ECON 337901
FINANCIAL ECONOMICS

Peter Ireland

Boston College

January 29, 2019

Consumer Optimization

1. Graphical Analysis
2. Algebraic Analysis
3. Time Dimension
4. Risk Dimension

Consumer Optimization

Alfred Marshall, *Principles of Economics*, 1890. – supply and demand

Francis Edgeworth, *Mathematical Psychics*, 1881.

Vilfredo Pareto, *Manual of Political Economy*, 1906. – indifference curves

Consumer Optimization

John Hicks, *Value and Capital*, 1939. – wealth and substitution effects

Paul Samuelson, *Foundations of Economic Analysis*, 1947. – mathematical reformulation

Irving Fisher, *The Theory of Interest*, 1930. – intertemporal extension.

Consumer Optimization

Gerard Debreu, *Theory of Value*, 1959.

Kenneth Arrow, "The Role of Securities in the Optimal Allocation of Risk Bearing," *Review of Economic Studies*, 1964.

Extensions to include risk and uncertainty.

Consumer Optimization: Graphical Analysis

Consider a consumer who likes two goods: apples and bananas.

$$Y = \text{income}$$
$$c_a = \text{consumption of apples}$$
$$c_b = \text{consumption of bananas}$$
$$p_a = \text{price of an apple}$$
$$p_b = \text{price of a banana}$$

The consumer's budget constraint is

$$Y \geq p_a c_a + p_b c_b$$

Consumer Optimization: Graphical Analysis

So long as the consumer always prefers more to less, the budget constraint will always bind:
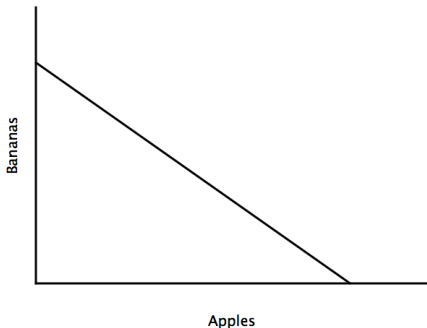
$$Y = p_a c_a + p_b c_b$$

or

$$c_b = \frac{Y}{p_b} - \left(\frac{p_a}{p_b} \right) c_a$$

Which shows that the graph of the budget constraint will be a straight line with slope $-(p_a/p_b)$ and intercept Y/p_b.

Consumer Optimization: Graphical Analysis

The budget constraint is a straight line with slope $-(p_a/p_b)$ and intercept Y/p_b.

Consumer Optimization: Graphical Analysis

The budget constraint describes the consumer's market opportunities.

Francis Edgeworth (Ireland, 1845-1926) and Vilfredo Pareto (Italy, 1848-1923) were the first to use indifference curves to describe the consumer's preferences.
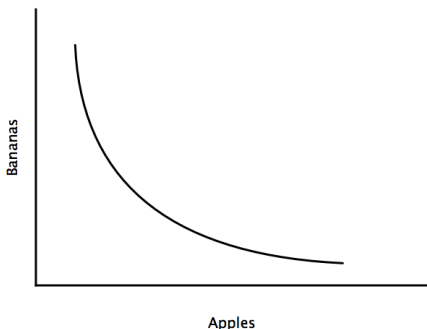
Each indifference curve traces out a set of combinations of apples and bananas that give the consumer a given level of utility or satisfaction.
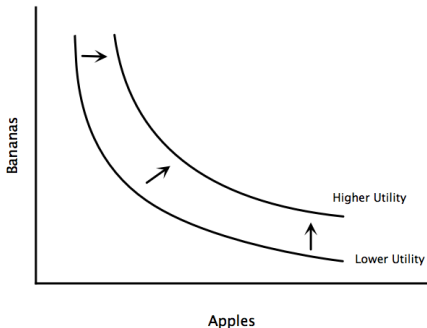
Consumer Optimization: Graphical Analysis
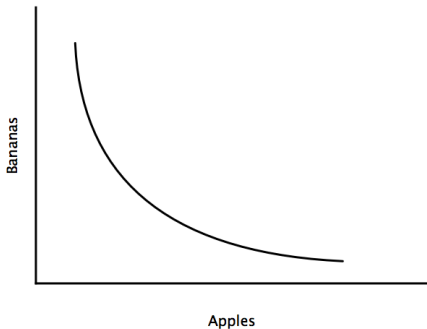
Each indifference curve traces out a set of combinations of apples and bananas that give the consumer a given level of utility.
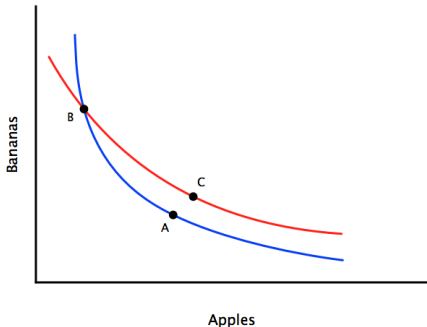
Consumer Optimization: Graphical Analysis

Each indifference curve slopes down, since the consumer requires more apples to compensate for a loss of bananas and more bananas to compensate for a loss of apples, if more is preferred to less.

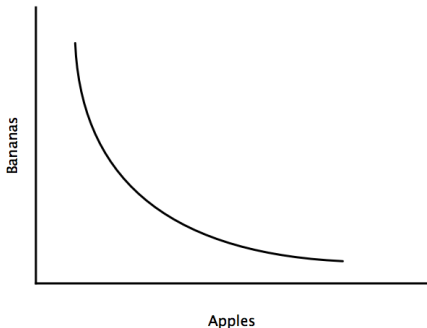# Consumer Optimization: Graphical Analysis

Indifference curves farther away from the origin represent higher levels of utility, if more is preferred to less.
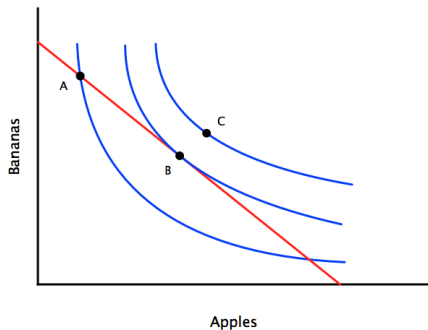
Consumer Optimization: Graphical Analysis

A and B yield the same level of utility, and B and C yield the same level of utility, but C is preferred to A if more is preferred to less. Indifference curves cannot intersect.

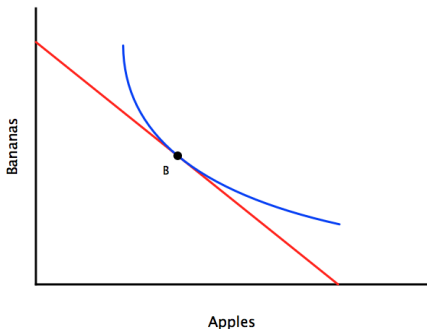# Consumer Optimization: Graphical Analysis

Bananas

Apples

Indifference curves are convex to the origin if consumers have a preference for diversity.

Consumer Optimization: Graphical Analysis

A is suboptimal and C is infeasible. B is optimal.

Consumer Optimization: Graphical Analysis

At B, the optimal choice, the indifference curve is tangent to the budget constraint.

Consumer Optimization: Graphical Analysis

Recall that the budget constraint

$$Y = p_a c_a + p_b c_b$$

or

$$c_b = \frac{Y}{p_b} - \left(\frac{p_a}{p_b}\right) c_a$$

has slope $-(p_a/p_b)$. But what is the slope of the indifference curve?