ECON 337901
FINANCIAL ECONOMICS

Peter Ireland

Boston College

March 12, 2019

Midterm Exam

Tuesday, March 19: 10:30 - 11:45am

Short answer questions like those from the homeworks, covering the material discussed in class, up through and including bond pricing (Overview of Asset Pricing Theory).

Bring a pen or pencil. You probably won't need a calculator, but you can use one if you want to.

Midterm Exam

For Extra Practice:

- Midterm from Fall 2013: Question 1
- Midterm from Spring 2014: Questions 1, 2
- Final (two versions) from Spring 2014: Question 4
- Midterm from Spring 2015: Questions 1, 3
- Midterm from Fall 2015: Questions 1, 2, 3
- Midterm from Spring 2016: Questions 1, 2, 3
- Midterm from Spring 2017: Questions 1, 2, 3, 4
- Midterm from Fall 2017: Questions 1, 2, 3, 4, 5
- Midterm from Spring 2018: Questions 1, 2, 3, 4, 5

2 Overview of Asset Pricing Theory

A Pricing Safe Cash Flows
B Pricing Risky Cash Flows
C Two Perspectives on Asset Pricing

Next: Making Choices in Risky Situations

Pricing Safe Cash Flows

A T-year discount bond is an asset that pays off \$1, for sure, T years from now.

If this bond sells for \$ P_T today, the annualized return from buying the bond today and holding it to maturity is

$$1 + r_T = \left(\frac{1}{P_T}\right)^{1/T}.$$

Hence, the bond price and the interest rate are related via

$$P_T = \frac{1}{(1 + r_T)^T}.$$

Pricing Safe Cash Flows

Since, for a T-period discount bond,

$$P_T = \frac{1}{(1 + r_T)^T},$$

the interest rate equates today's price of the bond to the present discounted value of the future payments made by the bond.

US Treasury bills, that is, US government bonds with maturities less than one year, are structured as discount bonds.

Pricing Safe Cash Flows

A T-year coupon bond is an asset that makes an annual interest (coupon) payment of \$$C$ each year, every year, for the next T years, and then pays off \$$F$ (face or par value), for sure, T years from now.

US Treasury notes and bonds, with maturities of more than one year, are structured as coupon bonds.

Pricing Safe Cash Flows

Notice that a coupon bond can be viewed as a bundle, or portfolio of discount bonds, since the cash flows from a T-year coupon bond can be replicated by buying

C one-year discount bonds

C two-year discount bonds

...

C T-year discount bonds

F more T-year discount bonds

Pricing Safe Cash Flows

No arbitrage implies that the price P_T^C of the coupon bond must equal the cost of the portfolio of discount bonds:

$$\begin{aligned} P_T^C &= CP_1 + CP_2 + \ldots + CP_T + FP_T \\ &= \frac{C}{1+r_1} + \frac{C}{(1+r_2)^2} + \ldots + \frac{C}{(1+r_T)^T} + \frac{F}{(1+r_T)^T} \end{aligned}$$

Today's price of the coupon bond equals the present discounted value of the future payments made by the bond.

Pricing Safe Cash Flows

Next, consider an asset that generates an arbitrary stream of safe (riskless) cash flows $C_1, C_2, \ldots, C_T$, over the next T years.

To simplify the task of "pricing" this asset, we might view it as a portfolio of more basic assets: one that pays C_1 for sure in one year, one that pays C_2 for sure in two years, …, and one that pays C_T for sure in T years.

The price of the multi-period asset must equal the sum of the prices of the more basic assets.

Pricing Safe Cash Flows

We've now reduced the problem of pricing any riskless asset to the simpler problem of pricing a more basic asset that pays C_t for sure t years from now.

But this more basic asset has the same payoff as C_t t-year discount bonds. Its price P_t^A today must equal

$$P_t^A = C_t P_t = \frac{C_t}{(1 + r_t)^t},$$

the present discounted value of its cash flow.

Pricing Safe Cash Flows

Suppose you want to borrow \$1 $n-1$ years from now ...

Repay with interest n years from now ...

And lock in the "n-year forward rate" r_n^f today.

Pricing Safe Cash Flows

To get \$1 $n-1$ years from now, you can buy an $(n-1)$-year discount bond.

The problem is, this costs P_{n-1} today.

Suppose, in addition, you sell short P_{n-1}/P_n n-year discount bonds today. This will give you

$$\left(\frac{P_{n-1}}{P_n}\right) P_n = P_{n-1}$$

dollars today, to offset the cost of buying the $(n-1)$-year discount bond.

Pricing Safe Cash Flows

But then you have to repay P_{n-1}/P_n dollars n years from now.

