Consumer Optimization: The Risk Dimension

To implement state-contingent consumption plans, Arrow and Debreu imagined that the consumer would trade contingent claims for both future states.

A contingent claim for the good state costs $q^G$ today, and delivers one unit of consumption next year in the good state and zero units of consumption next year in the bad state.

A contingent claim for the bad state costs $q^B$ today, and delivers one unit of consumption next year in the bad state and zero units of consumption next year in the good state.
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Payoffs for the contingent claim for the good state (a long position).
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Payoffs for the contingent claim for the bad state (a long position).
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Payoffs for a short position in the contingent claim for the good state.
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Payoffs for a short position in the contingent claim for the bad state.
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<table>
<thead>
<tr>
<th>Trading Strategy</th>
<th>Claim</th>
<th>Cash Flow at $t = 0$</th>
<th>Cash Flow in Good State at $t = 1$</th>
<th>Cash Flow in Bad State at $t = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long</td>
<td>Good</td>
<td>$-q^G$</td>
<td>+1</td>
<td>0</td>
</tr>
<tr>
<td>Long</td>
<td>Bad</td>
<td>$-q^B$</td>
<td>0</td>
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<td>Short</td>
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</tbody>
</table>

Like a sophisticated form of saving and borrowing, where the investor can "fine-tune" the future state in which payments are received or made.
Today, the consumer divides his or her income up into an amount to be consumed and amounts used to purchase the two contingent claims:

\[ Y_0 \geq c_0 + q^G s^G + q^B s^B, \]

where \( s^G \) and \( s^B \) denote the number of each contingent claim purchased or sold short.

If either \( s^G \) or \( s^B \) is negative, the consumer is taking a short position in that claim.
Next year, the consumer simply spends his or her income, including payoffs on contingent claims:

\[ Y_1^G + s^G \geq c_1^G \]

in the good state and

\[ Y_1^B + s^B \geq c_1^B \]

in the bad state.
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\[ Y_0 \geq c_0 + q^G s^G + q^B s^B \]
\[ Y_1^G + s^G \geq c_1^G \]
\[ Y_1^B + s^B \geq c_1^B \]

Multiply both sides of the second equation by \( q^G \) and both sides of the third equation by \( q^B \), Then add them all up to get the lifetime budget constraint

\[ Y_0 + q^G Y_1^G + q^B Y_1^B \geq c_0 + q^G c_1^G + q^B c_1^B. \]
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The problem is to choose $c_0$, $c_G^1$, and $c_B^1$ to maximize expected utility

$$u(c_0) + \beta \pi u(c_G^1) + \beta (1 - \pi) u(c_B^1),$$

subject to the budget constraint

$$Y_0 + q^G Y_1^G + q^B Y_1^B \geq c_0 + q^G c_G^1 + q^B c_B^1.$$

This was Arrow and Debreu’s key insight: that finance is like grocery shopping. Mathematically, making decisions over time and under uncertainty is no different from choosing apples, bananas, and pears!