

# ECON 337901

# FINANCIAL ECONOMICS

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# Midterm Exam

Tuesday, March 19: 10:30 - 11:45am

Short answer questions like those from the homeworks, covering the material discussed in class, up through and including bond pricing (Overview of Asset Pricing Theory).

Bring a pen or pencil. You probably won't need a calculator, but you can use one if you want to.

# Midterm Exam

For Extra Practice:

Midterm from Fall 2013: Question 1

Midterm from Spring 2014: Questions 1, 2

Final (two versions) from Spring 2014: Question 4

Midterm from Spring 2015: Questions 1, 3

Midterm from Fall 2015: Questions 1, 2, 3

Midterm from Spring 2016: Questions 1, 2, 3

Midterm from Spring 2017: Questions 1, 2, 3, 4

Midterm from Fall 2017: Questions 1, 2, 3, 4, 5

Midterm from Spring 2018: Questions 1, 2, 3, 4, 5

## Criteria for Choice Over Risky Prospects

	Percentage Return in			
	Good State	Bad State	$E(R)$	$\sigma(R)$
Asset 1	20	5	12.5	7.5
Asset 2	60	-50	5	55
Asset 3	60	5	32.5	27.5

Asset 3 exhibits **state-by-state dominance** over assets 1 and 2. Any investor who prefers more to less would always choose asset 3 above the others.

## Criteria for Choice Over Risky Prospects

In general, one asset displays state-by-state dominance over another if:

1. It pays off at least as much in all states

AND

2. It pays off more in at least one state,  
so investors who prefer more to less will never regret buying it.

## Criteria for Choice Over Risky Prospects

	Percentage Return in			
	Good State	Bad State	$E(R)$	$\sigma(R)$
Asset 1	20	5	12.5	7.5
Asset 2	60	-50	5	55
Asset 3	60	5	32.5	27.5

Although asset 3 is unambiguously the best, the choice between assets 1 and 2 seems less clear cut.

## Criteria for Choice Over Risky Prospects

	Percentage Return in			
	Good State	Bad State	$E(R)$	$\sigma(R)$
Asset 1	20	5	12.5	7.5
Asset 2	60	-50	5	55
Asset 3	60	5	32.5	27.5

Asset 1 exhibits **mean-variance dominance** over asset 2, since it offers a higher expected return with lower variance.

## Criteria for Choice Over Risky Prospects

In general, one asset displays mean-variance dominance over another if:

1.  $E(R_1) > E(R_2)$  and  $\sigma(R_1) \leq \sigma(R_2)$

so that it offers a higher expected return with no greater standard deviation,

OR

2.  $E(R_1) \geq E(R_2)$  and  $\sigma(R_1) < \sigma(R_2)$

so that it offers a smaller standard deviation and no less expected return.



## Criteria for Choice Over Risky Prospects

	Percentage Return in			
	Good State	Bad State	$E(R)$	$\sigma(R)$
Asset 1	20	5	12.5	7.5
Asset 2	60	-50	5	55
Asset 3	60	5	32.5	27.5

But notice that by the mean-variance criterion, asset 3 dominates asset 2 but not asset 1, even though on a state-by-state basis, asset 3 is clearly to be preferred.

## Criteria for Choice Over Risky Prospects

Consider two more assets:

	Percentage Return in			
	Good State	Bad State	$E(R)$	$\sigma(R)$
Asset 4	5	3		
Asset 5	8	2		

Again, neither exhibits state-by-state dominance, so let's try to use the mean-variance criterion again.

## Criteria for Choice Over Risky Prospects

	Percentage Return in		$E(R)$	$\sigma(R)$
	Good State	Bad State		
Asset 4	5	3		
Asset 5	8	2		

$$E(R_4) = (1/2)5 + (1/2)3 = 4$$

$$\sigma(R_4) = [(1/2)(5 - 4)^2 + (1/2)(3 - 4)^2]^{1/2} = 1$$

$$E(R_5) = (1/2)8 + (1/2)2 = 5$$

$$\sigma(R_5) = [(1/2)(8 - 5)^2 + (1/2)(2 - 5)^2]^{1/2} = 3$$

## Criteria for Choice Over Risky Prospects

	Percentage Return in			
	Good State	Bad State	$E(R)$	$\sigma(R)$
Asset 4	5	3	4	1
Asset 5	8	2	5	3

Neither asset exhibits mean-variance dominance either.

## Criteria for Choice Over Risky Prospects

	Percentage Return in			
	Good State	Bad State	$E(R)$	$\sigma(R)$
Asset 4	5	3	4	1
Asset 5	8	2	5	3

William Sharpe (US, b.1934, Nobel Prize 1990) suggested that in these circumstances, it can help to compare the two assets' **Sharpe ratios**, defined as  $E(R)/\sigma(R)$ .

## Criteria for Choice Over Risky Prospects

**Note:** in practice, the Sharpe ratio is usually defined as the expected “excess return” above the risk-free rate  $r_f$  divided by the standard deviation:

$$\frac{E(R) - r_f}{\sigma(R)}.$$

For these preliminary examples, we are either assuming that  $r_f = 0$  or using  $E(R)/\sigma(R)$  as a simplified definition of the Sharpe ratio.

