Problem Set 10

ECON 337901 - Financial Economics
Boston College, Department of Economics

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For Extra Practice - Not Collected or Graded

1. Interpreting Measures of Risk Aversion

Consider an investor who has vN-M expected utility with Bernoulli utility function

$$u(Y) = \frac{Y^{1-\gamma} - 1}{1-\gamma},$$

where, as we now know, $\gamma > 0$ represents the coefficient of relative risk aversion.

Suppose that the investor's initial wealth is $Y_0 = 10$ and that he or she is confronted with the lottery $(0.1, -0.1, \pi)$. For values of γ equal to 1/2, 2, 3, 10, and 20, compute the exact value of π^* that makes the investor indifferent between accepting or rejecting the bet. Recall from class that this value of π^* can be approximated by

$$\pi^* \approx \frac{1}{2} + \frac{1}{4}\left[-\frac{Yu''(Y)}{u'(Y)}\right]k$$

where k is the size of the bet as a fraction of initial wealth: 0.01, or one percent, in this case. Compare your exact answers to the values implied by the formula to see how well the approximation works.

2. Insurance, Part I

Suppose that you own a business worth \$100000. With probability $\pi_1 = 0.05$, a disaster – a fire, let's say – occurs that reduces the value of the business to \$50000. Let x denote the premium on an insurance policy that will protect you fully against that loss.

Your choices are as follows. You can take out the insurance policy, in which case your wealth will be $\$(100000 - x)$ no matter what: you'll have to pay the premium of x up front, but the insurance company will pay you \$50000 to compensate for the loss if it occurs. Or you can forego buying insurance and take your chances, in which case your wealth will be \$100000 with probability 0.95 and \$50000 with probability 0.05.

Assuming that your preferences are described by a vN-M expected utility function with logarithmic Bernoulli utility function

$$u(Y) = \ln(Y),$$

what is the maximum premium x that you will be willing to pay for the insurance policy? (*Note:* To solve this problem, you may need to use the fact that if $x = \ln(y)$, then $y = \exp(x)$. That is, the exponential function is the inverse function of the natural logarithm.

3. Insurance, Part II

Extending the example from question 2, above, suppose that in addition to the 5 percent chance of fire, there is an even smaller probability of an even bigger disaster: a flood, let's say, that occurs with probability $\pi_2 = 0.01$ but reduces the value of your business to \$1.

Your choices are now as follows. You can take out the insurance policy, in which case your wealth will be $\$(100000 - x)$ no matter what. Or you can forgo buying insurance, in which case your wealth will be \$100000 with probability 0.94, \$50000 with probability 0.05, and \$1 with probability 0.01.

Still assuming you have vN-M expected utility with logarithmic Bernoulli utility function, what is the maximum premium x that you will be willing to pay for insurance now?