1. Optimal Consumption Growth

Consider a consumer who receives income $Y$ at the beginning of period $t = 0$, which he or she divides up into an amount $c_0$ to be consumed and an amount $s$ to be saved, subject to $Y \geq c_0 + s$.

Suppose that the consumer receives no additional income in period $t = 1$, so that all of his or her consumption during that period has to be purchased with the savings and the interest earned on savings from period $t = 0$. Letting $r$ denote the interest rate, this means that $(1 + r)s \geq c_1$.

As in class, we can combine these two single-period constraints to obtain the consumer’s present-value, or “lifetime,” budget constraint

$Y \geq c_0 + \frac{c_1}{1 + r}$.

Suppose, further, that the consumer’s preferences over consumption during the two periods are described by the utility function

$\ln(c_0) + \beta \ln(c_1)$,

where $\beta$, satisfying $0 < \beta < 1$, determines how patient ($\beta$ larger) or impatient ($\beta$ smaller) the consumer is. Set up the Lagrangian for this consumer’s problem: choose $c_0$ and $c_1$ to maximize the utility function subject to the lifetime budget constraint. Then use the first-order conditions to obtain an expression that shows how the consumer’s optimal choice of consumption growth $c_1^*/c_0^*$ depends on the discount factor $\beta$ and the interest rate $r$. 