

ECON 337901

FINANCIAL ECONOMICS

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January 27, 2022

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Constrained Optimization: Example 1

For the problem

$$\max_x \left(-\frac{1}{2} \right) (x - 5)^2 \text{ subject to } 7 \geq x,$$

$F(x) = (-1/2)(x - 5)^2$, $c = 7$, and $G(x) = x$. The Lagrangian is

$$L(x, \lambda) = \left(-\frac{1}{2} \right) (x - 5)^2 + \lambda(7 - x).$$

Constrained Optimization: Example 1

With

$$L(x, \lambda) = \left(-\frac{1}{2}\right) (x - 5)^2 + \lambda(7 - x),$$

the first-order condition

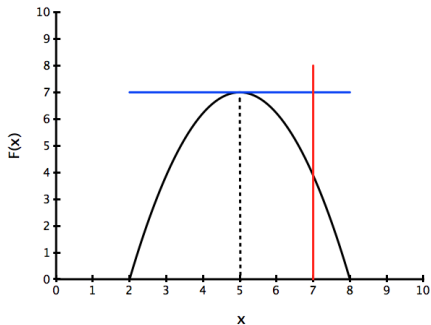
$$-(x^* - 5) - \lambda^* = 0$$

and the complementary slackness condition

$$\lambda^*(7 - x^*) = 0$$

are satisfied with $x^* = 5$, $F'(x^*) = 0$, $\lambda^* = 0$, and $7 > x^*$.

Constrained Optimization: Example 1



Here, the solution has $F'(x^*) = 0$ since the constraint is nonbinding.

Constrained Optimization: Example 2

For the problem

$$\max_x \left(-\frac{1}{2} \right) (x - 5)^2 \text{ subject to } 4 \geq x,$$

$F(x) = (-1/2)(x - 5)^2$, $c = 4$, and $G(x) = x$. The Lagrangian is

$$L(x, \lambda) = \left(-\frac{1}{2} \right) (x - 5)^2 + \lambda(4 - x).$$

Constrained Optimization: Example 2

With

$$L(x, \lambda) = \left(-\frac{1}{2}\right) (x - 5)^2 + \lambda(4 - x),$$

the first-order condition

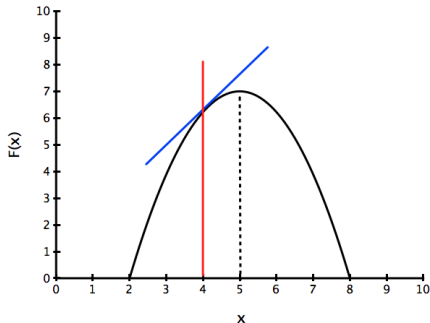
$$-(x^* - 5) - \lambda^* = 0$$

and the complementary slackness condition

$$\lambda^*(4 - x^*) = 0$$

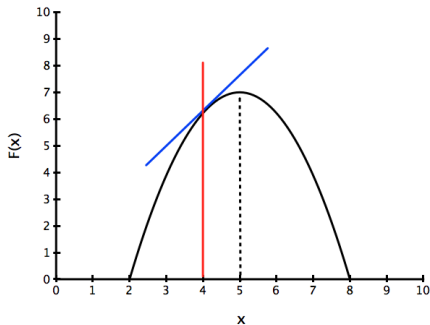
are satisfied with $x^* = 4$ and $F'(x^*) = \lambda^* = 1 > 0$.

Constrained Optimization: Example 2



Here, the solution has $F'(x^*) = \lambda^* G'(x^*) > 0$ since the constraint is binding. $F'(x^*) > 0$ indicates that we'd like to increase the value of x , but the constraint won't let us.

Constrained Optimization: Example 2



With a binding constraint, $F'(x^*) \neq 0$ but $F'(x^*) - \lambda^* G'(x^*) = 0$. The value x^* that solves the problem is a critical point, not of the objective function $F(x)$, but instead of the entire Lagrangian $F(x) + \lambda[c - G(x)]$.

Consumer Optimization

1. Graphical Analysis
2. Algebraic Analysis
3. Time Dimension
4. Risk Dimension

Consumer Optimization

Alfred Marshall, *Principles of Economics*, 1890. – supply and demand

Francis Edgeworth, *Mathematical Psychics*, 1881.

Vilfredo Pareto, *Manual of Political Economy*, 1906. – indifference curves

Consumer Optimization

John Hicks, *Value and Capital*, 1939. – wealth and substitution effects

Paul Samuelson, *Foundations of Economic Analysis*, 1947. – mathematical reformulation

Irving Fisher, *The Theory of Interest*, 1930. – intertemporal extension.

Consumer Optimization

Gerard Debreu, *Theory of Value*, 1959.

Kenneth Arrow, “The Role of Securities in the Optimal Allocation of Risk Bearing,” *Review of Economic Studies*, 1964.

Extensions to include risk and uncertainty.