ECON 337901
FINANCIAL ECONOMICS

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March 15, 2022

2 Overview of Asset Pricing Theory

A Pricing Safe Cash Flows

B Pricing Risky Cash Flows

C Two Perspectives on Asset Pricing

Pricing Safe Cash Flows

A T-year discount bond is an asset that pays off \$1, for sure, T years from now.

If this bond sells for \$ P_T today, the annualized return from buying the bond today and holding it to maturity is

$$1 + r_T = \left(\frac{1}{P_T}\right)^{1/T}.$$

Hence, the bond price and the interest rate are related via

$$P_T = \frac{1}{(1 + r_T)^T}.$$

Pricing Safe Cash Flows

Since, for a T-period discount bond,

$$P_T = \frac{1}{(1 + r_T)^T},$$

the interest rate equates today's price of the bond to the present discounted value of the future payments made by the bond.

US Treasury bills, that is, US government bonds with maturities less than one year, are structured as discount bonds.

Pricing Safe Cash Flows

A T-year coupon bond is an asset that makes an annual interest (coupon) payment of \$$C$ each year, every year, for the next T years, and then pays off \$$F$ (face or par value), for sure, T years from now.

US Treasury notes and bonds, with maturities of more than one year, are structured as coupon bonds.

Pricing Safe Cash Flows

Notice that a coupon bond can be viewed as a bundle, or portfolio of discount bonds, since the cash flows from a T-year coupon bond can be replicated by buying

C one-year discount bonds

C two-year discount bonds

...

C T-year discount bonds

F more T-year discount bonds

Pricing Safe Cash Flows

And if both discount and coupon bonds are traded, then the price of the coupon bond must equal the price of the portfolio of discount bonds.

If the coupon bond was cheaper than the portfolio of discount bonds, one could sell the discount bonds, buy the coupon bond, and thereby profit.

If the coupon bond was more expensive than the portfolio of discount bonds, one could sell the coupon bond, buy the discount bonds, and thereby profit.

Pricing Safe Cash Flows

Building on this insight, the price P_T^C of the coupon bond must satisfy

$$
\begin{aligned}
P_T^C &= CP_1 + CP_2 + \ldots + CP_T + FP_T \\
&= \frac{C}{1+r_1} + \frac{C}{(1+r_2)^2} + \ldots + \frac{C}{(1+r_T)^T} + \frac{F}{(1+r_T)^T}
\end{aligned}
$$

Today's price of the coupon bond equals the present discounted value of the future payments made by the bond.

Pricing Safe Cash Flows

$$P_T^C = \frac{C}{1+r_1} + \frac{C}{(1+r_2)^2} + \ldots + \frac{C}{(1+r_T)^T} + \frac{F}{(1+r_T)^T}$$

Note that the interest rates used to compute the present value are those on the discount bonds

The yield to maturity defined by the value r that satisfies

$$P_T^C = \frac{C}{1+r} + \frac{C}{(1+r)^2} + \ldots + \frac{C}{(1+r)^T} + \frac{F}{(1+r)^T}$$

is a measure of the interest rate on the coupon bond.

Pricing Safe Cash Flows

In fact, the US Treasury allows financial institutions to break US Treasury coupon bonds down into portfolios of separately-traded discount bonds.

These securities are called US Treasury STRIPS (Separate Trading of Registered Interest and Principal of Securities).

Pricing Safe Cash Flows

Next, consider an asset that generates an arbitrary stream of safe (riskless) cash flows $C_1, C_2, \ldots, C_T$, over the next T years.

To simplify the task of "pricing" this asset, we might view it as a portfolio of more basic assets: one that pays C_1 for sure in one year, one that pays C_2 for sure in two years, ..., and one that pays C_T for sure in T years.

The price of the multi-period asset must equal the sum of the prices of the more basic assets.

Pricing Safe Cash Flows

We've now reduced the problem of pricing any riskless asset to the simpler problem of pricing a more basic asset that pays C_t for sure t years from now.

But this more basic asset has the same payoff as C_t t-year discount bonds. Its price P_t^A today must equal

$$P_t^A = C_t P_t = \frac{C_t}{(1+r_t)^t},$$

the present discounted value of its cash flow.

Application to Forward Rates

Suppose you want to borrow \$1 $n-1$ years from now.

Repay with interest n years from now.

Lock in the "n-year forward rate" r_n^f today.

Application to Forward Rates

Suppose you want to borrow \$1 $n-1$ years from now.

To receive \$1 $n-1$ years from now: buy an $(n-1)$-year discount bond.

The problem is, this costs P_{n-1} today.

Application to Forward Rates

To receive \$1 $n-1$ years from now: buy an $(n-1)$-year discount bond.

The problem is, this costs P_{n-1} today.

By selling short P_{n-1}/P_n n-year discount bonds today, you'll receive

$$\left(\frac{P_{n-1}}{P_n}\right) P_n = P_{n-1}$$

to pay for the $(n-1)$-year discount bond.

Application to Forward Rates

To receive \$1 $n-1$ years from now: buy an $(n-1)$-year discount bond.

By selling short P_{n-1}/P_n n-year discount bonds today, you'll have to repay $\$(P_{n-1}/P_n)$ n years from now.

$$1 + r_n^f = \frac{\text{amount repaid } n \text{ years from now}}{\$1 \text{ received } (n-1) \text{ years from now}} = \frac{P_{n-1}}{P_n}$$

Application to Forward Rates

$$1 + r_n^f = \frac{\text{amount repaid } n \text{ years from now}}{\$1 \text{ received } (n-1) \text{ years from now}} = \frac{P_{n-1}}{P_n}$$

$$r_n^f = \frac{P_{n-1}}{P_n} - 1$$

Note that forward rates can be computed from discount bond prices that are observed today. No forecasting is necessary.

Pricing Safe Cash Flows

Using discount bond prices, we can compute:

1. Interest rates on discount bonds.
2. Prices of coupon bonds.
3. Prices of more complex risk-free assets.
4. Forward rates.