ECON 337901
FINANCIAL ECONOMICS

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Consumer Optimization

1. Graphical Analysis
2. Algebraic Analysis
3. Time Dimension
4. Risk Dimension

Consumer Optimization

Alfred Marshall, *Principles of Economics*, 1890. – supply and demand

Francis Edgeworth, *Mathematical Psychics*, 1881.

Vilfredo Pareto, *Manual of Political Economy*, 1906. – indifference curves

Consumer Optimization

John Hicks, *Value and Capital*, 1939. – wealth and substitution effects

Paul Samuelson, *Foundations of Economic Analysis*, 1947. – mathematical reformulation

Irving Fisher, *The Theory of Interest*, 1930. – intertemporal extension.

Consumer Optimization

Gerard Debreu, *Theory of Value*, 1959.

Kenneth Arrow, "The Role of Securities in the Optimal Allocation of Risk Bearing," *Review of Economic Studies*, 1964.

Extensions to include risk and uncertainty.

Consumer Optimization: Graphical Analysis

Consider a consumer who likes two goods: apples and bananas.

$$
\begin{aligned}
Y &= \text{income} \\
c_a &= \text{consumption of apples} \\
c_b &= \text{consumption of bananas} \\
p_a &= \text{price of an apple} \\
p_b &= \text{price of a banana}
\end{aligned}
$$

The consumer's budget constraint is

$$Y \geq p_a c_a + p_b c_b$$

Consumer Optimization: Graphical Analysis

So long as the consumer always prefers more to less, the budget constraint will always bind:

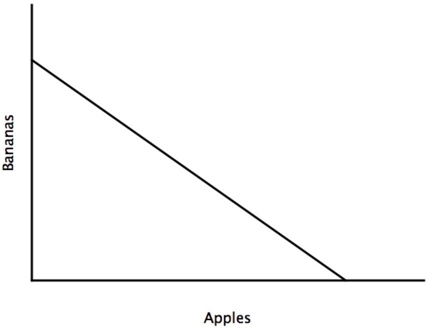
$$Y = p_a c_a + p_b c_b$$

or

$$c_b = \frac{Y}{p_b} - \left(\frac{p_a}{p_b}\right) c_a$$

Which shows that the graph of the budget constraint will be a straight line with slope $-(p_a/p_b)$ and intercept Y/p_b.

Consumer Optimization: Graphical Analysis



The budget constraint is a straight line with slope $-(p_a/p_b)$ and intercept Y/p_b.

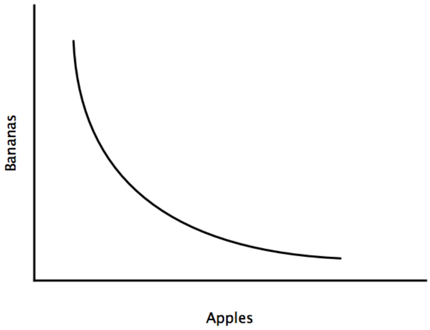
Consumer Optimization: Graphical Analysis

The budget constraint describes the consumer's market opportunities.

Francis Edgeworth (Ireland, 1845-1926) and Vilfredo Pareto (Italy, 1848-1923) were the first to use indifference curves to describe the consumer's preferences.

Each indifference curve traces out a set of combinations of apples and bananas that give the consumer a given level of utility or satisfaction.

Consumer Optimization: Graphical Analysis



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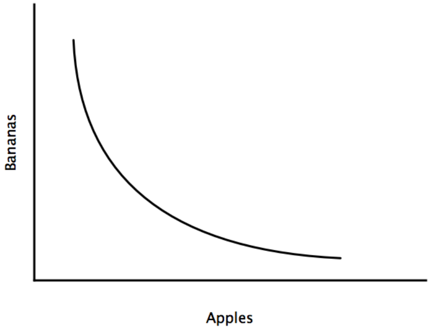
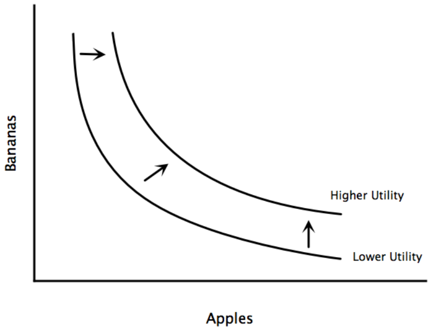
Consumer Optimization: Graphical Analysis



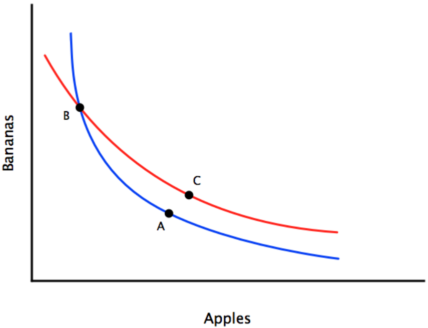
Each indifference curve slopes down, since the consumer requires more apples to compensate for a loss of bananas and more bananas to compensate for a loss of apples, if more is preferred to less.

