Problem Set 8

ECON 337901 - Financial Economics
Boston College, Department of Economics

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For Extra Practice - Not Collected or Graded

1. The Term Structure of Interest Rates

The *term structure of interest rates* refers to relationship between the annualized interest rates on zero-coupon bonds with different terms to maturity. Suppose the prices on discount bonds paying off \$1 for sure at maturity are selling for the prices shown in the table below:

Term to Maturity T in Years	Bond Price P_T in dollars
1	0.990
2	0.961
3	0.915
4	0.858
5	0.784
6	0.705
7	0.623
8	0.540
9	0.460
10	0.386

Using these data, compute the annualized interest rate on each of the ten discount bonds. (*Note:* If you round to the nearest percent, you should end up with nice numbers.)

2. Bond Pricing

Recall that any coupon bonds can always be regarded a portfolio of discount bonds. Using this insight, and the same data from question 1, above, find the price of a ten-year coupon bond that makes an annual interest (coupon) payment of \$100 at the end of each year, every year, for the next ten years before making a final payment of \$1000 (given by the bond's *par* or *face* value) at maturity ten years from now.

3. Forward Rates

Financial market participants define the *n-year forward rate* as the interest rate at which it is possible to arrange by contract, today, for a one-year loan in which the funds are received $n-1$ years from now and therefore repaid n years from now. It turns out that forward rates can be inferred from the term structure of interest rates on discount bonds of various maturities or, equivalently but in this case more directly, from the prices of discount bonds various maturities.

To see how, suppose you want to receive a loan of \$1 one year from now, to be repaid with interest r_2^f two years from now, where r_2^f therefore denotes two-year forward rate. The cash flows from this loan are: you receive \$1 one year from now and pay back $\$(1+r_2^f)$ two years from now. An easier way to arrange to receive \$1 one year from now is to buy a one-year discount bond today for price P_1. The problem with this strategy, however, is that it requires you to pay P_1 today, when you really don't want to make any payments until two years from now.

Suppose, however, that you simultaneously *sell* P_1/P_2 two-year discount bonds today, where P_2 is the price of a two-year discount bond today. This second transaction provides you with the $(P_1/P_2) \times P_2 = P_1$ that will offset the cost of buying the one-year discount bond today, but requires you to make a payment of P_1/P_2 two years from now, when the P_1/P_2 two-year discount bonds mature.

Now we are finished, except for one final calculation. The loan you want provides you with \$1 one year from now and requires you to pay back $\$(1+r_2^f)$ two years from now. The two bond market transactions outlined above provide you with \$1 one year from now and require you to make the payment P_1/P_2 two years from now. Thus, the two-year forward rate implied by the discount bond prices or, equivalently if you prefer, the term structure of interest rates, is given by

$$1 + r_2^f = \frac{P_1}{P_2},$$

or

$$r_2^f = \frac{P_1}{P_2} - 1.$$

Similar reasoning confirms that for any value of $n \geq 2$, the n-year forward rate is

$$r_n^f = \frac{P_{n-1}}{P_n} - 1,$$

where P_n is the price of an n-year discount bond and P_{n-1} is the price of an $(n-1)$-year discount bond. Finally, for the sake of completeness, we can note that the one-year forward rate must always equal the interest rate on a one-year discount bond, since arranging today for a one-year loan that is made today and repaid one year from now is the same as selling a one-year discount bond.

Using the same data on bond prices from question 1, above, compute the implied n-year forward rates for $n = 1$ through $n = 10$.

Note: To find the implied forward rate when $n = 1$, you need to know the price of a "zero-year discount bond," since for $n = 1$, $P_{n-1} = P_0$. Since that bond pays off \$1 "zero years from now," it actually pays off \$1 today; and since the "price" of \$1 today is \$1, you can use $P_0 = 1$ in the formula for r_1^f.