1. The Gains From Diversification

Consider portfolios formed from two risky assets, the first with expected return equal to \( \mu_1 = 8 \) and standard deviation of its return equal to \( \sigma_1 = 8 \) and the second with expected return equal to \( \mu_2 = 4 \) and standard deviation of its return equal to \( \sigma_2 = 4 \). Let \( w \) denote the fraction of wealth in the portfolio allocated to asset 1 and \( 1 - w \) the corresponding fraction of wealth allocated to asset 2. Suppose first that there is zero correlation between the two returns, so that \( \rho_{12} = 0 \), and compute the expected return on the portfolio and the standard deviation of the return on the portfolio for values of \( w \) equal to 0, 0.2, 0.4, 0.6, 0.8, and 1. Then repeat the calculations with \( \rho_{12} = -0.50 \). For which of these two values of \( \rho_{12} \) are the gains from diversification larger?

2. The Efficient Frontier

Suppose that in addition to the two risky assets from question 1, above, we add a third, with expected return equal to \( \mu_3 = 6 \) and standard deviation of its return equal to \( \sigma_3 = 6 \). Obviously, a 6 percent expected return can be achieved with a standard deviation of 6 percent simply by holding this third asset. The question is how much better one can do, in terms of a lower standard deviation, by holding the optimal diversified portfolio. To find out, suppose that the three asset returns are uncorrelated, so that \( \rho_{12} = \rho_{13} = \rho_{23} = 0 \). Then consider the problem: choose fractions \( w_1, w_2 \) and \( 1 - w_1 - w_2 \) of a portfolio allocated to assets 1, 2, and 3 in order to maximize \(-\sigma_P^2\), minus one times the variance of the portfolio’s return, subject to the constraint that \( \mu_P \), the expected return on the portfolio, equals \( \bar{\mu} = 6 \). In class, we derived the first-order conditions for this problem. Use those first-order conditions together with the constraint to solve for the optimal choices of \( w_1, w_2 \) and \( 1 - w_1 - w_2 \). Then calculate the minimized value of \( \sigma_P \). How does this minimized standard deviation compare to standard deviation \( \sigma_3 = 6 \) that an investor would have to accept by holding asset 3 alone?