Solutions to Problem Set 2

ECON 337901 - Financial Economics
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1. Utility Maximization

The Lagrangian for the consumer's problem

$$L = \alpha \ln(c_a) + (1-\alpha)\ln(c_b) + \lambda(Y - p_a c_a - p_b c_b)$$

leads to the first-order conditions

$$\frac{\alpha}{c_a^*} - \lambda^* p_a = 0$$

and

$$\frac{1-\alpha}{c_b^*} - \lambda^* p_b = 0$$

Together with the budget constraint

$$Y = p_a c_a^* + p_b c_b^*,$$

these first-order conditions form a system of three equations in the three unknowns: c_a^*, c_b^*, and λ^*.

There are many ways of solving this three-equation system, but perhaps the easiest is to rewrite the first-order conditions as

$$c_a^* = \frac{\alpha}{\lambda^* p_a}$$

and

$$c_b^* = \frac{1-\alpha}{\lambda^* p_b}$$

and substitute these expressions into the budget constraint to obtain

$$Y = \frac{\alpha}{\lambda^*} + \frac{1-\alpha}{\lambda^*} = \frac{1}{\lambda^*}.$$

This last equation leads directly to

$$\lambda^* = \frac{1}{Y},$$

which can then be substituted back into the previous expressions for c_a^* and c_b^* to obtain

$$c_a^* = \frac{\alpha Y}{p_a}$$

and

$$c_b^* = \frac{(1-\alpha)Y}{p_b}$$

These last two equations show how the consumer's purchases of apples rises when income rises and falls when the price of apples rises; similarly, the consumer's purchases of bananas rises when income rises and falls when the price of bananas rises. Interestingly, these expressions also show that the parameter α from the utility function in this example also measures the fraction $p_a c_a^*/Y$ of total income that consumer spends on apples; similarly, $1-\alpha$ measures the fraction $p_b c_b^*/Y$ of income that the consumer spends on bananas.