Problem Set 5

In class, we saw how a consumer optimizing under risk and uncertainty solves a problem that is mathematically identical to a static (single point-in-time) problem with three goods.

$$\max_{c_0, c_1^G, c_1^B} \; u(c_0) + \beta\pi u(c_1^G) + \beta(1-\pi)u(c_1^B)$$

subject to

$$Y_0 + q^G Y_1^G + q^B Y_1^B \geq c_0 + q^G c_1^G + q^B c_1^B$$

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Problem set 5 presents a special case of this problem that highlights this: the solution procedure is similar to the one you've already used for problem set 2.

In addition, the example will help us see what Kenneth Arrow called the "role of securities" in helping consumers manage risk.

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Specialize the general problem by setting

$$u(c) = \ln(c)$$

so that more is preferred to less and so that the consumer is risk averse

Then set

$$\beta = 3/4$$

and

$$\pi = 1 - \pi = 1/2$$

These simple fractions will make the math easier.

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Set

$$Y_0 = 90, Y_1^G = 160, Y_1^B = 20$$

Notice that the consumer faces a lot of income risk: ± 70 from a base of 90

Finally, set

$$q^G = 1/4, q^B = 1/2$$

Simple fractions, but since $q^B > q^G$, goods are “scarce” in the bad state – the bad state must be bad for everyone and the high price q^B signals this to each individual consumer.

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$$\max_{c_0, c_1^G, c_1^B} \; u(c_0) + \beta \pi u(c_1^G) + \beta(1-\pi) u(c_1^B)$$

subject to

$$Y_0 + q^G Y_1^G + q^B Y_1^B \geq c_0 + q^G c_1^G + q^B c_1^B$$

$$\max_{c_0, c_1^G, c_1^B} \; \ln(c_0) + (3/4)(1/2)\ln(c_1^G) + (3/4)(1/2)\ln(c_1^B)$$

subject to

$$90 + (1/4)160 + (1/2)20 \geq c_0 + (1/4)c_1^G + (1/2)c_1^B$$

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$$\max_{c_0, c_1^G, c_1^B} \ln(c_0) + (3/4)(1/2)\ln(c_1^G) + (3/4)(1/2)\ln(c_1^B)$$

subject to

$$90 + (1/4)160 + (1/2)20 \geq c_0 + (1/4)c_1^G + (1/2)c_1^B$$

$$\max_{c_0, c_1^G, c_1^B} \ln(c_0) + (3/8)\ln(c_1^G) + (3/8)\ln(c_1^B)$$

subject to

$$140 \geq c_0 + (1/4)c_1^G + (1/2)c_1^B$$

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$$\max_{c_0, c_1^G, c_1^B} \; \ln(c_0) + (3/8)\ln(c_1^G) + (3/8)\ln(c_1^B)$$

subject to

$$140 \geq c_0 + (1/4)c_1^G + (1/2)c_1^B$$

$$\begin{aligned} L(c_0, c_1^G, c_1^B, \lambda) = \; & \ln(c_0) + (3/8)\ln(c_1^G) + (3/8)\ln(c_1^B) \\ & + \lambda[140 - c_0 - (1/4)c_1^G - (1/2)c_1^B] \end{aligned}$$

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$$L(c_0, c_1^G, c_1^B, \lambda) = \ln(c_0) + (3/8)\ln(c_1^G) + (3/8)\ln(c_1^B) + \lambda[140 - c_0 - (1/4)c_1^G - (1/2)c_1^B]$$

FOCs:

$$\frac{1}{c_0^*} - \lambda^* = 0$$

$$\frac{3/8}{c_1^{G*}} - \lambda^*(1/4) = 0$$

$$\frac{3/8}{c_1^{B*}} - \lambda^*(1/2) = 0$$

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Together with the binding constraint

$$140 = c_0^* + (1/4)c_1^{G*} + (1/2)c_1^{B*}$$

the FOCs

$$\frac{1}{c_0^*} - \lambda^* = 0$$

$$\frac{3/8}{c_1^{G*}} - \lambda^*(1/4) = 0$$

$$\frac{3/8}{c_1^{B*}} - \lambda^*(1/2) = 0$$

form a system of 4 equations in 4 unknowns: c_0^*, c_1^{G*}, c_1^{B*}, λ^*

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Rewrite

$$\frac{1}{c_0^*} - \lambda^* = 0 \Rightarrow c_0^* = \frac{1}{\lambda^*}$$

$$\frac{3/8}{c_1^{G*}} - \lambda^*(1/4) = 0 \Rightarrow c_1^{G*} = \frac{3/8}{(1/4)\lambda^*} = \frac{3}{2\lambda^*}$$

$$\frac{3/8}{c_1^{B*}} - \lambda^*(1/2) = 0 \Rightarrow c_1^{B*} = \frac{3/8}{(1/2)\lambda^*} = \frac{3}{4\lambda^*}$$

Then substitute these expressions for c_0^*, c_1^{G*}, c_1^{B*} into the budget constraint

$$140 = c_0^* + (1/4)c_1^{G*} + (1/2)c_1^{B*}$$

to find the numerical value of λ^*.

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$$140 = c_0^* + (1/4)c_1^{G*} + (1/2)c_1^{B*}$$

$$140 = \frac{1}{\lambda^*} + \frac{1}{4}\left(\frac{3}{2\lambda^*}\right) + \frac{1}{2}\left(\frac{3}{4\lambda^*}\right)$$

$$140 = \frac{1}{\lambda^*}\left(1 + \frac{3}{8} + \frac{3}{8}\right) = \frac{1}{\lambda^*}\left(\frac{14}{8}\right)$$

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The substitute the numerical value of λ^* back into

$$c_0^* = \frac{1}{\lambda^*}$$

$$c_1^{G*} = \frac{3}{2\lambda^*}$$

$$c_1^{B*} = \frac{3}{4\lambda^*}$$

to find the numerical values of c_0^*, c_1^{G*}, c_1^{B*}

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Finally, compare your solutions for c_0^*, c_1^{G*}, c_1^{B*} to the original pattern of income:

$$Y_0^* = 90$$

$$Y_1^{G*} = 160$$

$$Y_1^{B*} = 20$$

In terms of consumption, $c_1^{B*} < c_1^{G*}$, so the bad state is still bad, but not nearly as bad for consumption as for income. The consumer is using contingent claims to partially insure against the bad state.