Solutions to Problem Set 4

ECON 337901 - Financial Economics
Boston College, Department of Economics

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1. Optimal Consumption Growth

The Lagrangian for the consumer's problem

$$L = \ln(c_0) + \beta \ln(c_1) + \lambda \left(Y - c_0 - \frac{c_1}{1+r} \right)$$

leads to the first-order conditions

$$\frac{1}{c_0^*} - \lambda^* = 0$$

and

$$\frac{\beta}{c_1^*} - \frac{\lambda^*}{1+r} = 0.$$

Rewrite the first-order condition for c_0^* as

$$\lambda_0^* = \frac{1}{c_0^*}$$

and substitute this expression for λ^* into the first-order condition for c_1^* to obtain

$$\frac{\beta}{c_1^*} = \frac{1}{(1+r)c_0^*}$$

or

$$\frac{c_1^*}{c_0^*} = \beta(1+r).$$

This last expression shows how the optimal rate of consumption growth depends on the discount factor β and the interest rate r. Intuitively, the consumer postpones consumption from period $t = 0$ and $t = 1$, so that consumption grows faster, when β is larger, so that he or she is more patient, and when the interest rate r is higher.