

ECON 337901

FINANCIAL ECONOMICS

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Consumer Optimization: The Risk Dimension

$$Y_0 \geq c_0 + q^G s^G + q^B s^B$$

$$Y_1^G + s^G \geq c_1^G$$

$$Y_1^B + s^B \geq c_1^B$$

Multiply both sides of the second equation by q^G and both sides of the third equation by q^B , Then add them all up to get the lifetime budget constraint

$$Y_0 + q^G Y_1^G + q^B Y_1^B \geq c_0 + q^G c_1^G + q^B c_1^B.$$

Consumer Optimization: The Risk Dimension

The problem is to choose c_0 , c_1^G , and c_1^B to maximize expected utility

$$u(c_0) + \beta\pi u(c_1^G) + \beta(1 - \pi)u(c_1^B),$$

subject to the budget constraint

$$Y_0 + q^G Y_1^G + q^B Y_1^B \geq c_0 + q^G c_1^G + q^B c_1^B.$$

This was Arrow and Debreu's key insight: that finance is like grocery shopping. Mathematically, making decisions over time and under uncertainty is no different from choosing apples, bananas, and pears!

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The Lagrangian is

$$L = u(c_0) + \beta\pi u(c_1^G) + \beta(1 - \pi)u(c_1^B) \\ + \lambda (Y_0 + q^G Y_1^G + q^B Y_1^B - c_0 - q^G c_1^G - q^B c_1^B),$$

and the first-order conditions are

$$u'(c_0^*) - \lambda^* = 0 \\ \beta\pi u'(c_1^{G*}) - \lambda^* q^G = 0 \\ \beta(1 - \pi)u'(c_1^{B*}) - \lambda^* q^B = 0$$

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The first-order conditions

$$u'(c_0^*) - \lambda^* = 0$$

$$\beta\pi u'(c_1^{G*}) - \lambda^* q^G = 0$$

$$\beta(1 - \pi)u'(c_1^{B*}) - \lambda^* q^B = 0$$

imply that marginal rates of substitution equal relative prices:

$$\frac{u'(c_0^*)}{\beta\pi u'(c_1^{G*})} = \frac{1}{q^G} \text{ and } \frac{u'(c_0^*)}{\beta(1 - \pi)u'(c_1^{B*})} = \frac{1}{q^B}$$

$$\text{and } \frac{\pi u'(c_1^{G*})}{(1 - \pi)u'(c_1^{B*})} = \frac{q^G}{q^B}.$$

Consumer Optimization: The Risk Dimension

Do we really observe consumers trading in contingent claims?

Yes, if we think of financial assets as “bundles” of contingent claims.

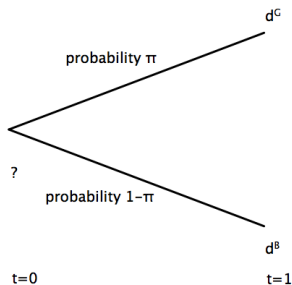
This insight is also Arrow and Debreu's.

Consumer Optimization: The Risk Dimension

A “stock” is a risky asset that pays dividend d^G next year in the good state and d^B next year in the bad state.

These payoffs can be replicated by buying d^G contingent claims for the good state and d^B contingent claims for the bad state.

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Payoffs for the stock.

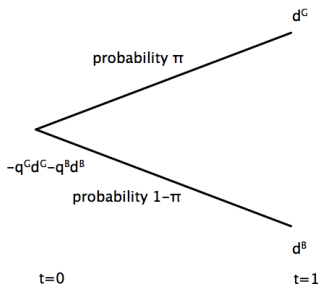
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If we start with knowledge of the contingent claims prices q^G and q^B , then we can infer that the stock must sell today for

$$q^{stock} = q^G d^G + q^B d^B.$$

Since if the stock cost more than the equivalent bundle of contingent claims, traders could make profits for sure by short selling the stock and buying the contingent claims; and if the stock cost less than the equivalent bundle of contingent claims, traders could make profits for sure by buying the stock and selling the contingent claims.

Consumer Optimization: The Risk Dimension



“Pricing” the stock.