Solutions to Problem Set 11

ECON 337901 - Financial Economics — Peter Ireland
Boston College, Department of Economics — Spring 2024

For Extra Practice - Not Collected or Graded

1. The Gains From Diversification

With two assets, one with $\mu_1 = 8$ and $\sigma_1 = 8$ and the other with $\mu_2 = 4$ and $\sigma_2 = 4$, and with the share w allocated to asset 1, the expected return of the portfolio is

$$\mu_P = w\mu_1 + (1 - w)\mu_2$$

and the standard deviation of the portfolio is

$$\sigma_P = \sqrt{w^2\sigma_1^2 + (1 - w)^2\sigma_2^2 + 2w(1 - w)\sigma_1\sigma_2\rho_{12}},$$

where ρ_{12} is the correlation between the two returns. Probably, the easiest way to compute μ_P and σ_P for a range of values for w is using a spreadsheet or some other computer program, although the values can also be found with a handheld calculator. In any case, the results are tabulated below.

w	μ_P	σ_P with $\rho_{12} = 0$	σ_P with $\rho_{12} = -0.5$
0.0	4.0	4.00	4.00
0.2	4.8	3.58	2.77
0.4	5.6	4.00	2.88
0.6	6.4	5.06	4.23
0.8	7.2	6.45	6.04
1.0	8.0	8.00	8.00

The table confirms that in both cases there are gains to diversification, in the sense that there are portfolios of the two assets combined that have *both* higher expected returns and lower standard deviations than asset 2 alone. But the table also confirms that the gains from diversification are strongest when the correlation between the two asset returns is negative.