Problem Set 3

Problem set 3 confirms that the graphical and algebraic approaches to consumer optimization lead to the same solutions.

As, of course, they should!

Problem Set 3

In problem set 2, you found the solution to the problem

$$\max_{c_a, c_b} \; \alpha \ln(c_a) + (1-\alpha)\ln(c_b) \text{ subject to } Y \geq p_a c_a + p_b c_b$$

is

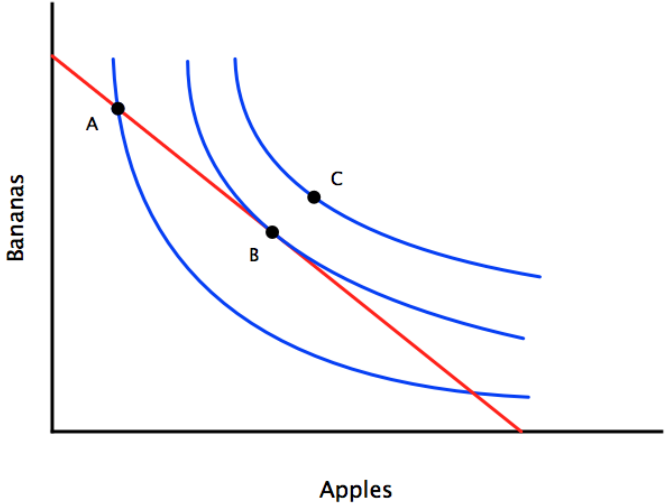
$$c_a^* = \frac{\alpha Y}{p_a}$$

$$c_b^* = \frac{(1-\alpha) Y}{p_b}$$

using the method of Lagrange multipliers.

Now let's use the graph to obtain the same solutions.

Problem Set 3



Problem Set 3

The tangency point B lies on the budget constraint:

$$c_b^* = \frac{Y}{p_b} - \left(\frac{p_a}{p_b}\right) c_a^*$$

In addition, at B, the slope of the budget constraint equals the slope of the indifference curve:

$$-\frac{p_a}{p_b} = -\frac{MU_a}{MU_b} = -\frac{\alpha / c_a^*}{(1-\alpha)/c_b^*}$$

Problem Set 3

The tangency condition implies

$$\frac{p_a}{p_b} = \frac{\alpha / c_a^*}{(1-\alpha)/c_b^*} = \frac{\alpha}{c_a^*} \times \frac{c_b^*}{1-\alpha} = \left(\frac{\alpha}{1-\alpha}\right)\frac{c_b^*}{c_a^*}$$

$$(1-\alpha) p_a c_a^* = \alpha p_b c_b^*$$

Problem Set 3

Now take

$$(1-\alpha)p_a c_a^* = \alpha p_b c_b^*$$

and substitute in the budget constraint

$$c_b^* = \frac{Y}{p_b} - \left(\frac{p_a}{p_b}\right) c_a^*$$

$$(1-\alpha)p_a c_a^* = \alpha p_b \left[\frac{Y}{p_b} - \left(\frac{p_a}{p_b}\right) c_a^*\right]$$

Problem Set 3

Use

$$(1-\alpha)p_a c_a^* = \alpha p_b \left[\frac{Y}{p_b} - \left(\frac{p_a}{p_b}\right) c_a^*\right]$$

to find c_a^* in terms of p_a, Y, and α.

Then use

$$c_b^* = \frac{Y}{p_b} - \left(\frac{p_a}{p_b}\right) c_a^*$$

to find c_b^* in terms of p_b, Y, and $1-\alpha$.

Your answers should be the same as in problem set 2!