Consumer Optimization: Graphical Analysis



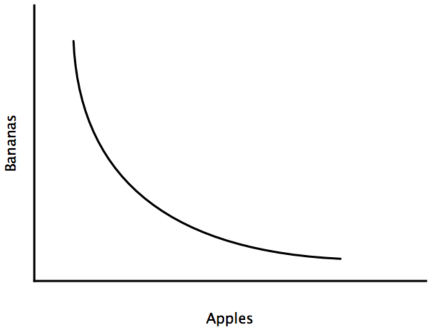
Indifference curves farther away from the origin represent higher levels of utility, if more is preferred to less.

Consumer Optimization: Graphical Analysis



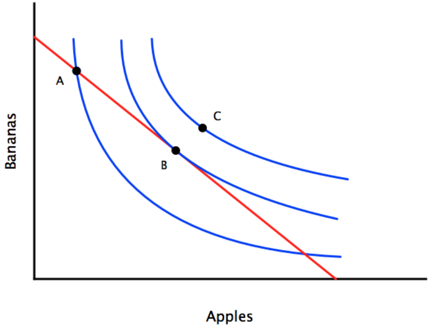
A and B yield the same level of utility, and B and C yield the same level of utility, but C is preferred to A if more is preferred to less. Indifference curves cannot intersect.

Consumer Optimization: Graphical Analysis



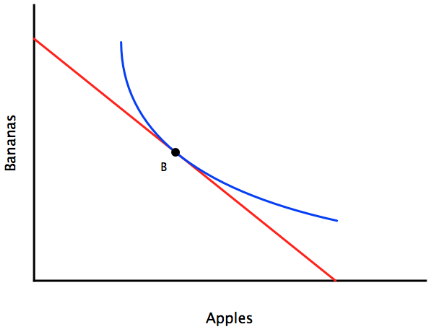
Indifference curves are convex to the origin if consumers have a preference for diversity.

Consumer Optimization: Graphical Analysis



A is suboptimal and C is infeasible. B is optimal.

Consumer Optimization: Graphical Analysis



At B, the optimal choice, the indifference curve is tangent to the budget constraint.

Consumer Optimization: Graphical Analysis

Recall that the budget constraint

$$Y = p_a c_a + p_b c_b$$

or

$$c_b = \frac{Y}{p_b} - \left(\frac{p_a}{p_b}\right) c_a$$

has slope $-(p_a/p_b)$.

Consumer Optimization: Graphical Analysis

Suppose that the consumer's preferences are also described by the utility function

$$u(c_a) + \beta u(c_b).$$

The function u is increasing, with $u'(c) > 0$, so that more is preferred to less, and concave, with $u''(c) < 0$, so that marginal utility falls as consumption rises.

The parameter β measures how much more (if $\beta > 1$) or less (if $\beta < 1$) the consumer likes bananas compared to apples.

Consumer Optimization: Graphical Analysis

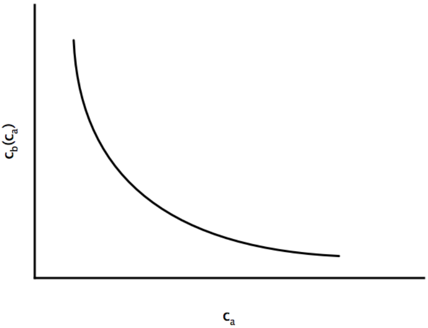
Since an indifference curve traces out the set of (c_a, c_b) combinations that yield a given level of utility $\bar{U}$, the equation for an indifference curve is

$$\bar{U} = u(c_a) + \beta u(c_b).$$

Use this equation to define a new function, $c_b(c_a)$, describing the number of bananas needed, for each number of apples, to keep the consumer on this indifference curve:

$$\bar{U} = u(c_a) + \beta u[c_b(c_a)].$$

Consumer Optimization: Graphical Analysis



The function $c_b(c_a)$ satisfies $\bar{U} = u(c_a) + \beta u[c_b(c_a)]$.

Consumer Optimization: Graphical Analysis

Differentiate both sides of

$$\bar{U} = u(c_a) + \beta u[c_b(c_a)]$$

to obtain

$$0 = u'(c_a) + \beta u'[c_b(c_a)]c_b'(c_a)$$

or

$$c_b'(c_a) = -\frac{u'(c_a)}{\beta u'[c_b(c_a)]}.$$

Consumer Optimization: Graphical Analysis

This last equation,

$$c_b'(c_a) = -\frac{u'(c_a)}{\beta u'[c_b(c_a)]},$$

written more simply as

$$c_b'(c_a) = -\frac{u'(c_a)}{\beta u'(c_b)},$$

measures the slope of the indifference curve: the consumer's marginal rate of substitution.

Consumer Optimization: Graphical Analysis

Thus, the tangency of the budget constraint and indifference curve can be expressed mathematically as

$$\frac{p_a}{p_b} = \frac{u'(c_a)}{\beta u'(c_b)}.$$

The marginal rate of substitution equals the relative prices.