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FINANCIAL ECONOMICS

Peter Ireland

Boston College

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Interpreting the Measures of Risk Aversion

How accurate is the approximation?

$$\pi^* \approx \frac{1}{2} + \frac{1}{4}kR_R(Y)$$

To see, consider an example where an investor has initial income

$$Y_0 = 10$$

and chooses whether to accept or reject the lottery

$$(x, y, \pi) = (0.1, -0.1, \pi)$$

A bet over $k = 0.1/10 = 0.01$, or 1 percent of income.

Interpreting the Measures of Risk Aversion

Suppose that

$$u(Y) = \frac{Y^{1-\gamma} - 1}{1 - \gamma}$$

so that

$$R_R(Y) = \gamma.$$

With $k = 0.01$, the approximation becomes

$$\pi^* \approx \frac{1}{2} + \frac{1}{4} k R_R(Y)$$

$$\pi^* \approx \frac{1}{2} + \frac{0.01\gamma}{4}.$$

Try computing the approximations to π^* for $\gamma = 1/2, 2, 3, 10, 20$.

Interpreting the Measures of Risk Aversion

The exact value of π^* satisfies

$$
\begin{aligned}
u(Y_0) &= \pi^* u(Y_0 + Y_0 k) + (1-\pi^*) u(Y_0 - Y_0 k) \\
u(10) &= \pi^* u(10.1) + (1-\pi^*) u(9.9) \\
\frac{10^{1-\gamma}-1}{1-\gamma} &= \pi^* \left(\frac{10.1^{1-\gamma}-1}{1-\gamma}\right) + (1-\pi^*) \left(\frac{9.9^{1-\gamma}-1}{1-\gamma}\right) \\
10^{1-\gamma} &= \pi^*(10.1^{1-\gamma}) + 9.9^{1-\gamma} - \pi^*(9.9^{1-\gamma})
\end{aligned}
$$

Interpreting the Measures of Risk Aversion

The exact value of π^* satisfies

$$10^{1-\gamma} = \pi^*(10.1^{1-\gamma}) + 9.9^{1-\gamma} - \pi^*(9.9^{1-\gamma})$$

$$\pi^* = \frac{10^{1-\gamma} - 9.9^{1-\gamma}}{10.1^{1-\gamma} - 9.9^{1-\gamma}}$$

Compute the exact values of π^* for $\gamma = 1/2, 2, 3, 10, 20$ and compare them to the approximations. They will be quite close.

Insurance

We can also use the expected utility framework to consider a consumer's decisions about whether or not to buy insurance against a loss.

As an example, consider a consumer with income $Y = 100000$, who faces a $\pi_1 = 0.05$ probability of suffering a 50000 loss.

Suppose this consumer's preferences are described by a von Neumann-Morgenstern expected utility function with logarithmic Bernoulli utility function $u(Y) = \ln(Y)$. We know this implies a coefficient of relative risk aversion equal to one.

Insurance

What is the most this consumer will be willing to pay for insurance?

Let x^* denote the insurance premium that makes the consumer indifferent between buying and not buying insurance.

If the actual insurance premium is less than x^*, the consumer will buy the insurance; if the premium is greater than x^*, the consumer will choose to remain uninsured.

Insurance

x^* satisfies

$$u(Y - x^*) = \pi_1 u(Y - 50000) + (1 - \pi_1) u(Y)$$

$$\ln(100000 - x^*) = 0.05 \ln(50000) + 0.95 \ln(100000) \approx 11.48$$

$$\exp[\ln(100000 - x^*)] = \exp[0.05 \ln(50000) + 0.95 \ln(100000)]$$

$$100000 - x^* = \exp[0.05 \ln(50000) + 0.95 \ln(100000)]$$

$$x^* = 100000 - \exp[0.05 \ln(50000) + 0.95 \ln(100000)]$$

Just remember: $e^{y+z} \neq e^y + e^z$

Insurance

Calculate x^* again when there is

$\pi_1 = 0.05$ probability of a 50000 loss

$\pi_2 = 0.01$ probability of a 99999 loss

$1 - \pi_1 - \pi_2 = 0.94$ probability of no loss

The consumer will be willing to pay (a lot) more for insurance.

Certainty Equivalent

Suppose your income is $Y = 50000$ and you are offered a risky asset that pays 50000 with probability $1/2$ and 0 with probability $1/2$.

This asset has

$$E(\tilde{Z}) = (1/2)50000 + (1/2)0 = 25000.$$

Certainty Equivalent

Which would you choose: taking the risk with $\tilde{Z}$, or receiving $E(\tilde{Z})$ for sure?

No risk averse investor would give up $E(\tilde{Z})$ for sure to take the risk.

But suppose the risk-free alternative was less attractive? How much less would you accept to avoid risk?

Certainty Equivalent

The "certainty equivalent" $CE(\tilde{Z})$ is the smallest amount than an investor would accept, for sure, to avoid the risk associated with $\tilde{Z}$.

In this case, where $\tilde{Z}$ is a coin flip over 50000 versus nothing, $CE(\tilde{Z})$ satisfies

$$u[Y + CE(\tilde{Z})] = (1/2)u(Y + 50000) + (1/2)u(Y)$$

$$u[50000 + CE(\tilde{Z})] = (1/2)u(100000) + (1/2)u(50000)$$

Certainty Equivalent

$$u[50000 + CE(\tilde{Z})] = (1/2)u(100000) + (1/2)u(50000)$$

Suppose that

$$u(Y) = \frac{Y^{1-\gamma} - 1}{1 - \gamma}$$

so that relative risk aversion equals γ.

Certainty Equivalent

$$u[50000 + CE(\tilde{Z})] = (1/2)u(100000) + (1/2)u(50000)$$

$$\begin{aligned} & \frac{[50000 + CE(\tilde{Z})]^{1-\gamma} - 1}{1-\gamma} \\ = \quad & \frac{1}{2}\left(\frac{100000^{1-\gamma} - 1}{1-\gamma}\right) + \frac{1}{2}\left(\frac{50000^{1-\gamma} - 1}{1-\gamma}\right) \end{aligned}$$

$$[50000 + CE(\tilde{Z})]^{1-\gamma} = (1/2)(100000^{1-\gamma}) + (1/2)(50000^{1-\gamma})$$

$$CE(\tilde{Z}) = [(1/2)(100000^{1-\gamma}) + (1/2)(50000^{1-\gamma})]^{\frac{1}{1-\gamma}} - 50000$$

Assessing the Level of Risk Aversion

Certainty equivalent for an asset that pays 50000 with probability 1/2 and 0 with probability 1/2 when income is 50000 and the coefficient of relative risk aversion is γ.

γ	$CE(\tilde{Z})$	
0	25000	("risk neutrality," Pascal)
1	20711	(log utility, D Bernoulli)
2	16667	
3	13246	
4	10571	
5	8566	
10	3991	
20	1858	
50	712	