Problem Set 4

In his 1930 book, *The Theory of Interest*, Irving Fisher extended microeconomic theory along the intertemporal dimension by reinterpreting apples c_a and bananas c_b as consumption c_0 today $(t = 0)$ and consumption c_1 next year $(t = 1)$.

Following in his footsteps, let's suppose the consumer's preferences over consumption at different dates can be described by the utility function

$$u(c_0) + \beta u(c_1)$$

where higher values of the discount factor β correspond to greater degrees of patience.

Problem Set 4

In the utility function

$$u(c_0) + \beta u(c_1)$$

$u'(c) > 0$ reflects the assumption that more is preferred to less, while $u''(c) < 0$ reflects the assumption of diminishing marginal utility and, in this case, a preference for smoothness in consumption over time (dislike of business cycles).

The special case

$$u(c) = \ln(c)$$

satisfies both of these assumptions.

Problem Set 4

As in class, let

$$Y_0 = \text{ income today}$$

$$Y_1 = \text{ income next year}$$

$$s = \text{ amount saved (or borrowed, if negative) this year}$$

Budget constraints:

$$Y_0 \geq c_0 + s$$

$$Y_1 + (1 + r)s \geq c_1$$

Problem Set 4

Again as in class, combine the two budget constraints

$$Y_0 \geq c_0 + s$$

$$Y_1 + (1 + r)s \geq c_1$$

Into a consolidated, present-value budget constraint:

$$Y_0 + \frac{Y_1}{1 + r} \geq c_0 + \frac{c_1}{1 + r}$$

and, for simplicity, set $Y_0 = Y$ and $Y_1 = 0$.

Problem Set 4

Now the consumer's problem can be stated as

$$\max_{c_0, c_1} \ln(c_0) + \beta \ln(c_1) \text{ subject to } Y \geq c_0 + \frac{c_1}{1+r}$$

Form the Lagrangian

$$L(c_0, c_1, \lambda) = \ln(c_0) + \beta \ln(c_1) + \lambda \left(Y - c_0 - \frac{c_1}{1+r} \right)$$

Problem Set 4

Use the Lagrangian

$$L(c_0, c_1, \lambda) = \ln(c_0) + \beta \ln(c_1) + \lambda \left(Y - c_0 - \frac{c_1}{1+r} \right)$$

To derive the first-order conditions:

$$\frac{1}{c_0^*} - \lambda^* = 0$$

$$\frac{\beta}{c_1^*} - \lambda^* \left(\frac{1}{1+r} \right) = 0$$

Problem Set 4

At this point, you could use the first-order conditions:

$$\frac{1}{c_0^*} = \lambda^*$$

$$\frac{\beta}{c_1^*} = \lambda^* \left(\frac{1}{1+r} \right)$$

together with the binding budget constraint

$$Y = c_0^* + \frac{c_1^*}{1+r}$$

to solve for c_0^*, c_1^*, and λ^* in terms of β, $1+r$, and Y.

Problem Set 4

Instead divide

$$\frac{1}{c_0^*} = \lambda^*$$

by

$$\frac{\beta}{c_1^*} = \lambda^* \left(\frac{1}{1+r} \right)$$

to eliminate λ^*.

Then you can see how consumption growth

$\frac{c_1^*}{c_0^*}$ depends on patience β and the interest rate $1+r$.