

Problem Set 4

ECON 337901 - Financial Economics
Boston College, Department of Economics

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Due Tuesday, February 13

1. Optimal Consumption Growth

Consider a consumer who receives income Y at the beginning of period $t = 0$, which he or she divides up into an amount c_0 to be consumed and an amount s to be saved, subject to

$$Y \geq c_0 + s.$$

Suppose that the consumer receives no additional income in period $t = 1$, so that all of his or her consumption during that period has to be purchased with the savings and the interest earned on savings from period $t = 0$. Letting r denote the interest rate, this means that

$$(1 + r)s \geq c_1.$$

As in class, we can combine these two single-period constraints to obtain the consumer's present-value, or "lifetime," budget constraint

$$Y \geq c_0 + \frac{c_1}{1 + r}.$$

Suppose, further, that the consumer's preferences over consumption during the two periods are described by the utility function

$$\ln(c_0) + \beta \ln(c_1),$$

where β , satisfying $0 < \beta < 1$, determines how patient (β larger) or impatient (β smaller) the consumer is. Set up the Lagrangian for this consumer's problem: choose c_0 and c_1 to maximize the utility function subject to the lifetime budget constraint. Then use the first-order conditions to obtain an expression that shows how the consumer's optimal choice of consumption *growth* c_1^*/c_0^* depends on the discount factor β and the interest rate r .