniel Bernoulli in the 1700s.

Expected Utility Functions

Concavity of the Bernoulli utility function u in the expected utility function

$$U(p_G, p_B) = \pi u(p_G) + (1 - \pi)u(p_B)$$

captures the investor's risk aversion.

Since concavity is reflected by the condition

$$u''(p) < 0$$

is it the case that a “more concave” Bernoulli utility function as reflected in a “more negative” value of $u''(p)$ captures a greater degree of risk aversion?