ECON 337901
FINANCIAL ECONOMICS

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Unconstrained Optimization: Example 2

Consider maximizing a function of three variables:

$$\max_{x_1,x_2,x_3} F(x_1, x_2, x_3)$$

Even if each variable can take on only 1,000 values, there are one billion possible combinations of (x_1, x_2, x_3) to search over!

This is an example of what Richard Bellman (US, 1920-1984) called the "curse of dimensionality."

Unconstrained Optimization: Example 2

Consider the problem:

$$\max_{x_1,x_2,x_3} \left(-\frac{1}{2}\right)(x_1-\tau)^2 + \left(-\frac{1}{2}\right)(x_2-x_1)^2 + \left(-\frac{1}{2}\right)(x_3-x_2)^2.$$

Now the three first-order conditions

$$-(x_1^*-\tau) + (x_2^*-x_1^*) = 0$$
$$-(x_2^*-x_1^*) + (x_3^*-x_2^*) = 0$$
$$-(x_3^*-x_2^*) = 0$$

lead us to the solution: $x_1^* = x_2^* = x_3^* = \tau$.

Constrained Optimization

To find the value of x that solves

$$\max_x F(x) \text{ subject to } c \geq G(x)$$

you can:

1. Try out every possible value of x.
2. Use calculus.

Since search could take forever, let's use calculus instead.

Constrained Optimization

A method for solving constrained optimization problems like

$$\max_x F(x) \text{ subject to } c \geq G(x)$$

was developed by Joseph-Louis Lagrange (France/Italy, 1736-1813) and extended by Harold Kuhn (US, 1925-2014) and Albert Tucker (US, 1905-1995).

Constrained Optimization

Associated with the problem:

$$\max_x F(x) \text{ subject to } c \geq G(x)$$

Define the Lagrangian

$$L(x, \lambda) = F(x) + \lambda[c - G(x)],$$

where λ is the Lagrange multiplier.

Constrained Optimization

Then, look for a critical point of the full Lagrangian

$$L(x, \lambda) = F(x) + \lambda[c - G(x)],$$

instead of just the objective function F by itself.

That is, use the FOC

$$F'(x^*) - \lambda^* G'(x^*) = 0.$$