# ECON 337901
# FINANCIAL ECONOMICS

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# Preferences and Utility Functions

Consumers have preferences.

Economists describe those preferences with a utility function.

What exactly does this mean? And what exactly do economists assume when they describe or "represent" preferences with a utility function?

# Preferences and Utility Functions

In the standard, static setting without uncertainty, let $c^1 = (c_a^1, c_b^1)$ and $c^2 = (c_a^2, c_b^2)$ denote two bundles of apples and bananas.

"More preferred to less" is enough to predict which bundle the consumer will choose if one of the bundles has more of both goods than the other.

Even if there is a trade-off, however, we should expect the consumer to be able to say which bundle is preferred or to express indifference.

# Preferences and Utility Functions

Economists say that the consumer's preferences are represented by a utility function $U$ when:

The consumer says "I prefer $c^1$ to $c^2$"

if and only if

$$U(c_a^1, c_b^1) > U(c_a^2, c_b^2)$$

# Preferences and Utility Functions

Under what assumptions can preferences be represented by a utility function? Gerard Debreu answered this question in 1954:

**Cowles Foundation Paper 97**
**Reprinted from Thrall, Davis and Coombs , eds.,**
**Decision Processes, John Wiley, 1954**

CHAPTER XI

**REPRESENTATION OF A PREFERENCE ORDERING BY A NUMERICAL FUNCTION***

by

Gerard Debreu

COWLES COMMISSION FOR RESEARCH IN ECONOMICS

Theorem If preferences are complete, transitive, and continuous, then they can be represented by a continuous, real-valued utility function.

# Preferences and Expected Utility Functions

Under certainty, the "goods" are described by consumption baskets with known characteristics.

Under uncertainty, the "goods" are random (state-contingent payoffs.

Accordingly, let $p^1 = (p^1_G, p^1_B)$ and $p^2 = (p^2_G, p^2_B)$ denote the payoffs from two assets in a good state that occurs with probability $\pi$ and a bad state with probability $1 - \pi$.

# Preferences and Expected Utility Functions

In 1947, John von Neumann and Oskar Morgenstern worked out the conditions under which investors' preferences over risky payoffs could be described by an expected utility function such as

$$U(p) = E[u(p)] = \pi u(p_G) + (1-\pi)u(p_B),$$

where the Bernoulli utility function (named after Daniel Bernoulli, from the 1700s) over payoffs $u$ is increasing and concave and the von Neumann-Morgenstern expected utility function $U$ is linear in the probabilities.

# Preferences and Expected Utility Functions

Economists say that an investor's preferences are represented by an expected utility function $U$ when:

The investor says "I prefer $p^1$ to $p^2$"

if and only if

$$\begin{aligned} U(p_G^1, p_B^1) &= \pi u(p_G^1) + (1-\pi)u(p_B^1) \\ &> \pi u(p_G^2) + (1-\pi)u(p_B^2) \\ &= U(p_G^2, p_B^2) \end{aligned}$$

# Preferences and Expected Utility Functions

Debreu's 1954 theorem says that if preferences over state-contingent payoffs (assets or "lotteries") are complete, transitive, and continuous, then they can be represented by some utility function.

von Neumann and Morgenstern identified the extra assumptions needed for that utility function to take the special form of an expected utility function.

Maurice Allais (Nobel Prize 1988) showed that these extra assumptions are quite "strong," in the sense that it is not difficult to find examples where perfectly reasonable and rational investors make choices that violate those assumptions.

# Preferences and Expected Utility Functions

Expected utility remains the dominant framework for analyzing economic decision-making under uncertainty.

But a very active line of ongoing research continues to explore alternatives and generalizations.

# Preferences and Expected Utility Functions

For more detail on these issues, see:

Danthine and Donaldson, Ch 3 ("Making Choices in Risky Situations").

Mark Machina, "Choice Under Uncertainty: Problems Solved and Unsolved," *Journal of Economic Perspectives* (Summer 1987).

# Expected Utility Functions

We have already seen that concavity of the Bernoulli utility function $u$ in the expected utility function

$$U(p_G, p_B) = \pi u(p_G) + (1 - \pi)u(p_B)$$

captures the investor's risk aversion.

In fact, this insight was first made by Gabriel Cramer and Daniel Bernoulli in the 1700s.

# Expected Utility Functions

Concavity of the Bernoulli utility function $u$ in the expected utility function

$$U(p_G, p_B) = \pi u(p_G) + (1 - \pi)u(p_B)$$

captures the investor's risk aversion.

Since concavity is reflected by the condition

$$u''(p) < 0$$

is it the case that a "more concave" Bernoulli utility function as reflected in a "more negative" value of $u''(p)$ captures a greater degree of risk aversion?

# Expected Utility Functions

Unfortunately, no. To see why, suppose that an investor's preferences are represented by the expected utility function $U$, so that

The investor says "I prefer $p^1$ to $p^2$"

if and only if

$$\begin{aligned} U(p_G^1, p_B^1) &= \pi u(p_G^1) + (1-\pi)u(p_B^1) \\ &> \pi u(p_G^2) + (1-\pi)u(p_B^2) \\ &= U(p_G^2, p_B^2) \end{aligned}$$

# Expected Utility Functions

Then it will also be the case that

The investor says "I prefer $p^1$ to $p^2$"

if and only if

$$V(p_G^1, p_B^1) > V(p_G^2, p_B^2)$$

where

$$V(p_G, p_B) = \alpha U(p_G, p_B)$$

for any value of $\alpha > 0$.

# Expected Utility Functions

Since

$$\begin{aligned} V(p_G, p_B) &= \alpha U(p_G, p_B) \\ &= \alpha[\pi u(p_G) + (1-\pi)u(p_B)] \\ &= \pi\alpha u(p_G) + (1-\pi)\alpha u(p_B) \\ &= \pi v(p_G) + (1-\pi)v(p_B) \end{aligned}$$

where $v(p) = \alpha u(p)$ and therefore $v''(p) = \alpha u''(p)$.

By choosing $\alpha$ to be large or small, we can make $v''(p)$ "more" or "less" negative, yet the underlying degree of risk aversion – that is, the investor's preferences – are the same.

# Expected Utility Functions

Because, in economics, utility is an ordinal, not a cardinal, concept, we will have to work a little harder to quantify risk aversion in the expected utility framework.