Problem Set 2

ECON 337901 - Financial Economics — Peter Ireland
Boston College, Department of Economics — Spring 2024

Due Tuesday, February 6

1. Utility Maximization

Consider a consumer who uses his or her income Y to purchase c_a apples at the price of p_a per apple and c_b bananas at the price of p_b per banana, subject to the budget constraint

$$Y \geq p_a c_a + p_b c_b.$$

Suppose that the consumer's preferences over apples and bananas are described by the utility function

$$\alpha \ln(c_a) + (1 - \alpha) \ln(c_b)$$

where $\ln(c)$ denotes the natural logarithm of c and α, satisfying $0 < \alpha < 1$, determines how much the consumer likes apples relative to bananas. Note that the choice of the function $u(c) = \ln(c)$ to help describe how much utility the consumer gets from each good implies that $u'(c) = 1/c$, an expression that will help in solving for consumer's optimal choices.

Set up the Lagrangian for this constrained optimization problem: choose c_a and c_b to maximize the utility function subject to the budget constraint. Then, using the first-order conditions together with the budget constraint, see if you can obtain equations that show how the consumer's optimal choices c_a^* and c_b^* depend on his or her income Y as well as the prices p_a and p_b and the parameter α from the utility function. Note that to do this, you will also have to find an equation that shows how the value λ^* of the Lagrange multiplier associated with the solution to the consumer's problem depends on Y and/or p_a, p_b, and α. Using these equations, do you notice any relationship between $p_a c_a^*/Y$ and $p_b c_b^*/Y$, measuring the fractions of income that the consumer optimally spends on apples and bananas, and the weights α and $1 - \alpha$ from the utility function?