Problem Set 3

ECON 337901 - Financial Economics
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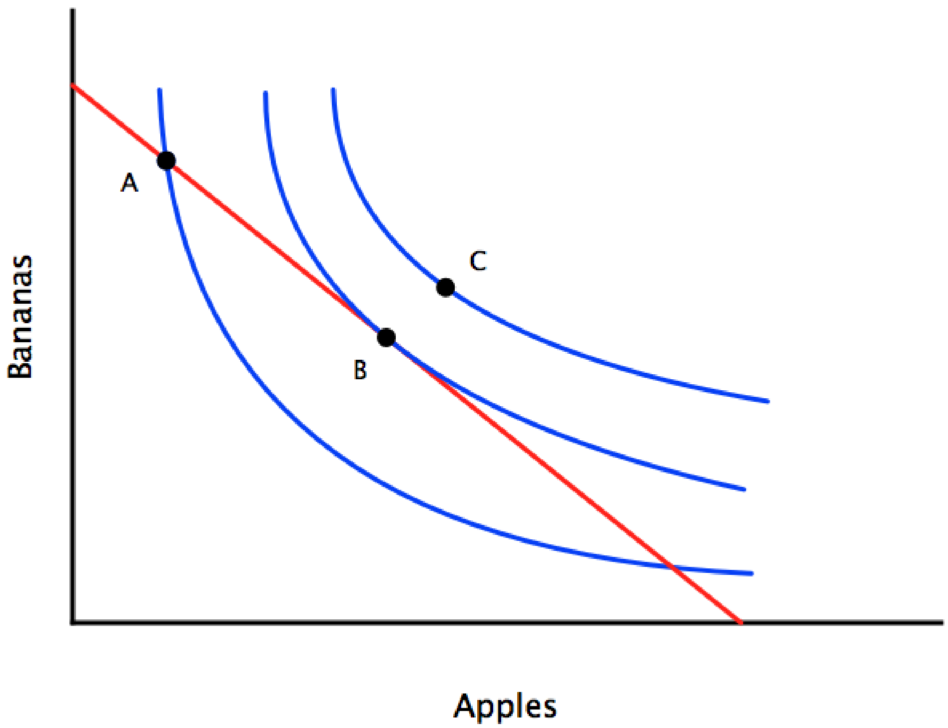
For Extra Practice - Not Collected or Graded

1. Consumer Optimization

The figure below illustrates the familiar tangency condition between an optimizing consumer's indifference curve and budget constraint. In the graph, bundle C is infeasible (unaffordable), bundle A is suboptimal, and bundle B is optimal.



Let c_a^* and c_b^* denote the number of apples and bananas included in the optimal bundle B. From the graph, there are two conditions that these quantities should satisfy.

First, c_a^* and c_b^* are on the consumer's budget constraint. This means that

$$c_b^* = \frac{Y}{p_b} - \left(\frac{p_a}{p_b}\right) c_a^*, \tag{1}$$

where Y is the consumer's income and p_a and p_b are the prices of apples and bananas.

Second, the slope of the indifference curve equations the slope of the budget constraint. The slope of the budget constraint is $-(p_a/p_b)$. Suppose that consumer's utility function takes the form

$$\alpha u(c_a) + (1-\alpha)u(c_b),$$

where α satisfies $0 < \alpha < 1$ and measures how much the consumer likes apples relative to bananas. With this utility function, the slope of the indifference curve equals the marginal rate of substitution

$$-\frac{\alpha u'(c_a^*)}{(1-\alpha)u'(c_b^*)}.$$

If the consumer prefers more to less and has a preference for variety, then the function $u(c)$ that measures utility from each of the two goods separately should be increasing and concave. A convenient choice that has both of these properties is the natural log function $u(c) = \ln(c)$. With this function, the marginal rate of substitution becomes, more simply,

$$-\frac{\alpha c_b^*}{(1-\alpha)c_a^*}.$$

This last expression lets us write the condition summarizing the equality of the slopes of the indifference curve and budget constraint as

$$-\frac{p_a}{p_b} = -\frac{\alpha c_b^*}{(1-\alpha)c_a^*},$$

or, canceling the minus signs and cross-multiplying to get rid of the fractions,

$$(1-\alpha)p_a c_a^* = \alpha p_b c_b^*. \tag{2}$$

Equations (1) and (2) form a system of two equations that can be used to solve for the optimal consumptions c_a^* and c_b^* in terms of income Y, the prices p_a and p_b, and the weight α from the utility function. Although there are many ways of doing this, probably the easiest is to substitute the expression for c_b^* given by equation (1) into the right-hand side of equation (2), and solve algebraically for c_a^*. Then, you can substitute this solution for c_a^* back into (1) to find the solution for c_b^*. Try it out and see. Because the special utility function here is the same as the one from question 2 on the first problem set, your answers should turn out to be the same, too.