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FINANCIAL ECONOMICS

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Pricing Safe Cash Flows

Consider an asset that generates an arbitrary stream of safe (riskless) cash flows $C_1, C_2, \ldots, C_T$, over the next T years.

Each of the payoffs can be large or small, positive or negative.

The only requirement is that they must be known for sure.

Pricing Safe Cash Flows

Consider an asset that generates an arbitrary stream of safe (riskless) cash flows $C_1, C_2, \ldots, C_T$, over the next T years.

To simplify the task of "pricing" this asset, we might view it as a portfolio of more basic assets: one that pays C_1 for sure in one year, one that pays C_2 for sure in two years, ..., and one that pays C_T for sure in T years.

The price of the multi-period asset must equal the sum of the prices of the more basic assets.

Pricing Safe Cash Flows

We've now reduced the problem of pricing any riskless asset to the simpler problem of pricing a more basic asset that pays C_t for sure t years from now.

But this more basic asset has the same payoff as C_t t-year discount bonds.

If P_t is the price of each t-year discount bond, then no arbitrage requires

$$P_t^A = C_t P_t$$

Pricing Safe Cash Flows

For a t-year discount bond, the price P_t and interest rate r_t are related via

$$P_t = \frac{1}{(1 + r_t)^t}$$

Therefore, price of a risk-free asset that pays off C_t t years from now must also satisfy:

$$P_t^A = C_t P_t = \frac{C_t}{(1 + r_t)^t},$$

the present discounted value of its cash flow.

Pricing Safe Cash Flows

Now go back to the original asset, which generates an arbitrary stream of safe (riskless) cash flows $C_1, C_2, \ldots, C_T$, over the next T years.

Its price must also equal the present discounted value of the future cash flows

$$\begin{aligned} P^A &= P_1 C_1 + P_2 C_2 + \ldots + P_T C_T \\ &= \frac{C_1}{1+r_1} + \frac{C_2}{(1+r_2)^2} + \ldots + \frac{C_T}{(1+r_T)^T} \end{aligned}$$

where the interest rates used to compute the present value are the interest rates on discount bonds with different terms to maturity.

Pricing Safe Cash Flows

For example: Suppose the prices of discount bonds are $P_1 = 0.90$ and $P_2 = 0.80$.

Consider a risk free asset that pays \$100 one year from now but requires a payment of \$100 two years from now.

These cash flows can be replicated by buying 100 one-year discount bonds and selling short 100 two-year discount bonds. No arbitrage implies

$$P^A = 100P_1 - 100P_2 = 90 - 80 = 10.$$

Pricing Risky Cash Flows

Now consider a risky asset, with cash flows $\tilde{C}_1, \tilde{C}_2, \ldots, \tilde{C}_T$ over the next T years that are random variables with values that are unknown today.

Again, we might simplify the task of pricing this asset, by viewing it as a portfolio of more basic assets, each of which makes a random payment $\tilde{C}_t$ after t years, then summing up the prices of all of these more basic assets.

Pricing Risky Cash Flows

But we still have to deal with the fact that the payoff $\tilde{C}_t$ is risky.

And that is what the modern theory of asset pricing, on which this course is based, is really all about.

Pricing Risky Cash Flows

One possibility is to break down the random payoff $\tilde{C}_t$ into separate components $C_{t,1}, C_{t,2}, \ldots, C_{t,n}$ delivered in n different "states of the world" that can prevail t years from now.

The risky asset that delivers the random payoff $\tilde{C}_t$ t years from now can itself be viewed as a portfolio of contingent claims: $C_{t,1}$ contingent claims for state 1, $C_{t,2}$ contingent claims for state 2, ..., and $C_{t,n}$ contingent claims for state n.

Pricing Risky Cash Flows

This Arrow-Debreu approach to asset pricing then computes

$$P_t^A = q_{t,1}C_{t,1} + q_{t,2}C_{t,2} + \ldots + q_{t,n}C_{t,n}$$

where $q_{t,i}$ is the price today of a contingent claim that delivers one dollar if state i occurs t years from now and zero otherwise.

This approach uses contingent claims as the "basic building blocks" for risky assets, in the same way that discount bonds can be viewed as the building blocks for coupon bonds.

Pricing Risky Cash Flows

In probability theory, if a random variable $\tilde{X}$ can take on n possible values, $X_1, X_2, \ldots, X_n$, with probabilities $\pi_1, \pi_2, \ldots, \pi_n$, then the expected value of $\tilde{X}$ is

$$E(\tilde{X}) = \pi_1 X_1 + \pi_2 X_2 + \ldots + \pi_n X_n.$$

Pricing Risky Cash Flows

More traditional approaches to asset pricing replace the random payoff $\tilde{C}_t$ with its expected value $E(\tilde{C}_t)$ and then "penalize" the fact that the payoff is random by discounting it at a higher rate

$$P_t^A = \frac{E(\tilde{C}_t)}{(1 + r_t + \psi_t)^t}$$

The capital asset pricing model (CAPM) will give us a way of determining values for the risk premium ψ_t.

Two Perspectives on Asset Pricing

Although all are designed to accomplish the same basic goal – to value risky cash flows – different theories of asset pricing can be grouped under two broad headings.

No-arbitrage theories take the prices of some assets as given and use those to determine the prices of other assets.

Equilibrium theories price all assets based on the principles of microeconomic theory.

Two Perspectives on Asset Pricing

No-arbitrage theories rest on the "weak" assumption that more is preferred to less. They don't require us to say anything about investors' aversion to risk.

But they deliver implications only for relative valuations, and therefore raise questions that only equilibrium theories can answer.

What determines the prices of all assets considered as a group? And how do those asset prices relate to economic fundamentals?

Two Perspectives on Asset Pricing

What determines the prices of all assets considered as a group? And how do those asset prices relate to economic fundamentals?

As an equilibrium theory of asset pricing, the CAPM will also help us answer these questions.

But to do so, the CAPM requires us to make assumptions about how investors trade off risk versus return.

Preferences and Utility Functions

Consumers have preferences.

Economists describe those preferences with a utility function.

What exactly does this mean? And what exactly do economists assume when they describe or “represent” preferences with a utility function?

Preferences and Utility Functions

In the standard, static setting without uncertainty, let $c^1 = (c_a^1, c_b^1)$ and $c^2 = (c_a^2, c_b^2)$ denote two bundles of apples and bananas.

"More preferred to less" is enough to predict which bundle the consumer will choose if one of the bundles has more of both goods than the other.

Even if there is a trade-off, however, we should expect the consumer to be able to say which bundle is preferred or to express indifference.

Preferences and Utility Functions

Economists say that the consumer's preferences are represented by a utility function U when:

The consumer says "I prefer c^1 to c^2"

if and only if

$$U(c_a^1, c_b^1) > U(c_a^2, c_b^2)$$