Solutions to Problem Set 5

ECON 337901 - Financial Economics — Peter Ireland
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1. Consumer Optimization Under Uncertainty

The Lagrangian for the consumer's problem

$$L = \ln(c_0) + \left(\frac{3}{8}\right)\ln(c_1^G) + \left(\frac{3}{8}\right)\ln(c_1^B) + \lambda\left[140 - c_0 - \left(\frac{1}{4}\right)c_1^G - \left(\frac{1}{2}\right)c_1^B\right]$$

leads to the first-order conditions

$$\frac{1}{c_0^*} - \lambda^* = 0,$$

$$\frac{3}{8c_1^{G*}} - \frac{\lambda^*}{4} = 0,$$

and

$$\frac{3}{8c_1^{B*}} - \frac{\lambda^*}{2} = 0.$$

Rewrite these three conditions as

$$c_0^* = \frac{1}{\lambda^*},$$

$$c_1^{G*} = \frac{3}{2\lambda^*},$$

and

$$c_1^{B*} = \frac{3}{4\lambda^*},$$

and substitute them expressions into the binding budget constraint to obtain

$$140 = \frac{1}{\lambda^*} + \frac{3}{8\lambda^*} + \frac{3}{8\lambda^*} = \frac{8+3+3}{8\lambda^*} = \frac{14}{8\lambda^*},$$

which implies that

$$\lambda^* = \frac{1}{80} \text{ or } \frac{1}{\lambda^*} = 80.$$

Substituting this solution for λ^* back into the previous expressions for c_0^*, c_1^{G*}, and c_1^{B*} provides the solutions

$$c_0^* = 80,$$

$$c_1^{G*} = 120,$$

and

$$c_1^{B*} = 60.$$

Finally, comparing these solutions to the original values for income $Y_0 = 90$, $Y_1^G = 160$, and $Y_1^B = 20$ shows that the consumer does succeed, to some extent, in smoothing out his or her consumption relative to income, both over time and across states. In particular, by consuming less today and in the good state next year, the consumer is able to consume more in the bad state next year.