Solutions to Problem Set 3

ECON 337901 - Financial Economics
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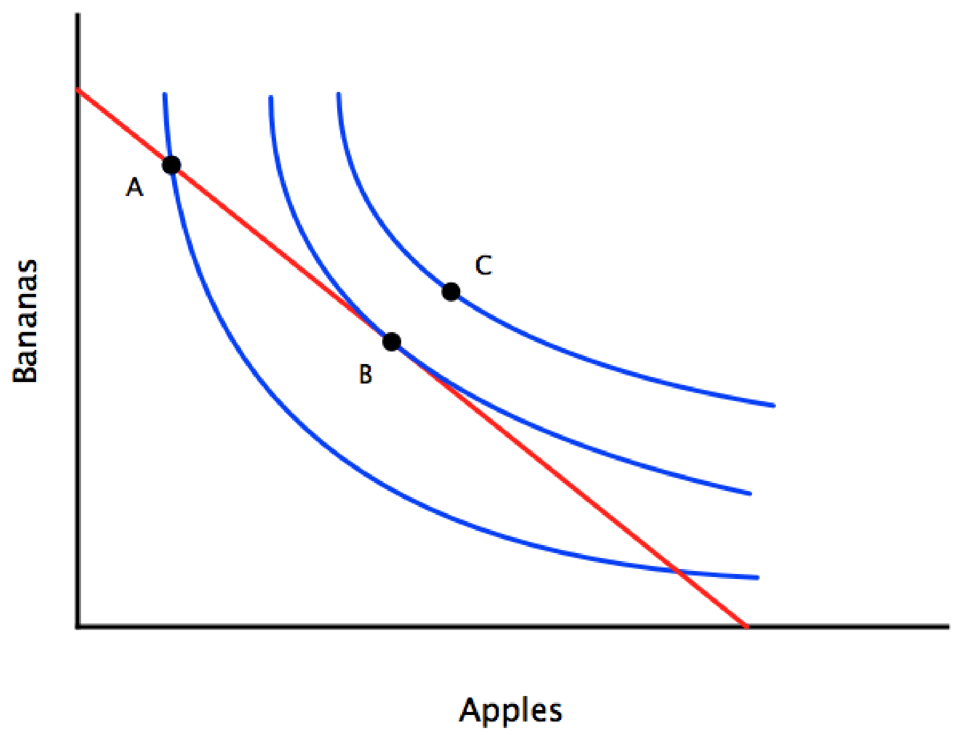
For Extra Practice - Not Collected or Graded

1. Consumer Optimization

In the graph, c_a^* and c_b^* from bundle B are on the consumer's budget constraint. This means that

$$c_b^* = \frac{Y}{p_b} - \left(\frac{p_a}{p_b}\right) c_a^*, \tag{1}$$

where Y is the consumer's income and p_a and p_b are the prices of apples and bananas.



Also in the graph, the budget constraint is tangent to the indifference curve. If the consumer's utility function is the same as in question 2 from the first problem set, namely

$$\alpha \ln(c_a) + (1-\alpha)\ln(c_b),$$

this tangency condition requires that

$$(1-\alpha)p_a c_a^* = \alpha p_b c_b^*. \tag{2}$$

Take the expression for c_b^* given by equation (1) and substitute it into the right-hand side of (2) to get

$$(1-\alpha)p_a c_a^* = \alpha p_b \left[\frac{Y}{p_b} - \left(\frac{p_a}{p_b}\right) c_a^*\right].$$

Next, use the distributive law of multiplication on the right-hand side and re-arrange the resulting terms to get

$$(1-\alpha)p_a c_a^* = \alpha Y - \alpha p_a c_a^*$$

or

$$p_a c_a^* = \alpha Y,$$

which shows that, with the logarithmic utility function, the consumer allocates the fraction α of his or her income Y to spending $p_a c_a^*$ on apples. Dividing by p_a now yields the solution

$$c_a^* = \frac{\alpha Y}{p_a}.$$

To finish the problem, substitute this solution for c_a^* back into equation (1) to get

$$c_b^* = \frac{Y}{p_b} - \left(\frac{p_a}{p_b}\right)\left(\frac{\alpha Y}{p_a}\right) = \frac{Y}{p_b} - \frac{\alpha Y}{p_b} = \frac{(1-\alpha)Y}{p_b}.$$

of course, these solutions for c_a^* and c_b^* are the same as those from question 2 on the first problem set.