

Problem Set 3

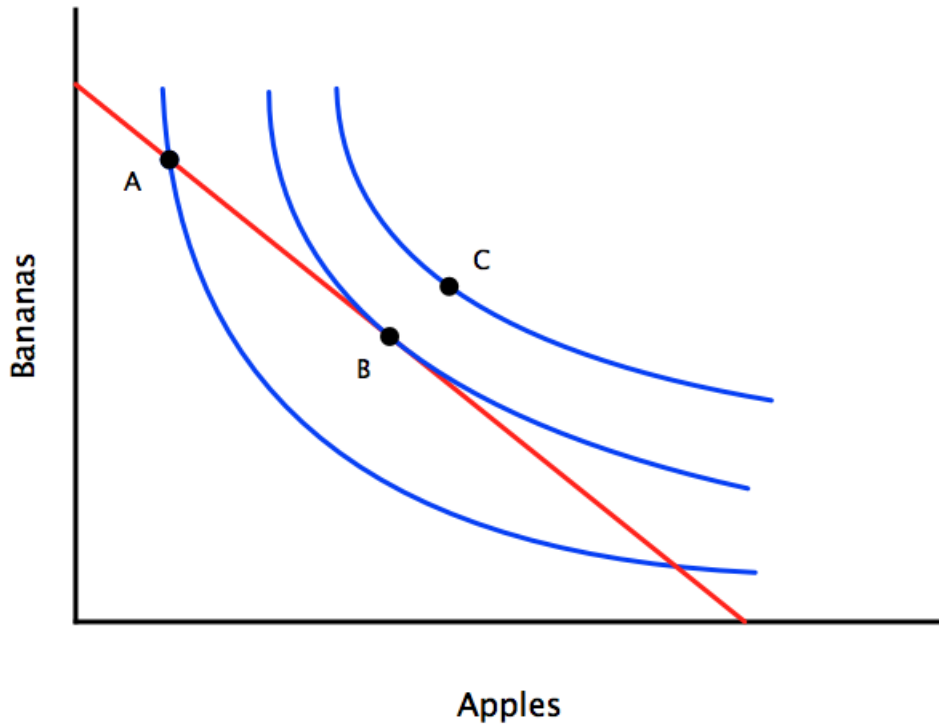
ECON 337901 - Financial Economics
Boston College, Department of Economics

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Spring 2023

For Extra Practice - Not Collected or Graded

1. Consumer Optimization

The figure below illustrates the familiar tangency condition between an optimizing consumer's indifference curve and budget constraint. In the graph, bundle C is infeasible (unaffordable), bundle A is suboptimal, and bundle B is optimal.



Let c_a^* and c_b^* denote the number of apples and bananas included in the optimal bundle B. From the graph, there are two conditions that these quantities should satisfy.

First, c_a^* and c_b^* are on the consumer's budget constraint. This means that

$$c_b^* = \frac{Y}{p_b} - \left(\frac{p_a}{p_b}\right) c_a^*, \quad (1)$$

where Y is the consumer's income and p_a and p_b are the prices of apples and bananas.

Second, the slope of the indifference curve equations the slope of the budget constraint. The slope of the budget constraint is $-(p_a/p_b)$. Suppose that consumer's utility function takes

the form

$$\alpha u(c_a) + (1 - \alpha)u(c_b),$$

where α satisfies $0 < \alpha < 1$ and measures how much the consumer likes apples relative to bananas. With this utility function, the slope of the indifference curve equals the marginal rate of substitution

$$-\frac{\alpha u'(c_a^*)}{(1 - \alpha)u'(c_b^*)}.$$

If the consumer prefers more to less and has a preference for variety, then the function $u(c)$ that measures utility from each of the two goods separately should be increasing and concave. A convenient choice that has both of these properties is the natural log function $u(c) = \ln(c)$. With this function, the marginal rate of substitution becomes, more simply,

$$-\frac{\alpha c_b^*}{(1 - \alpha)c_a^*}.$$

This last expression lets us write the condition summarizing the equality of the slopes of the indifference curve and budget constraint as

$$-\frac{p_a}{p_b} = -\frac{\alpha c_b^*}{(1 - \alpha)c_a^*},$$

or, canceling the minus signs and cross-multiplying to get rid of the fractions,

$$(1 - \alpha)p_a c_a^* = \alpha p_b c_b^*. \tag{2}$$

Equations (1) and (2) form a system of two equations that can be used to solve for the optimal consumptions c_a^* and c_b^* in terms of income Y , the prices p_a and p_b , and the weight α from the utility function. Although there are many ways of doing this, probably the easiest is to substitute the expression for c_b^* given by equation (1) into the right-hand side of equation (2), and solve algebraically for c_a^* . Then, you can substitute this solution for c_a^* back into (1) to find the solution for c_b^* . Try it out and see. Because the special utility function here is the same as the one from problem set 2, your answers should turn out to be the same, too.