

Solutions to Problem Set 9

ECON 337901 - Financial Economics
Boston College, Department of Economics

Peter Ireland
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For Extra Practice - Not Collected or Graded

1. Interpreting Measures of Risk Aversion

With von Neumann-Morgenstern expected utility and Bernoulli utility function

$$u(Y) = \frac{Y^{1-\gamma} - 1}{1 - \gamma}$$

and initial wealth $Y = 10$, the probability π^* that makes the investor indifferent between accepting a rejecting a bet over one percent of income can be approximated by

$$\pi^* \approx \frac{1}{2} + \frac{\gamma}{400}.$$

The exact value of π^* , meanwhile, is given by

$$\pi^* = \frac{10^{1-\gamma} - 9.9^{1-\gamma}}{10.1^{1-\gamma} - 9.9^{1-\gamma}}.$$

The table below shows the exact and approximate values of π^* for five values of the relative risk aversion parameter γ . Results from the calculations confirm that more risk averse investors require a higher value of π^* and also confirm the accuracy of the approximation:

γ	Exact π^*	Approximate π^*
1/2	0.5013	0.5012
2	0.5050	0.5050
3	0.5075	0.5075
10	0.5250	0.5250
20	0.5498	0.5500

2. Insurance, Part I

With logarithmic Bernoulli utility function, your utility with the insurance policy is

$$\ln(100000 - x),$$

and your expected utility without insurance is

$$0.95 \ln(100000) + 0.05 \ln(50000).$$

Equating these two values to find the value of x^* that leaves you indifferent between buying and not buying insurance yields

$$\ln(100000 - x^*) = 0.95 \ln(100000) + 0.05 \ln(50000).$$

Hence

$$100000 - x^* = \exp [0.95 \ln(100000) + 0.05 \ln(50000)]$$

or

$$x^* = 100000 - \exp [0.95 \ln(100000) + 0.05 \ln(50000)] = 3406.37,$$

implying that you would be willing to pay a premium of up to \$3406.37 on the insurance policy.

3. Insurance, Part II

With the addition 1 percent chance of the biggest disaster, x^* will have to satisfy

$$\ln(100000 - x^*) = 0.94 \ln(100000) + 0.05 \ln(50000) + 0.01 \ln(1).$$

Since $\ln(1) = 0$,

$$100000 - x^* = \exp [0.94 \ln(100000) + 0.05 \ln(50000)]$$

or

$$x^* = 100000 - \exp [0.94 \ln(100000) + 0.05 \ln(50000)] = 13910.83,$$

implying that you would be willing to pay a premium of up to \$13910.83 on the insurance policy.