Problem Set 9

ECON 337901 - Financial Economics | Peter Ireland
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For Extra Practice - Not Collected or Graded

1. Ordinal Utility

Consider a consumer who uses his or her income Y to purchase c_a apples at the price of p_a per apple and c_b bananas at the price of p_b per banana, subject to the budget constraint

$$Y \geq p_a c_a + p_b c_b.$$

Suppose that the consumer preferences over apples and bananas are described by the utility function

$$c_a^{\alpha} c_b^{1-\alpha}$$

where α, satisfying $0 < \alpha < 1$, determines how much the consumer likes apples relative to bananas.

Set up the Lagrangian for this constrained optimization problem: choose c_a and c_b to maximize the utility function subject to the budget constraint. Then, verify that the first-order conditions and budget constraint are consistent with the optimal choices

$$c_a^* = \frac{\alpha Y}{p_a}$$

and

$$c_b^* = \frac{(1-\alpha)Y}{p_b}.$$

Notice that these are the same optimal choices that you derived in answer question 1 on problem set 2, where the consumer's utility function was

$$\alpha \ln(c_a) + (1-\alpha)\ln(c_b).$$

Can you explain why these two utility functions imply the same optimizing behavior?

2. Expected Utility and Aversion to Risk

Consider an investor with von Neumann-Morgenstern expected utility function

$$U(x, y, \pi) = \pi u(W_0 + x) + (1-\pi)u(W_0 + y),$$

defined over lotteries that offer payoff x with probability π and payoff y with probability $1-\pi$, where W_0 measures the investors wealth before accepting the lottery. As we discussed in class, concavity of the Bernoulli utility function u captures the investor's aversion to risk.

To verify this with an example, suppose that the Bernoulli utility function takes the specific form

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

so that

$$u'(c) = c^{-\gamma}$$

and

$$u''(c) = -\gamma c^{-\gamma-1}.$$

This last expression for $u''(c)$ shows that the investor will be risk averse if the preference parameter γ is positive and, will become in some sense, "more risk averse" the larger is γ.

Suppose that $W_0 = 10$, so that the investor starts out with \$10, and consider the following three lotteries:

1. Lottery one has $(x, y, \pi) = (5, 0, 1/2)$, so that the investor wins \$5 with probability 1/2 and gets nothing with probability 1/2.
2. Lottery two has $(x, y, \pi) = (2.5, 0, 1)$, so that the investor gets \$2.50 with probability one, that is, for sure.
3. Lottery three has $(x, y, \pi) = (2, 0, 1)$, so that the investor gets \$2 with probability one, that is, for sure.

Using these numbers, compute and compare the values of

$$U(x, y, \pi) = \pi u(W_0 + x) + (1-\pi)u(W_0 + y) = \pi \left[\frac{(W_0 + x)^{1-\gamma}}{1-\gamma}\right] + (1-\pi)\left[\frac{(W_0 + y)^{1-\gamma}}{1-\gamma}\right]$$

for all three lotteries when the investor has $\gamma = 1/2$. Then re-do the exercise with $\gamma = 2$ and $\gamma = 3$. Looking across the results for all values of γ, does the investor ever prefer the risky bet of \$5 versus 0 to the safe option that offers the average of \$2.50 for sure? Why or why not? Then compare the risky bet of \$5 versus 0 to the safe option that offers only \$2 for sure. For which value(s) of γ does the investor prefer the risky bet to the safe choice of \$2 for sure? For which value(s) of γ does the investor prefer the safe choice of \$2 to the risky bet? For which value(s) of γ is the investor indifferent between the risky bet and \$2 for sure?