Therefore

$$1 + r_n^f = \frac{P_{n-1}/P_n \text{ repaid } n \text{ years from now}}{\text{one dollar borrowed today}}$$

$$r_n^f = \frac{P_{n-1}}{P_n} - 1$$

Problem Set 8, Question 3: Use the discount bond prices from Q1 to compute forward rates with this formula.

Pricing Risky Cash Flows

Now consider a risky asset, with cash flows $\tilde{C}_1, \tilde{C}_2, \ldots, \tilde{C}_T$ over the next T years that are random variables with values that are unknown today.

Again, we might simplify the task of pricing this asset, by viewing it as a portfolio of more basic assets, each of which makes a random payment $\tilde{C}_t$ after t years, then summing up the prices of all of these more basic assets.

Pricing Risky Cash Flows

But we still have to deal with the fact that the payoff $\tilde{C}_t$ is risky.

And that is what the modern theory of asset pricing, on which this course is based, is really all about.

Pricing Risky Cash Flows

One possibility is to break down the random payoff $\tilde{C}_t$ into separate components $C_{t,1}, C_{t,2}, \ldots, C_{t,n}$ delivered in n different "states of the world" that can prevail t years from now.

The risky asset that delivers the random payoff $\tilde{C}_t$ t years from now can itself be viewed as a portfolio of contingent claims: $C_{t,1}$ contingent claims for state 1, $C_{t,2}$ contingent claims for state 2, ..., and $C_{t,n}$ contingent claims for state n.

Pricing Risky Cash Flows

This Arrow-Debreu approach to asset pricing then computes

$$P_t^A = q_{t,1}C_{t,1} + q_{t,2}C_{t,2} + \ldots + q_{t,n}C_{t,n}$$

where $q_{t,i}$ is the price today of a contingent claim that delivers one dollar if state i occurs t years from now and zero otherwise.

This approach uses contingent claims as the "basic building blocks" for risky assets, in the same way that discount bonds can be viewed as the building blocks for coupon bonds.

Pricing Risky Cash Flows

In probability theory, if a random variable $\tilde{X}$ can take on n possible values, $X_1, X_2, \ldots, X_n$, with probabilities $\pi_1, \pi_2, \ldots, \pi_n$, then the expected value of $\tilde{X}$ is

$$E(\tilde{X}) = \pi_1 X_1 + \pi_2 X_2 + \ldots + \pi_n X_n.$$

Pricing Risky Cash Flows

More traditional approaches to asset pricing replace the random payoff $\tilde{C}_t$ with its expected value $E(\tilde{C}_t)$ and then "penalize" the fact that the payoff is random by discounting it at a higher rate

$$P_t^A = \frac{E(\tilde{C}_t)}{(1 + r_t + \psi_t)^t}$$

The capital asset pricing model (CAPM) will give us a way of determining values for the risk premium ψ_t.

Two Perspectives on Asset Pricing

Although all are designed to accomplish the same basic goal – to value risky cash flows – different theories of asset pricing can be grouped under two broad headings.

No-arbitrage theories take the prices of some assets as given and use those to determine the prices of other assets.

Equilibrium theories price all assets based on the principles of microeconomic theory.

Two Perspectives on Asset Pricing

No-arbitrage theories require fewer assumptions and are sometimes easier to use.

We've already used no-arbitrage arguments, for example, to price stocks and bonds as portfolios of contingent claims and to price coupon bonds as portfolios of discount bonds.

Two Perspectives on Asset Pricing

But no-arbitrage theories raise questions that only equilibrium theories can answer.

Where do the prices of the basic securities come from? And how do asset prices relate to economic fundamentals?

As an equilibrium theory of asset pricing, the CAPM will also help us answer these questions.

3 Making Choices in Risky Situations

A Criteria for Choice Over Risky Prospects

B Preferences and Utility Functions

C Expected Utility Functions

D The Expected Utility Theorem

E The Allais Paradox

Criteria for Choice Over Risky Prospects

In the broadest sense, "risk" refers to uncertainty about the future cash flows provided by a financial asset.

A more specific way of modeling risk is to think of those cash flows as varying across different states of the world in future periods ...

... that is, to describe future cash flows as random variables.

Criteria for Choice Over Risky Prospects

Consider three assets:

		Payoffs Next Year in	
	Price Today	Good State	Bad State
Asset 1	-1000	1200	1050
Asset 2	-1000	1600	500
Asset 3	-1000	1600	1050

where the good and bad states occur with equal probability ($\pi = 1 - \pi = 1/2$).

