Problem Set 7

ECON 337901 - Financial Economics — Peter Ireland
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For Extra Practice - Not Collected or Graded

1. Stocks and Bonds as Portfolios of Contingent Claims

Consider an economic environment with risk in which there are two periods, $t = 0$ and $t = 1$, and two possible states at $t = 1$: a "good" state that occurs with probability $\pi = 1/2$ and a "bad" state that occurs with probability $1 - \pi = 1/2$. Suppose that, at first, investors trade two contingent claims. A contingent claim for the good state sells for $q^G = 0.30$ at $t = 0$ and pays off one dollar in the good state at $t = 1$ and zero in the bad state at $t = 1$. A contingent claim for the bad state sells for $q^B = 0.60$ at $t = 0$ and pays off one dollar in the bad state at $t = 1$ and zero in the good state at $t = 1$.

Now suppose two new assets are introduced. The first, a stock, pays a dividend of $d^G = 3$ in the good state at $t = 1$ and $d^B = 1$ in the bad state at $t = 1$. The second, a bond, pays off one dollar in both states, the good and bad, at $t = 1$.

a. Describe how an investor in this economy can form a portfolio of the two contingent claims that replicates the payoffs from the stock. How many claims of each type would the investor need to purchase? Given your answer to this question, and given the contingent claims prices $q^G = 0.30$ and $q^B = 0.60$, at what price q^s will the stock have to sell for $t = 0$ if there are to be no arbitrage opportunities across the markets for the contingent claims and the stock?

b. Next, describe how an investor in this economy can form a portfolio of the two contingent claims that replicates the payoffs from the bond. How many claims of each type would the investor need to purchase? Given your answer to this question, and given the contingent claims prices $q^G = 0.30$ and $q^B = 0.60$, at what price q^b will the bond have to sell for $t = 0$ if there are to be no arbitrage opportunities across the markets for the contingent claims and the bond?

c. Recall from class that the interest rate on the bond, which sells for q^b at $t = 0$ and pays off one dollar, for sure, at $t = 1$ can be computed as

$$r^b = \frac{1 - q^b}{q^b}.$$

Use your answer for q^b from part (b), above, together with this formula, to compute the interest rate on the bond.

2. Contingent Claims as Portfolios of Stocks and Bonds

Suppose that, instead of contingent claims, only the stock and bond described in question one trade in this economy. Exactly as above, the stock pays a dividend $d^G = 3$ in the good state at $t = 1$ and $d^B = 1$ in the bad state at $t = 1$ and the bond pays off one dollar for sure in both states, the good and bad, at $t = 1$. Suppose, too, that the stock price q^s and bond price q^b are also exactly the same as what you found in answering question one from above.

a. Describe how an investor in this economy can form a portfolio of the stock and the bond to replicate the payoffs from a contingent claim for the good state. How many shares of stock and how many bonds would the investor have to buy or sell short in order to form this portfolio? Given your answer to this question, and given the stock and bond prices that you found before, at what price q^G would the contingent claim for the good state have to sell for at $t = 0$ if there are to be no arbitrage opportunities across the markets for the stock, bond, and contingent claim?

b. Next, describe how an investor in this economy can form a portfolio of the stock and the bond to replicate the payoffs from a contingent claim for the bad state. How many shares of stock and how many bonds would the investor have to buy or sell short in order to form this portfolio? Given your answer to this question, and given the stock and bond prices that you found before, at what price q^B would the contingent claim for the bad state have to sell for at $t = 0$ if there are to be no arbitrage opportunities across the markets for the stock, bond, and contingent claim?

3. Arrow-Debreu No Arbitrage Pricing

Suppose, finally, that in addition to the stock and bond from questions 1 and 2, two more new assets are introduced. The first is another stock, which pays a dividend of $d_2^G = 4$ in the good state at $t = 1$ and a dividend of $d_2^B = 2$ in the bad state at $t = 1$. The second is an "exotic" asset, which makes a payment of 100 in the bad state at $t = 1$ but requires the holder to *make* a payment of 100 in the good state at $t = 1$.

a. From question 1, above, you know the prices q^s and q^b of the first stock and bond at $t = 0$. From question 2, above, you also know the contingent claims prices q^G and q^B implied by the prices of the stock and bond. In question 2, it should have turned out that implied contingent claims prices are the same as those originally given in question 1, namely, $q^G = 0.30$ and $q^B = 0.60$. Use this information to deduce the the price q_2^s at which the second stock will have to sell for at $t = 0$ if there are to be no arbitrage opportunities across markets for these various assets.

b. Similarly, use the information you already have about the stock, bond, and contingent claim prices to deduce the price q^A at which the exotic asset will have to sell for at $t = 0$ if there are to be no arbitrage opportunities across markets for these various assets.