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FINANCIAL ECONOMICS

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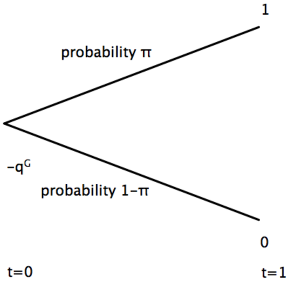
Consumer Optimization: The Risk Dimension

To implement state-contingent consumption plans, Arrow and Debreu imagined that the consumer would trade contingent claims for both future states.

A contingent claim for the good state costs q^G today, and delivers one unit of consumption next year in the good state and zero units of consumption next year in the bad state.

A contingent claim for the bad state costs q^B today, and delivers one unit of consumption next year in the bad state and zero units of consumption next year in the good state.

Consumer Optimization: The Risk Dimension



Payoffs for the contingent claim for the good state (a long position).

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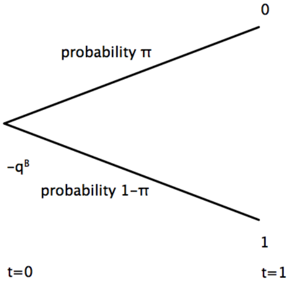

Payoffs for the contingent claim for the bad state (a long position).

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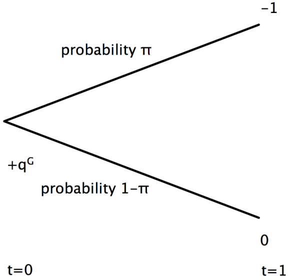

Payoffs for a short position in the contingent claim for the good state.

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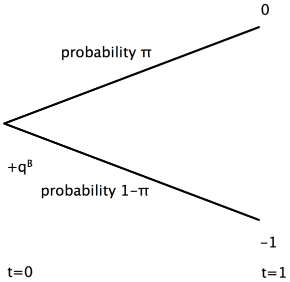

Payoffs for a short position in the contingent claim for the bad state.

Consumer Optimization: The Risk Dimension

Trading Strategy	Claim	Cash Flow at $t = 0$	Cash Flow in Good State at $t = 1$	Cash Flow in Bad State at $t = 1$
Long	Good	$-q^G$	$+1$	0
Long	Bad	$-q^B$	0	$+1$
Short	Good	$+q^G$	-1	0
Short	Bad	$+q^B$	0	-1

Like a sophisticated form of saving and borrowing, where the investor can "fine-tune" the future state in which payments are received or made.

Consumer Optimization: The Risk Dimension

Today, the consumer divides his or her income up into an amount to be consumed and amounts used to purchase the two contingent claims:

$$Y_0 \geq c_0 + q^G s^G + q^B s^B,$$

where s^G and s^B denote the number of each contingent claim purchased or sold short.

If either s^G or s^B is negative, the consumer is taking a short position in that claim.

Consumer Optimization: The Risk Dimension

Next year, the consumer simply spends his or her income, including payoffs on contingent claims:

$$Y_1^G + s^G \geq c_1^G$$

in the good state and

$$Y_1^B + s^B \geq c_1^B$$

in the bad state.

Consumer Optimization: The Risk Dimension

$$Y_0 \geq c_0 + q^G s^G + q^B s^B$$
$$Y_1^G + s^G \geq c_1^G$$
$$Y_1^B + s^B \geq c_1^B$$

Multiply both sides of the second equation by q^G and both sides of the third equation by q^B, Then add them all up to get the lifetime budget constraint

$$Y_0 + q^G Y_1^G + q^B Y_1^B \geq c_0 + q^G c_1^G + q^B c_1^B.$$

Consumer Optimization: The Risk Dimension

The problem is to choose c_0, c_1^G, and c_1^B to maximize expected utility

$$u(c_0) + \beta\pi u(c_1^G) + \beta(1-\pi)u(c_1^B),$$

subject to the budget constraint

$$Y_0 + q^G Y_1^G + q^B Y_1^B \geq c_0 + q^G c_1^G + q^B c_1^B.$$

This was Arrow and Debreu's key insight: that finance is like grocery shopping. Mathematically, making decisions over time and under uncertainty is no different from choosing apples, bananas, and pears!