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FINANCIAL ECONOMICS

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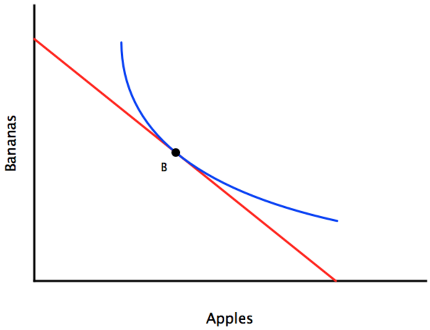
Consumer Optimization: Graphical Analysis



At B, the optimal choice, the indifference curve is tangent to the budget constraint.

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Recall that the budget constraint

$$Y = p_a c_a + p_b c_b$$

or

$$c_b = \frac{Y}{p_b} - \left(\frac{p_a}{p_b}\right) c_a$$

has slope $-(p_a/p_b)$. But what is the slope of the indifference curve?

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Suppose that the consumer's preferences are also described by the utility function

$$u(c_a) + \beta u(c_b).$$

The function u is increasing, with $u'(c) > 0$, so that more is preferred to less, and concave, with $u''(c) < 0$, so that marginal utility falls as consumption rises.

The parameter β measures how much more (if $\beta > 1$) or less (if $\beta < 1$) the consumer likes bananas compared to apples.

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Since an indifference curve traces out the set of (c_a, c_b) combinations that yield a given level of utility $\bar{U}$, the equation for an indifference curve is

$$\bar{U} = u(c_a) + \beta u(c_b).$$

Use this equation to define a new function, $c_b(c_a)$, describing the number of bananas needed, for each number of apples, to keep the consumer on this indifference curve:

$$\bar{U} = u(c_a) + \beta u[c_b(c_a)].$$

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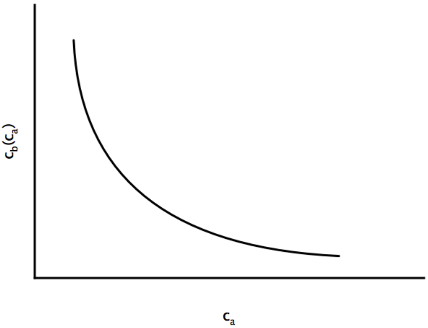

The function $c_b(c_a)$ satisfies $\bar{U} = u(c_a) + \beta u[c_b(c_a)]$.

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Differentiate both sides of

$$\bar{U} = u(c_a) + \beta u[c_b(c_a)]$$

to obtain

$$0 = u'(c_a) + \beta u'[c_b(c_a)]c_b'(c_a)$$

or

$$c_b'(c_a) = -\frac{u'(c_a)}{\beta u'[c_b(c_a)]}.$$

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This last equation,

$$c_b'(c_a) = -\frac{u'(c_a)}{\beta u'[c_b(c_a)]},$$

written more simply as

$$c_b'(c_a) = -\frac{u'(c_a)}{\beta u'(c_b)},$$

measures the slope of the indifference curve: the consumer's marginal rate of substitution.

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Thus, the tangency of the budget constraint and indifference curve can be expressed mathematically as

$$\frac{p_a}{p_b} = \frac{u'(c_a)}{\beta u'(c_b)}.$$

The marginal rate of substitution equals the relative prices.

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Returning to the more general expression

$$c_b'(c_a) = -\frac{u'(c_a)}{\beta u'[c_b(c_a)]},$$

we can see that $c_b'(c_a) < 0$, so that the indifference curve is downward-sloping, so long as the utility function u is strictly increasing, that is, if more is preferred to less.

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$$c_b'(c_a) = -\frac{u'(c_a)}{\beta u'[c_b(c_a)]}$$

Differentiating again yields

$$c_b''(c_a) = -\frac{\beta u'[c_b(c_a)]u''(c_a) - u'(c_a)\beta u''[c_b(c_a)]c_b'(c_a)}{\{\beta u'[c_b(c_a)]\}^2},$$

which is positive if u is strictly increasing (more is preferred to less) and concave (diminishing marginal utility). In this case, the indifference curve will be convex. Again, we see how concave functions have mathematical properties and economic implications that we like.