**Solutions to Problem Set 8**

ECON 337901 - Financial Economics — Peter Ireland
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For Extra Practice - Not Collected or Graded

## 1. The Term Structure of Interest Rates

Recall from class that the annualized interest rate $r_T$ on a $T$-year discount bond is related to the bond price $P_T$ via

$$r_T = \left(\frac{1}{P_T}\right)^{1/T} - 1.$$

Rounding to the nearest percentage point as suggested, the data from the table imply

| Term to Maturity $T$ in Years | Bond Price $P_T$ in Dollars | Interest Rate $r_T$ in Percent |
|---|---|---|
| 1 | 0.990 | 1 |
| 2 | 0.961 | 2 |
| 3 | 0.915 | 3 |
| 4 | 0.858 | 4 |
| 5 | 0.784 | 5 |
| 6 | 0.705 | 6 |
| 7 | 0.623 | 7 |
| 8 | 0.540 | 8 |
| 9 | 0.460 | 9 |
| 10 | 0.386 | 10 |

## 2. Bond Pricing

The coupon bond makes an annual coupon payment of \$100 each year for the next 10 years and returns its \$1000 face value upon maturity 10 years from now. There are two, fully equivalent ways of finding the price of this bond based on the data from question 1, above.

The first approach recognizes that the payoffs provided by this bond are the same as those provided by a portfolio consisting of 100 discount bonds of each maturity ranging from $T = 1$ to $T = 9$ and 1100 discount bonds with maturity $T = 10$. Hence, letting $P^C$ be the price of the coupon bond:

$$\begin{aligned} P^C &= 100 \times P_1 + 100 \times P_2 + 100 \times P_3 + 100 \times P_4 + 100 \times P_5 \\ &\quad + 100 \times P_6 + 100 \times P_7 + 100 \times P_8 + 100 \times P_9 + 1100 \times P_{10} \end{aligned}$$

Plugging in the prices $P_1$ through $P_{10}$ from the table yields $P^C = 1108.2$.

The second approach is to compute the coupon bond's price as the present discounted value of its future payments using the interest rates $r_1$ through $r_{10}$ that we calculated in answering question 1:

$$
\begin{aligned}
P^C &= \frac{100}{1+r_1} + \frac{100}{(1+r_2)^2} + \frac{100}{(1+r_3)^3} + \frac{100}{(1+r_4)^4} + \frac{100}{(1+r_5)^5} \\
&\quad + \frac{100}{(1+r_6)^6} + \frac{100}{(1+r_7)^7} + \frac{100}{(1+r_8)^8} + \frac{100}{(1+r_9)^9} + \frac{1100}{(1+r_{10})^{10}}
\end{aligned}
$$

Plugging in the interest rates $r_1$ through $r_{10}$ from the table yields $P^C = 1107.4$, but this slightly different answer reflects the fact that the interest rates were rounded to the nearest percent; using the interest rates implied by the formula without rounding yields $P^C = 1108.2$, just as before.

**3. Forward Rates**

The $n$-year forward rate can be inferred from the prices $P_n$ of an $n$-year discount bond and $P_{n-1}$ of an $n-1$-year discount bond using the formula

$$r_n^f = \frac{P_{n-1}}{P_n} - 1.$$

After rounding to the nearest tenth of a percent, the forward rates calculated using data from the table are as follows:

| Term to Maturity $T$ in Years | Bond Price $P_T$ in Dollars | Interest Rate $r_T$ in Percent | n-Year Forward Rate $r_n^f$ for $n = T$ in Percent |
|---|---|---|---|
| 1 | 0.990 | 1 | 1.0 |
| 2 | 0.961 | 2 | 3.0 |
| 3 | 0.915 | 3 | 5.0 |
| 4 | 0.858 | 4 | 6.6 |
| 5 | 0.784 | 5 | 9.4 |
| 6 | 0.705 | 6 | 11.2 |
| 7 | 0.623 | 7 | 13.2 |
| 8 | 0.540 | 8 | 15.4 |
| 9 | 0.460 | 9 | 17.4 |
| 10 | 0.386 | 10 | 19.2 |

The table reveals that, in this case, expectations of rising interest rates are reflected in both the term structure and in the pattern of forward rates.