Solutions to Problem Set 13

ECON 337901 - Financial Economics — Peter Ireland
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For Extra Practice - Not Collected or Graded

1. Portfolio Allocation with Mean-Variance Utility, Part I

The first-order condition for choosing σ_P to maximize

$$r_f + \left(\frac{\mu_r - r_f}{\sigma_r}\right)\sigma_P - \left(\frac{A}{2}\right)\sigma_P^2$$

is

$$\frac{\mu_r - r_f}{\sigma_r} - A\sigma_P^* = 0,$$

implying that the optimal choice is

$$\sigma_P^* = \frac{\mu_r - r_f}{A\sigma_r}$$

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2. Portfolio Allocation with Mean-Variance Utility, Part II

Since the portfolio with standard deviation σ_P allocates the fraction

$$w = \frac{\sigma_P}{\sigma_r}$$

of wealth to the risky asset, the solution

$$\sigma_P^* = \frac{\mu_r - r_f}{A\sigma_r}$$

derived above must allocate the share

$$w^* = \frac{\mu_r - r_f}{A\sigma_r^2}$$

to the risky asset. Intuitively, this share decreases if the investor is more risk averse, that is, if A is larger, or if the risky asset has a return that is more volatile, so that σ_r is larger, and increases when the risk premium on the risky asset $\mu_r - r_f$ increases.

3. Portfolio Allocation with Mean-Variance Utility, Part III

The first-order conditions for choosing w_1 and w_2 to maximize

$$(1 - w_1 - w_2)r_f + w_1\mu_1 + w_2\mu_2 - \left(\frac{A}{2}\right)(w_1^2\sigma_1^2 + w_2^2\sigma_2^2).$$

are

$$-r_f + \mu_1 - A\sigma_1^2 w_1^* = 0$$

and

$$-r_f + \mu_2 - A\sigma_2^2 w_2^* = 0.$$

Rearranging each of these two first-order conditions leads to the solutions

$$w_1^* = \frac{\mu_1 - r_f}{A\sigma_1^2}$$

and

$$w_2^* = \frac{\mu_2 - r_f}{A\sigma_2^2},$$

which show that the portfolio share for each risky asset decreases if the investor is more risk averse or if the risky asset has a return that is more volatile, and increases when the risk premium on the risky asset increases.

4. CAPM Betas and Expected Returns

With $r_f = 0.02$ and $E(\tilde{r}_M) = 0.08$, the CAPM implies that the expected return on each asset i is given by

$$E(\tilde{r}_i) = 0.02 + \beta_j \times 0.06.$$

Hence, for the eight stocks:

High Beta Stocks			Low Beta Stocks		
Company	Beta	$E(r_j)$	Company	Beta	$E(r_j)$
Intel	1.10	0.086	Kellogg	0.60	0.056
Ford Motor	1.15	0.089	Proctor & Gamble	0.65	0.059
Caterpillar	1.35	0.101	Wal-Mart	0.75	0.065
US Steel	2.20	0.152	Pfizer	0.95	0.077

Notice that the stocks with high betas are issued by companies that industries that are highly exposed to the business cycle, in the sense that those business become much more profitable during booms and much less profitable during recessions. The stocks with low betas, by contrast, are issued by industries that still do reasonably well, even during recessions. The examples underscore that investors "pay" for higher expected returns by exposing themselves to more aggregate risk.