## Criteria for Choice Over Risky Prospects

	Percentage Return in				
	Good State	Bad State	$E(R)$	$\sigma(R)$	$E/\sigma$
Asset 4	5	3	4	1	4
Asset 5	8	2	5	3	1.67

Comparing Sharpe ratios, asset 4 is preferred to asset 5.

## Criteria for Choice Over Risky Prospects

	Percentage Return in		$E(R)$	$\sigma(R)$	$E/\sigma$
	Good State	Bad State			
Asset 4	5	3	4	1	4
Asset 5	8	2	5	3	1.67

But using the Sharpe ratio to choose between assets means assuming that investors “weight” the mean and standard deviation equally, in the sense that a doubling of  $\sigma(R)$  is adequately compensated by a doubling of  $E(R)$ . Investors who are more or less averse to risk will disagree.



## Criteria for Choice Over Risky Prospects

1. State-by-state dominance is the most robust criterion, but often cannot be applied.
2. Mean-variance dominance is more widely-applicable, but can sometimes be misleading and cannot always be applied.
3. The Sharpe ratio can always be applied, but requires a very specific assumption about consumer attitudes towards risk.

We need a more careful and comprehensive approach to comparing random cash flows.

# Preferences and Utility Functions

Of course, economists face a more general problem of this kind.

Even if we accept that more (of everything) is preferred to less, how do consumers compare different “bundles” of goods that may contain more of one good but less of another?

Microeconomists have identified a set of conditions that allow a consumer's **preferences** to be described by a **utility function**.

# Preferences and Utility Functions

Gerard Debreu identified these conditions in 1954:

Cowles Foundation Paper 97  
Reprinted from Thrall, Davis and Coombs , eds.,  
Decision Processes, John Wiley, 1954

CHAPTER XI

**REPRESENTATION OF A PREFERENCE  
ORDERING BY A NUMERICAL FUNCTION\***

by

Gerard Debreu

COWLES COMMISSION FOR RESEARCH IN ECONOMICS

# Preferences and Utility Functions

Let  $a$ ,  $b$ , and  $c$  represent three bundles of goods.

These may be arbitrarily long lists, or **vectors** ( $a \in \mathbb{R}^N$ ), indicating how much of each of an arbitrarily large number of goods is included in the bundle.

A **preference relation**  $\succeq$  can be used to represent the consumer's preferences over different consumption bundles.

# Preferences and Utility Functions

The expression

$$a \succ b$$

indicates that the consumer strictly prefers  $a$  to  $b$ ,

$$a \sim b$$

indicates that the consumer is indifferent between  $a$  and  $b$ , and

$$a \succeq b$$

indicates that the consumer either strictly prefers or is indifferent between  $a$  and  $b$ .

# Preferences and Utility Functions

**A1** The preference relation is assumed to be **complete**: For any two bundles  $a$  and  $b$ , either  $a \succeq b$ ,  $b \succeq a$ , or both, and in the latter case  $a \sim b$ .

The consumer has to decide whether he or she prefers one bundle to another or is indifferent between the two. Ambiguous tastes are not allowed.

# Preferences and Utility Functions

**A2** The preference relation is assumed to be **transitive**: For any three bundles  $a$ ,  $b$ , and  $c$ , if  $a \succeq b$  and  $b \succeq c$ , then  $a \succeq c$ .

The consumer's tastes must be consistent in this sense. Together, (A1) and (A2) require the consumer to be fully informed and rational.

## Preferences and Utility Functions

A3 The preference relation is assumed to be **continuous**: If  $\{a_n\}$  and  $\{b_n\}$  are two sequences of bundles such that  $a_n \rightarrow a$ ,  $b_n \rightarrow b$ , and  $a_n \succeq b_n$  for all  $n$ , then  $a \succeq b$ .

Very small changes in consumption bundles cannot lead to large changes in preferences over those bundles.



## Preferences and Utility Functions

An two-good example that violates (A3) is the case of **lexicographic** preferences:

$$a = (a_1, a_2) \succ b = (b_1, b_2) \quad \text{if} \quad a_1 > b_1 \\ \text{or} \quad a_1 = b_1 \text{ and } a_2 > b_2.$$

It is not possible to represent these preferences with a utility function, since the preferences are fundamentally two-dimensional and the value of the utility function has to be one-dimensional.

# Preferences and Utility Functions

The following theorem was proven by Gerard Debreu in 1954.

**Theorem** If preferences are complete, transitive, and continuous, then they can be represented by a continuous, real-valued utility function. That is, if (A1)-(A3) hold, there is a continuous function  $u : \mathbb{R}^n \mapsto \mathbb{R}$  such that for any two consumption bundles  $a$  and  $b$ ,

$$a \succeq b \text{ if and only if } u(a) \geq u(b).$$

# Preferences and Utility Functions

Note that if preferences are represented by the utility function  $u$ ,

$$a \succeq b \text{ if and only if } u(a) \geq u(b),$$

then they are also represented by the utility function  $v$ , where

$$v(a) = F(u(a))$$

and  $F : \mathbb{R} \mapsto \mathbb{R}$  is **any** increasing function.

The concept of utility as it is used in standard microeconomic theory is **ordinal**, as opposed to **cardinal**.