Solutions to Problem Set 9

ECON 337901 - Financial Economics — Peter Ireland
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Due Thursday, April 2

1. Expected Utility and Aversion to Risk

With von Neumann-Morgenstern expected utility function

$$U(x, y, \pi) = \pi u(W_0 + x) + (1 - \pi)u(W_0 + y) = \pi \left[\frac{(W_0 + x)^{1-\gamma}}{1 - \gamma} \right] + (1 - \pi) \left[\frac{(W_0 + y)^{1-\gamma}}{1 - \gamma} \right]$$

and $W_0 = 10$, the table below compares the three lotteries $(x, y, \pi) = (5, 0, 1/2)$, $(x, y, \pi) = (2.5, 0, 1)$ and $(x, y, \pi) = (2, 0, 1)$ when $\gamma = 1/2$, $\gamma = 2$ and $\gamma = 3$.

γ	$U(5, 0, 1/2)$	$U(2.5, 0, 1)$	$U(2, 0, 1)$
1/2	7.0353	7.0711	6.9282
2	−0.0833	−0.0800	−0.0833
3	−0.0036	−0.0032	−0.0035

For all values of γ, the investor always prefers getting the average of 2.5 for sure to the alternative of 5 with probability 1/2 and 0 with probability 1/2. This first set of comparisons shows us once again how the concavity of the Bernoulli utility function represents the investor's aversion to risk. On the other hand, even a risk averse investor will be willing to accept gambles when the safer alternative offers less than the expected value of the bet. In this case, the investor with $\gamma = 1/2$ prefers the risky bet to receiving 2 for sure; the investor with $\gamma = 2$ is indifferent between the two options, and the investor with $\gamma = 3$ prefers receiving 2 for sure. This second set of comparisons suggests that γ is a measure of risk aversion, with higher values of γ implying more risk averse behavior.