ECON 337901
FINANCIAL ECONOMICS

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February 26, 2019

Midterm Exam

Tuesday, March 19: 10:30 - 11:45am

Short answer questions like those from the homeworks, covering the material discussed in class, up through and including bond pricing (Overview of Asset Pricing Theory).

Bring a pen or pencil. You probably won't need a calculator, but you can use one if you want to.

Midterm Exam

For Extra Practice:

Midterm from Fall 2013: Question 1
Midterm from Spring 2014: Questions 1, 2
Final (two versions) from Spring 2014: Question 4
Midterm from Spring 2015: Questions 1, 3
Midterm from Fall 2015: Questions 1, 2, 3
Midterm from Spring 2016: Questions 1, 2, 3
Midterm from Spring 2017: Questions 1, 2, 3, 4
Midterm from Fall 2017: Questions 1, 2, 3, 4, 5
Midterm from Spring 2018: Questions 1, 2, 3, 4, 5

Black-Scholes Option Pricing

Fischer Black (US, 1938-1995) and Myron Scholes (Canada/US, b.1941, Nobel Prize 1997) were the first to derive a formula for the price of an option.

Robert Merton (US, b.1944, Nobel Prize 1997) arrived at the same formula in a simpler way, by showing how options prices could be inferred from assumptions about and observations on the underlying stock price.

Their papers were both published in 1973.

Black-Scholes Option Pricing

Fischer Black and Myron Scholes, "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy* Vol.81 (May-June 1973): pp.637-654.

Robert Merton, "Theory of Rational Option Pricing," *The Bell Journal of Economics and Management Science* Vol.4 (Spring 1973): pp.141-183.

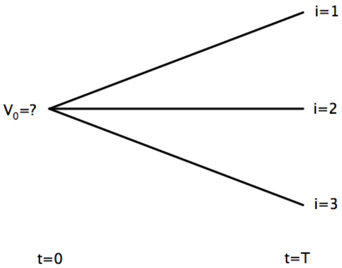
For an interesting postscript, see Roger Lowenstein, *When Genius Failed*.

Black-Scholes Option Pricing

Black and Scholes and Merton considered a more general setting, in which the option priced at $t = 0$ does not expire until $t = T$.

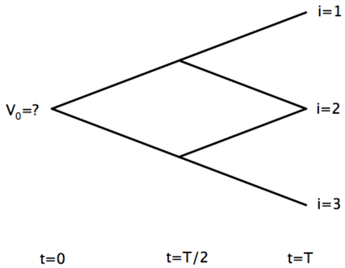
They also allowed for (many) more than two possible states at $t = T$.

Black-Scholes Option Pricing



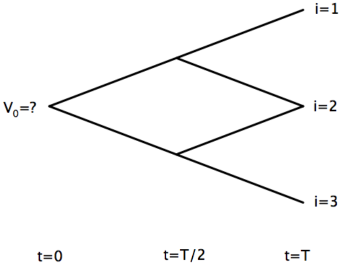
The technical problem is that with more than two states at $t = T$, more than two assets are needed to create a portfolio with the same payoffs as the option.

Black-Scholes Option Pricing



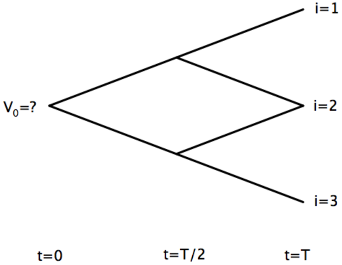
Black and Scholes and Merton realized that this problem can be solved by breaking the full period into sub-periods, so that there are only two states in each sub-period.

Black-Scholes Option Pricing



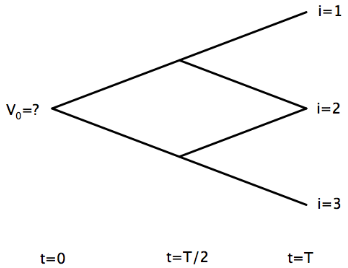
With three states at $t = T$, only two subperiods are needed, but with many states at $t = T$, many subperiods are needed.

Black-Scholes Option Pricing



A dynamic hedging strategy can then be used to track the payoffs on the option using a portfolio consisting only of the stock and bond . . .

Black-Scholes Option Pricing



. . . but where the number of shares and the number of bonds must be adjusted in each subperiod so that the portfolio can continue to track the option's payoffs.

Black-Scholes Option Pricing

Black and Scholes used methods in stochastic calculus developed by Kiyoshi Ito (Japan, 1915-2008) in the 1940s and early 1950s to show that in the more general case, the solution for the option price is

$$q^0 = N_1 q^s - N_2 q^b K = N_1 q^s - N_2 \left(\frac{K}{1 + r_f} \right)$$

where $N_1 = F(d_1)$ and $N_2 = F(d_2)$,

$$d_1 = \frac{\ln(q^s/K) + (r_f + \sigma^2/2)T}{\sigma\sqrt{T}} \text{ and } d_2 = d_1 - \sigma\sqrt{T}$$

σ is the standard deviation of the return on the stock, and F is the standard normal cumulative distribution function, so that $F(X)$ measures the probability that a random variable that is normally distributed with mean zero and variance one turns out to be less than or equal to X.

Black-Scholes Option Pricing

To use the Black-Scholes formula

$$q^0 = N_1 q^s - N_2 q^b K = N_1 q^s - N_2 \left(\frac{K}{1 + r_f} \right)$$

where $N_1 = F(d_1)$ and $N_2 = F(d_2)$,

$$d_1 = \frac{\ln(q^s/K) + (r_f + \sigma^2/2)T}{\sigma\sqrt{T}} \text{ and } d_2 = d_1 - \sigma\sqrt{T}$$

you need an estimate of σ, the standard deviation of the return on the stock.

Black-Scholes Option Pricing

Alternatively, if you see the price of a traded option, you can use the Black-Scholes formula

$$q^0 = N_1 q^s - N_2 q^b K = N_1 q^s - N_2 \left(\frac{K}{1 + r_f} \right)$$

where $N_1 = F(d_1)$ and $N_2 = F(d_2)$,

$$d_1 = \frac{\ln(q^s/K) + (r_f + \sigma^2/2)T}{\sigma\sqrt{T}} \text{ and } d_2 = d_1 - \sigma\sqrt{T}$$

to estimate σ, the standard deviation of the return on the stock. In fact, the VIX volatility index is similar to the σ implied by the price of an option on the S&P 500.