Criteria for Choice Over Risky Prospects

	Price Today	Payoffs Next Year in Good State	Bad State
Asset 1	−1000	1200	1050
Asset 2	−1000	1600	500
Asset 3	−1000	1600	1050

Asset 3 exhibits state-by-state dominance over assets 1 and 2. Any investor who prefers more to less would always choose asset 3 above the others.

Criteria for Choice Over Risky Prospects

In general, one asset displays state-by-state dominance over another if:

1. It pays off at least as much in all states

AND

2. It pays off more in at least one state,

so investors who prefer more to less will never regret buying it.

Criteria for Choice Over Risky Prospects

		Payoffs Next Year in	
	Price Today	Good State	Bad State
Asset 1	-1000	1200	1050
Asset 2	-1000	1600	500
Asset 3	-1000	1600	1050

Although asset 3 is unambiguously the best, the choice between assets 1 and 2 seems less clear cut.

Criteria for Choice Over Risky Prospects

It can often be helpful to convert prices and payoffs to percentage returns:

		Payoffs Next Year in	
	Price Today	Good State	Bad State
Asset 1	-1000	1200	1050
Asset 2	-1000	1600	500
Asset 3	-1000	1600	1050

	Percentage Return in	
	Good State	Bad State
Asset 1	20	5
Asset 2	60	-50
Asset 3	60	5

Criteria for Choice Over Risky Prospects

In probability theory, if a random variable $\tilde{X}$ can take on n possible values, $X_1, X_2, \ldots, X_n$, with probabilities $\pi_1, \pi_2, \ldots, \pi_n$, then the expected value of $\tilde{X}$ is

$$E(\tilde{X}) = \pi_1 X_1 + \pi_2 X_2 + \ldots + \pi_n X_n,$$

the variance of $\tilde{X}$ is

$$\begin{aligned}\sigma^2(\tilde{X}) &= \pi_1 [X_1 - E(\tilde{X})]^2 \\ &\quad + \pi_2 [X_2 - E(\tilde{X})]^2 + \ldots + \pi_n [X_n - E(\tilde{X})]^2,\end{aligned}$$

and the standard deviation of $\tilde{X}$ is $\sigma(\tilde{X}) = [\sigma^2(\tilde{X})]^{1/2}$.

Criteria for Choice Over Risky Prospects

	Percentage Return in	
	Good State	Bad State
Asset 1	20	5
Asset 2	60	-50
Asset 3	60	5

$$E(R_1) = (1/2)20 + (1/2)5 = 12.5$$

$$\sigma(R_1) = [(1/2)(20 - 12.5)^2 + (1/2)(5 - 12.5)^2]^{1/2} = 7.5$$

Criteria for Choice Over Risky Prospects

	Percentage Return in			
	Good State	Bad State	$E(R)$	$\sigma(R)$
Asset 1	20	5	12.5	7.5
Asset 2	60	-50		
Asset 3	60	5		

$$E(R_2) = (1/2)60 + (1/2)(-50) = 5$$

$$\sigma(R_2) = [(1/2)(60-5)^2 + (1/2)(-50-5)^2]^{1/2} = 55$$

Criteria for Choice Over Risky Prospects

	Percentage Return in			
	Good State	Bad State	$E(R)$	$\sigma(R)$
Asset 1	20	5	12.5	7.5
Asset 2	60	-50	5	55
Asset 3	60	5		

$$E(R_3) = (1/2)60 + (1/2)5 = 32.5$$

$$\sigma(R_3) = [(1/2)(60 - 32.5)^2 + (1/2)(5 - 32.5)^2]^{1/2} = 27.5$$

Criteria for Choice Over Risky Prospects

	Percentage Return in			
	Good State	Bad State	$E(R)$	$\sigma(R)$
Asset 1	20	5	12.5	7.5
Asset 2	60	-50	5	55
Asset 3	60	5	32.5	27.5

Asset 1 exhibits mean-variance dominance over asset 2, since it offers a higher expected return with lower variance.

Criteria for Choice Over Risky Prospects

In general, one asset displays mean-variance dominance over another if:

1. $E(R_1) > E(R_2)$ and $\sigma(R_1) \leq \sigma(R_2)$

so that it offers a higher expected return with no greater standard deviation,

OR

2. $E(R_1) \geq E(R_2)$ and $\sigma(R_1) < \sigma(R_2)$

so that it offers a smaller standard deviation and no less expected return.

Criteria for Choice Over Risky Prospects

	Percentage Return in			
	Good State	Bad State	$E(R)$	$\sigma(R)$
Asset 1	20	5	12.5	7.5
Asset 2	60	-50	5	55
Asset 3	60	5	32.5	27.5

But notice that by the mean-variance criterion, asset 3 dominates asset 2 but not asset 1, even though on a state-by-state basis, asset 3 is clearly to be preferred.