Problem Set 1

ECON 337901 - Financial Economics — Peter Ireland
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Due Tuesday, January 29

1. Profit Maximization

Consider a firm that hires n workers in order to produce y units of output according to the production function

$$y = n^{\alpha},$$

where α is a number between zero and one: $0 < \alpha < 1$. The restriction that $\alpha > 0$ guarantees that the firm can always produce more output by hiring more workers. Meanwhile, the restriction that $\alpha < 1$ implies that the firm faces diminishing returns to scale: the more workers the firm has already hired, the less output it can produce by hiring yet another worker. Let w denote the wage that the firm must pay each worker in a competitive labor market. Then the firm chooses n to maximize profits, defined as usual to equal revenues minus costs:

$$\max_{n} \; n^{\alpha} - wn.$$

This is an unconstrained optimization problem, with an objective function $F(n) = n^{\alpha} - wn$ that is concave, so that the first-order condition is both necessary and sufficient for the value n^* that maximizes profits.

Write down the first-order condition for n^*, using the rules of differentiation to find $F'(n^*)$ for this example. Then, see if you can rearrange the first-order condition to get an equation that shows how the firm's optimal choice n^* depends on the wage w and the parameter α from the production function. Given the assumptions that $0 < \alpha < 1$, see if you can tell whether the firm hires more or fewer workers when the wage w goes up.

2. Farming

Now consider a farmer, who works h hours in order to produce c units of food for his or her own consumption according to the production function

$$c = h^{\alpha},$$

where, as in question 1 above, α is a number between zero and one: $0 < \alpha < 1$. Suppose that the farmer gets utility from consuming more, but also suffers disutility from working; assume, in particular, that this trade-off is summarized by the utility function

$$\ln(c) - \beta h,$$

where $\ln(c)$ denotes the natural logarithm and $\beta > 0$ is a positive number that measures how much the farmer dislikes working. By substituting the production function into the utility function, we can characterize the farmer's optimal choice of hours worked by solving the problem

$$\max_{h} \ \alpha \ln(h) - \beta h.$$

This is another unconstrained optimization problem with an objective function that is concave, so that the first-order condition is both necessary and sufficient for the value h^* that maximizes the farmer's utility.

To derive the first-order condition for h^*, you'll need to use the rule for differentiating the natural log function: if $f(x) = \ln(x)$, then $f'(x) = 1/x$. After writing down this first-order condition, see if you can rearrange it to get an equation that shows how the farmer's optimal choice of h^* depends on the parameters α and β. Remember that when β rises, the farmer has a stronger distaste for working. Can you tell from your solution whether h^* goes up or down when β rises?