Problem Set 12

ECON 337901 - Financial Economics — Peter Ireland
Boston College, Department of Economics — Spring 2020

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1. The Gains From Diversification

Consider portfolios formed from two risky assets, the first with expected return equal to $\mu_1 = 8$ and standard deviation of its return equal to $\sigma_1 = 8$ and the second with expected return equal to $\mu_2 = 4$ and standard deviation of its return equal to $\sigma_2 = 4$. Let w denote the fraction of wealth in the portfolio allocated to asset 1 and $1 - w$ the corresponding fraction of wealth allocated to asset 2. Suppose first that there is zero correlation between the two returns, so that $\rho_{12} = 0$, and compute the expected return on the portfolio and the standard deviation of the return on the portfolio for values of w equal to 0, 0.2, 0.4, 0.6, 0.8, and 1. Then repeat the calculations with $\rho_{12} = -0.50$. For which of these two values of ρ_{12} are the gains from diversification larger?

2. The Efficient Frontier

Suppose that in addition to the two risky assets from question 1, above, we add a third, with expected return equal to $\mu_3 = 6$ and standard deviation of its return equal to $\sigma_3 = 6$. Obviously, a 6 percent expected return can be achieved with a standard deviation of 6 percent simply by holding this third asset. The question is how much better one can do, in terms of a lower standard deviation, by holding the optimal diversified portfolio. To find out, suppose that the three asset returns are uncorrelated, so that $\rho_{12} = \rho_{13} = \rho_{23} = 0$. Then consider the problem: choose fractions w_1, w_2 and $1 - w_1 - w_2$ of a portfolio allocated to assets 1, 2, and 3 in order to maximize $-\sigma_P^2$, minus one times the variance of the portfolio's return, subject to the constraint that μ_P, the expected return on the portfolio, equals $\bar{\mu} = 6$. In class, we derived the first-order conditions for this problem. Use those first-order conditions together with the constraint to solve for the optimal choices of w_1, w_2 and $1 - w_1 - w_2$. Then calculate the minimized value of σ_P. How does this minimized standard deviation compare to standard deviation $\sigma_3 = 6$ that an investor would have to accept by holding asset 3 alone?