

Problem Set 2

Problem set 2 presents an economic example that will give you practice solving a constrained optimization problem.

To make the problem more interesting, there will be two choice variables instead of just one.

And the results will give us a preview of consumer theory, which will be our focus next.

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Y = income

c_a, c_b = consumption of apples and bananas

p_a, p_b = prices of apples and bananas

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Budget constraint

$$Y \geq p_a c_a + p_b c_b$$

Utility:

$$\alpha \ln(c_a) + (1 - \alpha) \ln(c_b)$$

$0 \leq \alpha \leq 1$ is the weight on apples relative to bananas in the consumer's preferences

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The problem:

$$\max_{c_a, c_b} \alpha \ln(c_a) + (1 - \alpha) \ln(c_b) \text{ subject to } Y \geq p_a c_a + p_b c_b$$

The Lagrangian:

$$L(c_a, c_b, \lambda) = \alpha \ln(c_a) + (1 - \alpha) \ln(c_b) + \lambda(Y - p_a c_a - p_b c_b)$$

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The Lagrangian:

$$L(c_a, c_b, \lambda) = \alpha \ln(c_a) + (1 - \alpha) \ln(c_b) + \lambda(Y - p_a c_a - p_b c_b)$$

With two choice variables there will be two first-order conditions, each derived in the same way: by differentiating the Lagrangian by one choice variable, holding the other constant.

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$$L(c_a, c_b, \lambda) = \alpha \ln(c_a) + (1 - \alpha) \ln(c_b) + \lambda(Y - p_a c_a - p_b c_b)$$

FOC for c_a :

$$\frac{\alpha}{c_a^*} - \lambda^* p_a = 0$$

FOC for c_b :

$$\frac{1 - \alpha}{c_b^*} - \lambda^* p_b = 0$$

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The FOCs

$$\frac{\alpha}{c_a^*} - \lambda^* p_a = 0$$

$$\frac{1 - \alpha}{c_b^*} - \lambda^* p_b = 0$$

form a system of 2 equations in 3 unknowns: c_a^* , c_b^* , and λ^* .

Since the utility function $\alpha \ln(c_a) + (1 - \alpha) \ln(c_b)$ implies that “more is preferred to less,” we know in advance that the budget constraint will bind, providing a third equation

$$Y = p_a c_a^* + p_b c_b^*.$$

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In general, there are many ways to solve the three-equation system

$$\frac{\alpha}{c_a^*} - \lambda^* p_a = 0$$

$$\frac{1 - \alpha}{c_b^*} - \lambda^* p_b = 0$$

$$Y = p_a c_a^* + p_b c_b^*.$$

All will lead to the same solution!

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One approach is to use the FOCs to solve for c_a^* and c_b^* in terms of λ^* :

$$\frac{\alpha}{c_a^*} - \lambda^* p_a = 0 \Rightarrow c_a^* = \frac{\alpha}{\lambda^* p_a}$$

$$\frac{1 - \alpha}{c_b^*} - \lambda^* p_b = 0 \Rightarrow c_b^* = \frac{1 - \alpha}{\lambda^* p_b}$$

Then substitute these expressions for c_a^* and c_b^* into the binding constraint

$$Y = p_a c_a^* + p_b c_b^*.$$

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If you do this, you'll find that λ^* depends only on Y !

Then take your solution for λ^* and substitute it back into the two FOCs

$$c_a^* = \frac{\alpha}{\lambda^* p_a}$$

$$c_b^* = \frac{1 - \alpha}{\lambda^* p_b}$$

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Now you'll find that c_a^* depends on p_a , Y , and α .

And c_b^* depends on p_b , Y , and $1 - \alpha$.

These solutions for c_a^* and c_b^* are the (Marshallian) demand curves for apples and bananas.

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Now you'll find that c_a^* depends on p_a , Y , and α .

And c_b^* depends on p_b , Y , and $1 - \alpha$.

Both goods are “ordinary” goods (not Giffen goods): when the price goes up, quantity demanded goes down.

Problem Set 2

Now you'll find that c_a^* depends on p_a , Y , and α .

And c_b^* depends on p_b , Y , and $1 - \alpha$.

Both goods are “normal” goods: when the income goes up, quantity demanded goes up.

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Finally, use your solutions to see how

$$\frac{p_a c_a^*}{Y} = \text{share of income spent on apples}$$

depends on α .

And

$$\frac{p_b c_b^*}{Y} = \text{share of income spent on bananas}$$

depends on $1 - \alpha$.